

## Bode Plots

Bode plots are widely used as graphical portrayals of the sinusoidal steady-state behavior of dynamical systems. Bode plots show the relationship, in the sinusoidal steady-state, between the input and output phasors.

Recall that, in s-domain analysis the input and output are related via the system transfer function. To be specific, consider the input and output to both be voltages (of course, depending on the system, they could be temperatures, currents, or forces).

$$V_o(s) = TF(s) V_i(s)$$

The sinusoidal steady-state response is given when  $s \rightarrow j\omega$ .

Notice when  $s$  is replaced by  $j\omega$ , that  $V_o$ ,  $TF$ , and  $V_i$  all become complex functions of  $\omega$ . That is, they become phasor quantities.

$$V_o(\omega) = TF(\omega) V_i(\omega)$$

Think of these phasor quantities in polar form.

$$V_o \angle q_o = TF \angle q_{TF} V_i \angle q_i$$

$$TF \angle q_{TF} = \frac{V_o \angle q_o}{V_i \angle q_i} = \frac{V_o}{V_i} \angle (q_o - q_i)$$

$$TF = \frac{V_o}{V_i} \quad (TF \text{ is a function of } \omega)$$

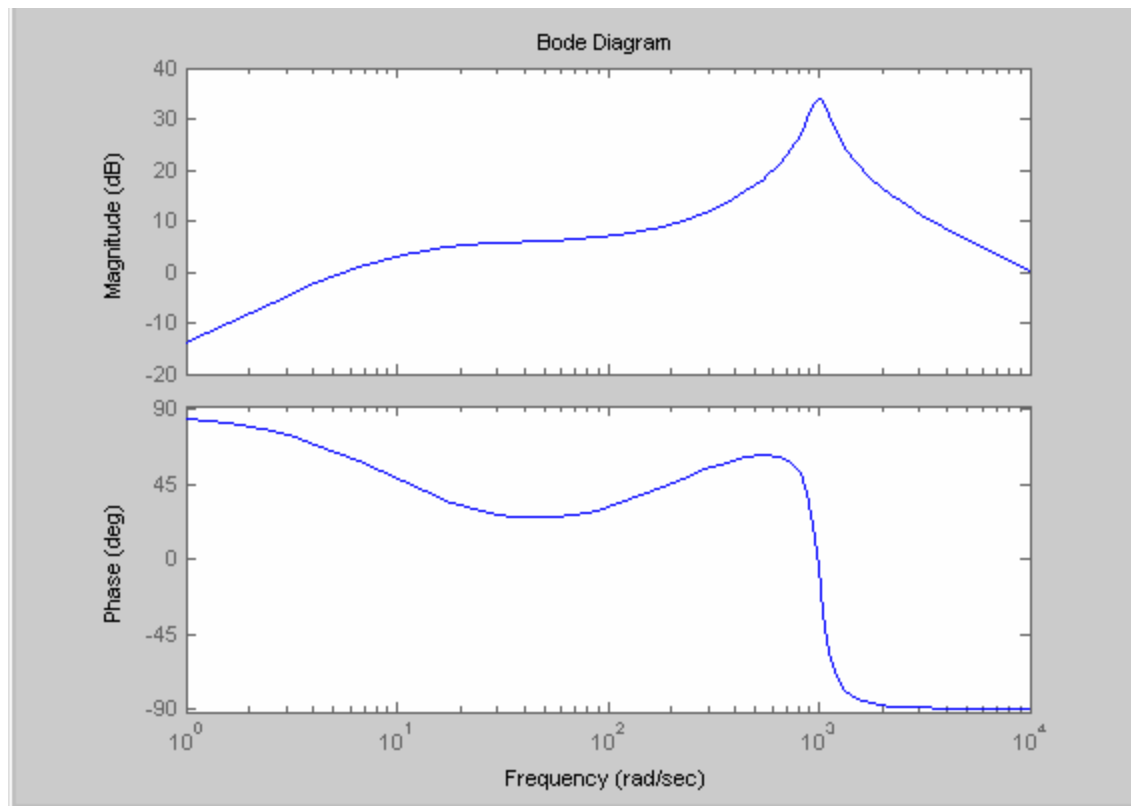
$$q_{TF} = q_o - q_i \quad (q_i \text{ is a function of } \omega)$$

There are two Bode plots. 1) The Bode magnitude plot gives the magnitude (in decibels) of  $TF$  as a function of  $\omega$ . 2) The Bode phase plot gives  $\theta_{TF}$  as a function of  $\omega$ .

