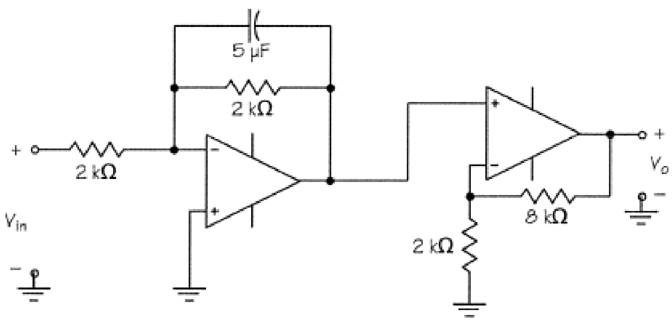
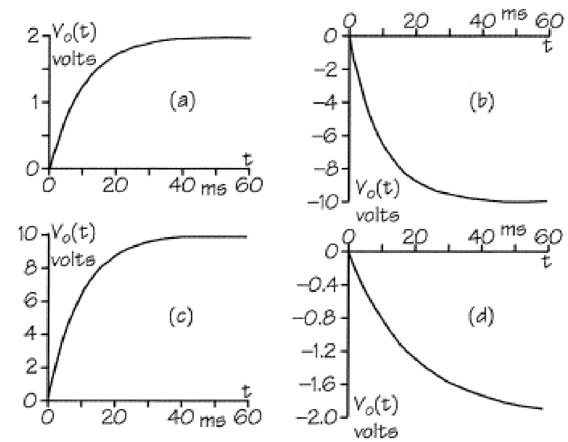
ECE207 ELEMENTS OF ELECTRICAL ENGINEERING
Homework Set 8 – Solutions

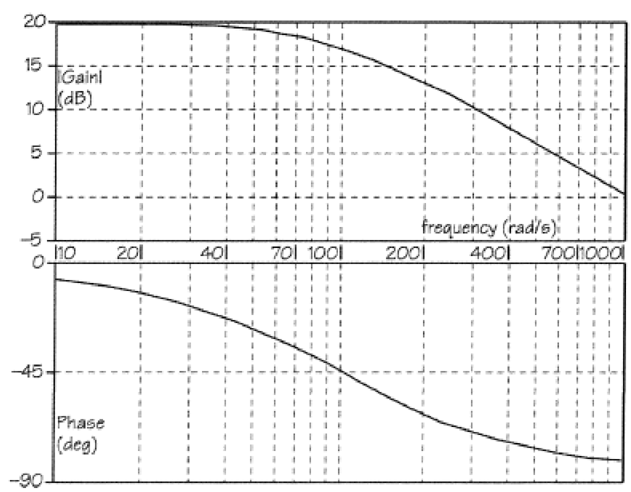
8-1 Which of the four responses shown best approximates the output voltage $V_o(t)$ for $0 < t < 60$ ms when V_{in} is a step with an amplitude of 2 volts. Fully justify your answer.





The op-amp on the left is a LPF with a dc gain of unity (inverted). The op-amp on the right is a non- inverting amplifier with a gain of 5. Therefore, the steady-state output will be 10 V and will be inverted, so (b) is the response.

8-2 The frequency response of a system is shown below. Which of the time functions shown is the best fit for the system's unit step response? Fully justify your answer.



a) $10(1 - e^{-100t})$

b) $1 - e^{-100t}$

c) $20(1 - e^{-100t})$

d) $20e^{-100t}$

e) none of the above.

The system is 1st order because it has a slope of -20 dB/dec. The 3 dB point occurs at 100 rad/s and the LF gain is 20 dB or $|H_{dc}| = 10$. This means the transfer function is:

$$H(s) = \frac{10}{1 + 0.01s}$$, which gives $\tau = 0.01$.

