Homework Set #1

Textbook: Chapter 6.1-6.2

Coverage: Capacitance and self inductance

DUE Thursday, March 8, 2012

For problems 3 and 4, you must do the calculus by hand, but you can use a calculator for the arithmetic.

1. Problem 6.1 on page 204 *(answers in back of book)*

$$0 \le t \le 2\,\mathrm{s}:$$

$$i_L = \frac{10^3}{2.5}\int_0^t 3\times 10^{-3} e^{-4x}\,dx + 1 = 1.2\frac{e^{-4x}}{-4}\Big|_0^t + 1$$

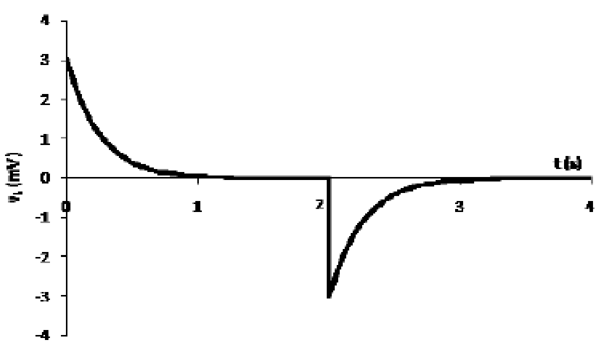
$$= -0.3e^{-4t} + 1.3\,\mathrm{A}, \qquad 0 \le t \le 2\,\mathrm{s}$$

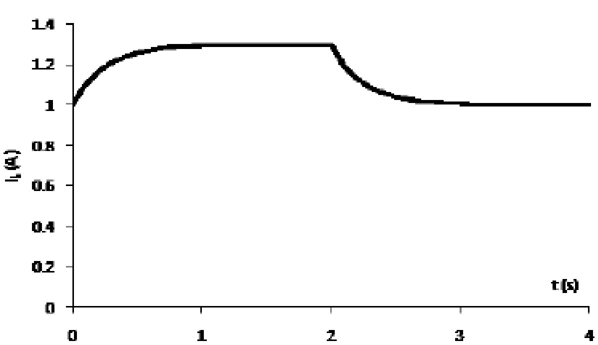
$$i_L(2) = -0.3e^{-8} + 1.3 = 1.3\,\mathrm{A}$$

$$t \ge 2\,\mathrm{s}:$$

$$i_L = \frac{10^3}{2.5}\int_2^t -3\times 10^{-3} e^{-4(x-2)}\,dx + 1.3 = -1.2\frac{e^{-4(x-2)}}{-4}\Big|_2^t + 1.3$$

$$= 0.3e^{-4(t-2)} + 1\,\mathrm{A}, \qquad t \ge 2\,\mathrm{s}$$



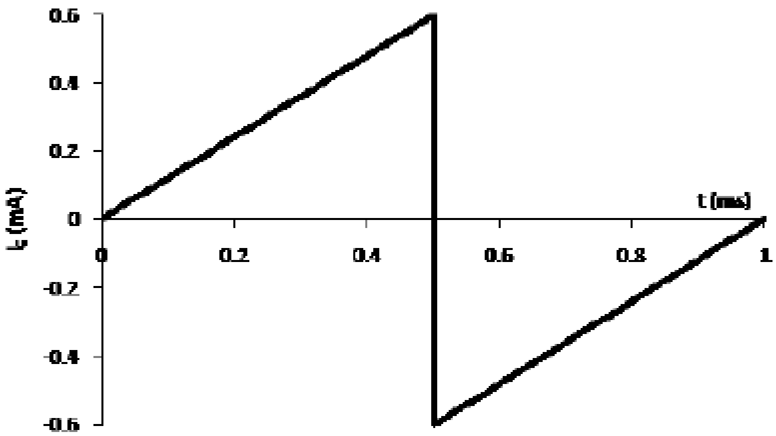


2. Problem 6.17 on page 206 *(answers in back of book)*

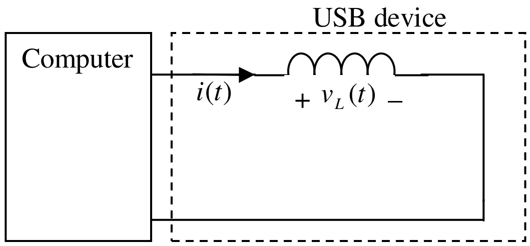
$$i_C = C(dv/dt)$$

$$0 < t < 0.5: \qquad i_C = 20 \times 10^{-6}(60)t = 1.2t\,\mathrm{mA}$$

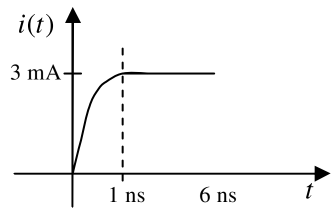
$$0.5 < t < 1: \qquad i_C = 20 \times 10^{-6}(60)(t-1) = 1.2(t-1)\,\mathrm{mA}$$



3. A computer communicates over wires (such as a USB cable) by sending very short pulses. The device at the far end of the cable (such as a memory stick) senses the pulses with electronics which have inductance and capacitance. They can be modeled with the appropriate valued device as shown below.