What good are Bode plots? What are they used for?

**Example**

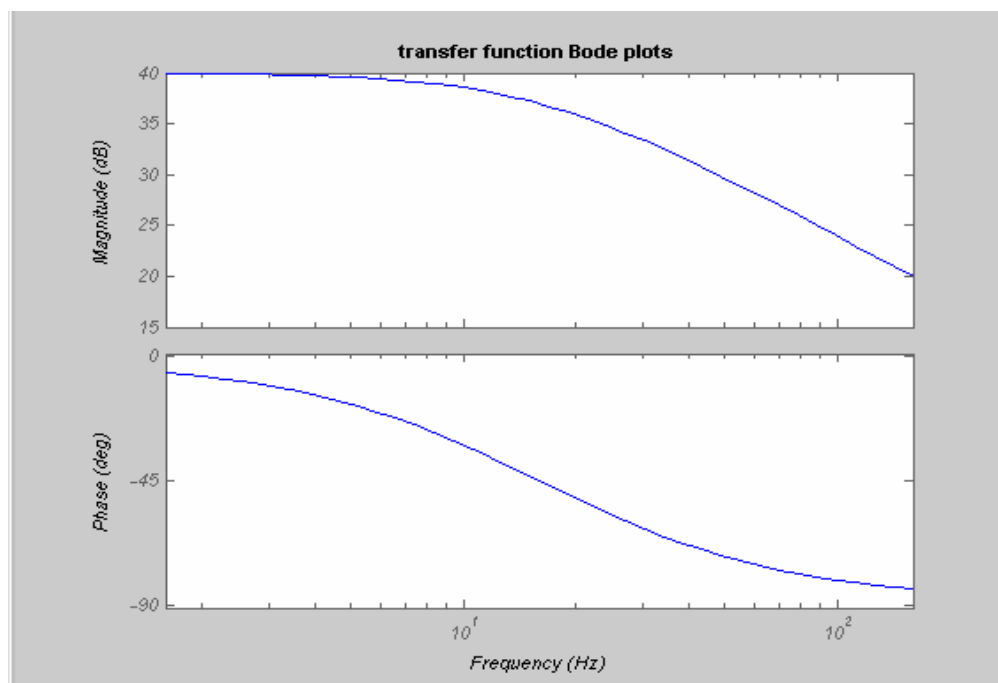
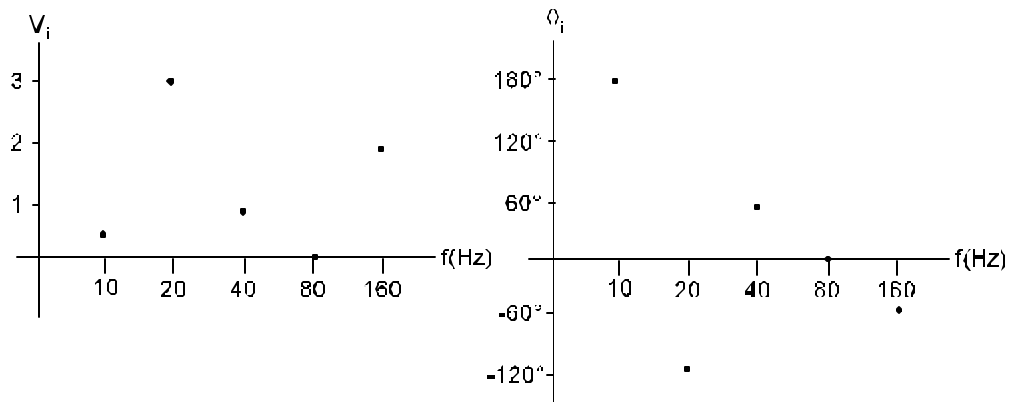
Suppose a system has the frequency response shown below.  
What is the output, if the input is  $V_i = 10 \cos(500t - 15^\circ)$  volts?