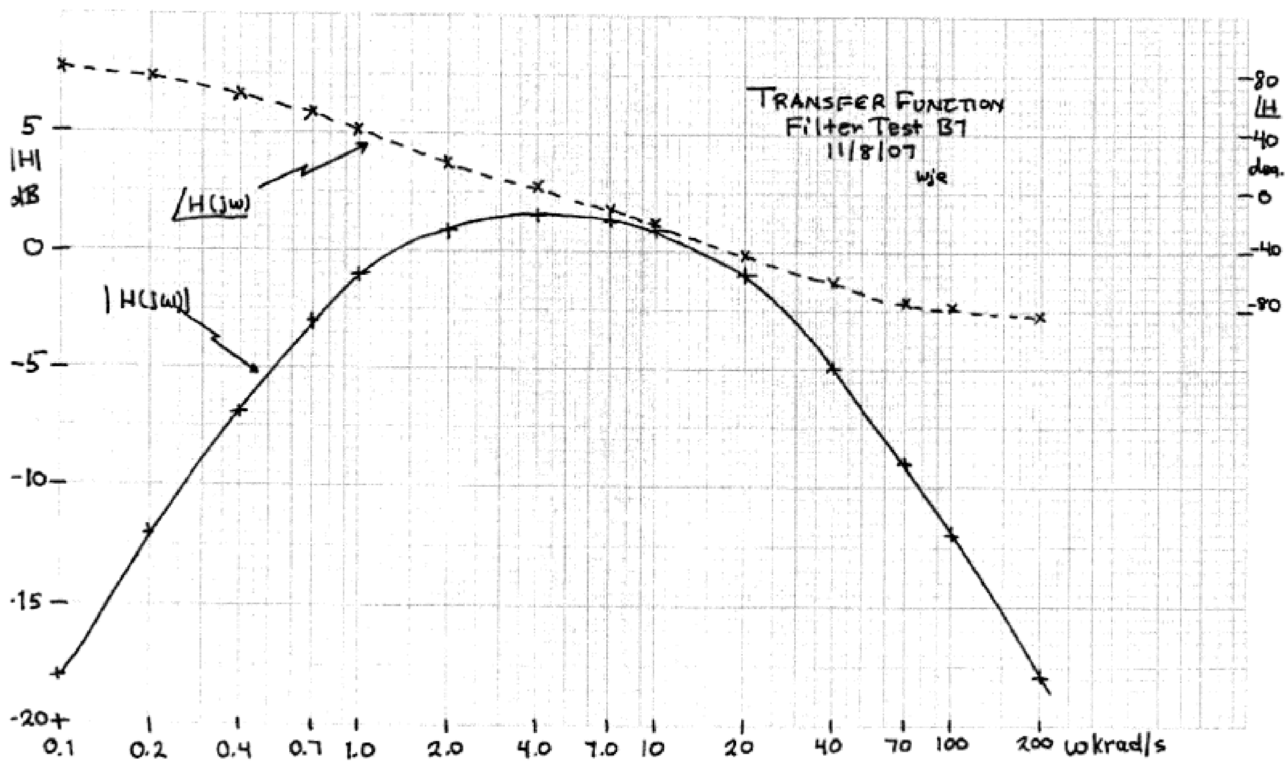
This gives a unit step response of: $10(1 - e^{-100t})$, which is **item (a).**

8-3 Data taken for a certain circuit are shown at the bottom of the page as a Bode plot of the magnitude and phase of the voltage transfer function $H(s) = V_o(s) / V_i(s)$.

a) If the input to this circuit is $v_i(t) = 6.0 \cos(20{,}000t + 50°)$ V, find the output $v_o(t)$ in the sinusoidal steady state.

b) If the observed output voltage $v_o(t)$ has an RMS value of 100 mV at a frequency of 60 Hz, what is the RMS value of the input voltage? [Clue: Note Hz!] [Clue²: 225 mV.]

c) Determine the transfer function H(s) represented by this Bode plot. Present your result for H(s) in the standard form where each term of the denominator is in the form (s/a + 1).



a)

The gain at 20 krad/s is -1 dB or $|H(20k)| = 10^{-1/20} = 0.8913$, with a phase shift of -40⁰.

$\boldsymbol{V_i}$ = 6/50 and ***H(20k)*** = 0.8913/-40 so $\boldsymbol{V_o}$ = 6 x 0.8913/50 - 40 = 5.348/10

$$v_0(t) = 5.38\cos(20000t + 10^0)$$

b) The gain at 377 rad/s is -7.0 dB or $|H(377)| = 10^{-7.0/20} = 0.4467$

$$|V_i| = \frac{100}{0.4467} = \boxed{224\text{ mV}}$$

c) The LF slope of +20 dB/dec means we have an “s” in the numerator. The two break points are both poles, so the form of H(s) is:

$$H(s) = \frac{K_b s}{(1 + s/p_1)(1 + s/p_2)}$$

Where $p_1 = 10^3$ rad/s, $p_2 = 20 \times 10^3$ rad/s and K_b is the gain at ω = 1 rad/s.

Since the gain at ω = 100 rad/s is -17.5 dB and the slope is 20 dB/dec, |H(1)| = -57.5 dB and $K_b = 10^{-57.5/20} = 1.334\text{x}10^{-3}$, which leads to:

$$H(s) = \frac{1.334\text{x}10^{-3} s}{(1 + 10^{-3} s)(1 + 50\text{x}10^{-6} s)}$$