**Example**

Suppose another system has a complicated periodic input for which we've found the Fourier series.

What is the system's output, in the sinusoidal steady state?



Working with Bode plots helps engineers develop a feel for frequency response. Bode plots are widely used. They will be used this quarter when discussing control systems.

For these reasons, engineers need to be able to sketch the frequency response for systems without having to resort to MATLAB.

### Bode plots

Begin with the s-domain transfer function. The transfer function can be written as a ratio of polynomials in s.

$$TF = \frac{a_n s^z + b_n s^{z-1} + \dots}{a_d s^p + b_d s^{p-1} + \dots}$$

The coefficients are just parameters of the system and so are real numbers. For polynomials with real coefficients, their factors are either real or occur in complex conjugate pairs. For this case, the transfer function can be manipulated in the form below. This form is called Bode form.

Bode form

$$TF = K_b \frac{s^n \left( \frac{s}{w_{bz1}} + 1 \right) \dots \left( \frac{s^2}{w_{nz1}^2} + \frac{2z_{z1}}{w_{nz1}} s + 1 \right) \dots}{s^m \left( \frac{s}{w_{bp1}} + 1 \right) \dots \left( \frac{s^2}{w_{np1}^2} + \frac{2z_{p1}}{w_{np1}} s + 1 \right) \dots}$$

s is replaced by j $\omega$ .

$$TF = K_b \frac{(j\omega)^n \left( \frac{j\omega}{w_{bz1}} + 1 \right) \dots \left( \frac{(j\omega)^2}{w_{nz1}^2} + \frac{2z_{z1}}{w_{nz1}} j\omega + 1 \right) \dots}{(j\omega)^m \left( \frac{j\omega}{w_{bp1}} + 1 \right) \dots \left( \frac{s^2}{w_{np1}^2} + \frac{2z_{p1}}{w_{np1}} j\omega + 1 \right) \dots}$$

Notice that the input-output relation, **TF**, is now complex.

Bode magnitude plot

$$|TF|_{\text{db}} = 20 \log_{10} |TF|$$

$$|TF|_{\text{db}} = 20 \log_{10} |K_b| + 20(n-m) \log_{10} |j\omega| + 20 \log \left| \frac{j\omega}{\omega_{bz1}} + 1 \right| + \dots$$

$$+ 20 \log \left| \frac{(j\omega)^2}{\omega_{nz1}^2} + \frac{2z_{z1}}{\omega_{nz1}} j\omega + 1 \right| + \dots - 20 \log \left| \frac{j\omega}{\omega_{bp1}} + 1 \right| - \dots$$

$$- 20 \log \left| \frac{s^2}{\omega_{np1}^2} + \frac{2z_{p1}}{\omega_{np1}} j\omega + 1 \right| - \dots$$

Bode phase plot

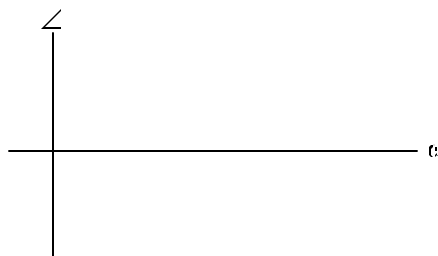
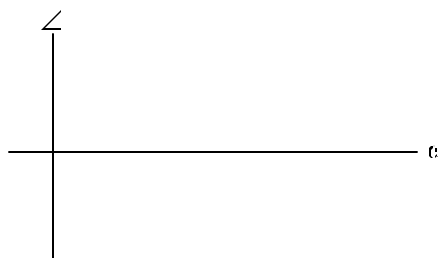
$$q = \angle K_b + (n-m) \angle(j\omega) + \angle \left( \frac{j\omega}{\omega_{bz1}} + 1 \right) + \dots$$

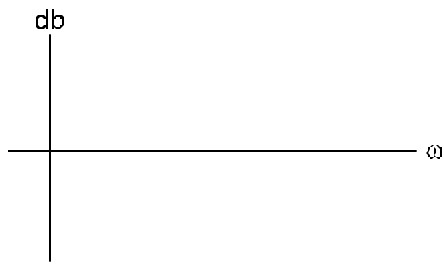
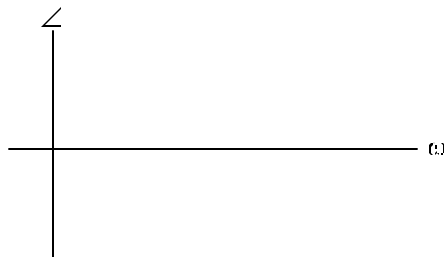
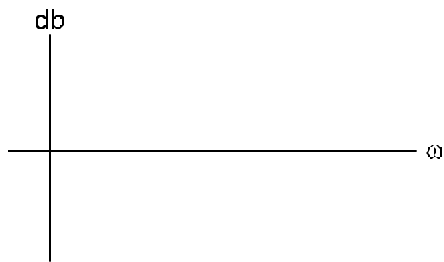
$$+ \angle \left( \frac{(j\omega)^2}{\omega_{nz1}^2} + \frac{2z_{z1}}{\omega_{nz1}} j\omega + 1 \right) + \dots - \angle \left( \frac{j\omega}{\omega_{bp1}} + 1 \right) - \dots$$

$$- \angle \left( \frac{s^2}{\omega_{np1}^2} + \frac{2z_{p1}}{\omega_{np1}} j\omega + 1 \right) - \dots$$

Both the magnitude and phase plots are sums of individual terms.  
Let's consider each of these terms in turn.

$K_b$

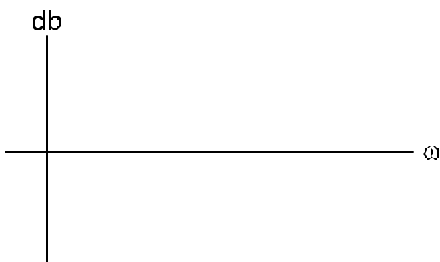


$1/s$  $\frac{s}{w_b} + 1$  $\frac{1}{\frac{s}{w_b} + 1}$ 

$$\frac{s^2}{\omega_n^2} + \frac{2z}{\omega_n}s + 1$$

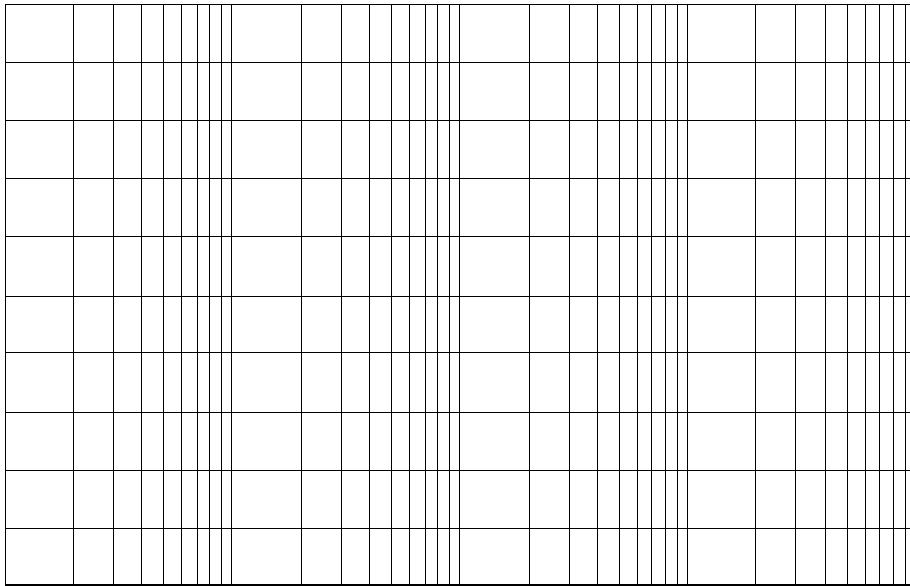


$$\frac{1}{\frac{s^2}{\omega_n^2} + \frac{2z}{\omega_n}s + 1}$$



**Example**

$$TF(s) = \frac{200s}{s + 10}$$



**Example**

$$TF(s) = \frac{10^5}{s^2 + 5100s + 500000}$$