L=2 nH



$$i(t) = \begin{cases} 0 & t \le 0ns \\ 3(1-e^{-4x10^9 t})mA & 0ns \le t < 1ns \\ 3mA & 1ns \le t < 6ns \end{cases}$$

(a) Find the voltage $v_L(t)$ for 0 ns < t ≤ 1 ns and plot your results.
(b) Find the power absorbed by the inductor at t = 0.5 ns.
(c) Find the energy stored in the inductor at t = 0.5 ns.

Solution:

For an inductor labeled with passive sign convention, $v = L\frac{di}{dt}$. Since we know $i(t)$, we just need to take a derivative.

$$v_L(t) = (2 \times 10^{-9}) \frac{d}{dt}\left[0.003\left(1 - e^{-4E+9\,t}\right)\right]$$

work in Amps

we need $\frac{d}{dt}\left[0.003 - 0.003\, e^{-4E+9t}\right]$

deriv. of const = 0

$\frac{d}{dt} e^u = e^u \frac{du}{dt}$

here, $\frac{du}{dt}$ is $-4E+9$

So the result is

$$\underbrace{-0.003\, e^{-4E+9\,t}}_{e^u}\ \underbrace{(-4E+9)}_{du/dt}$$

So, $\frac{d}{dt}\left[0.003 - 0.003\, e^{-4E+9t}\right]$

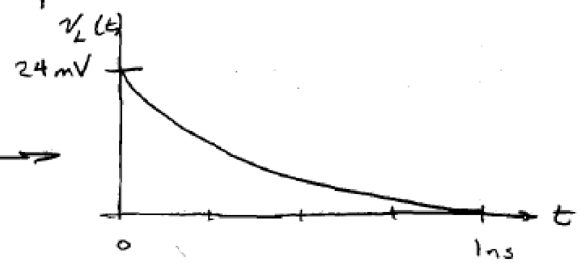
$$= (-0.003)(-4E+9)e^{-4E+9t} = (+12E6)\, e^{-4E+9t}$$

and

$$v_L(t) = (2E-9)(12E+6)e^{-4E+9t}\ \text{V}$$

$$\boxed{v_L(t) = (0.024)e^{-4E+9t}\ \text{V}} \quad 0 < t < 1\text{ns}$$

Since an exponential starts at 0 with a value of 1 and decays to zero in about 4 time constants, the plot should be



required →

($e^{-t/\tau}$ where τ is the time constant gives $\tau = \frac{1}{4E+9} = \frac{1}{4}$ ns)

b) $p(t) = v(t)\, i(t)$ (+ is absorbed for passive labeling)

$$v(0.5\text{ns}) = 0.024\left(e^{-4E+9(0.5E-9)}\right)$$

$$= 0.024\, e^{-2} = 0.003248\,\text{V} = 3.248\,\text{mV}$$

$$i(t) = 0.003\left(1 - e^{-4E+9(0.5E-9)}\right)$$

$$= 0.003\left(1 - e^{-2}\right) = 0.003\,(0.8647)$$

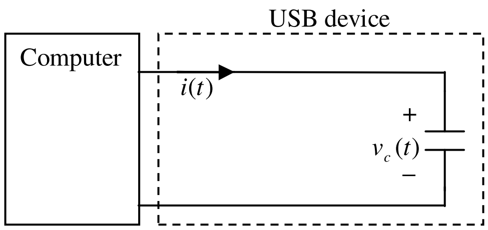
$$= 0.002594\,\text{A} = 2.594\,\text{mA}$$

$$\boxed{p(0.5\text{ns}) = 8.425\,\mu\text{W}}$$

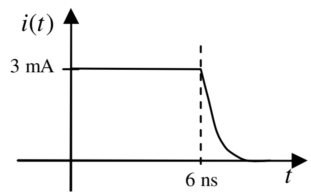
c) $W = \frac{1}{2} L i^2 = \frac{1}{2}(2E-9)(2.594E-3)^2$

$$\boxed{W = 6.729E-15\ \text{J}}$$ femto Joules!

4. A computer communicates over wires (such as a USB cable) by sending very short pulses. The device at the far end of the cable (such as a memory stick) senses the pulses with electronics which have inductance and capacitance. They can be modeled with the appropriate valued device as shown below.



C=5 pF
$v_c(0)$=0 V



$$i(t) = \begin{cases} 3mA & 0ns \le t < 6ns \\ 3e^{-4x10^9(t-6x10^{-9})}mA & 6ns \le t < 7ns \\ 0mA & t \ge 7ns \end{cases}$$

(a) Find the voltage $v_C(t)$ for 0 ns < t < 7 ns and plot your results.
(b) Find the power absorbed by the capacitor at t = 6.5 ns.
(c) Find the energy stored in the capacitor at t=2ns.

Solution: $v_c(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + v_c(0)$

for $0 \le i < 6$ ns, $i(t) = 3$mA, so

$$\int_0^t (3E-3)d\tau = (3E-3)t$$

and $v_c(t) = \frac{1}{5E-12}(3E-3)t + 0V$

$$v_c(t) = 600E6\, t \ \ V$$

at $t = 6$ ns, $v_c(t) = 600E6(6E-9) = 3.6V$

from $6ns \le t < 7ns$, $i(t) = 3e^{-4E9(t-6E-9)}$ √

$$v_c(t) = \frac{1}{C}\int_{6ns}^{t} i(\tau)d\tau + v_c(6ns)$$

here $\int e^u du = e^u$ we need $du = -4E9$

so $\int \frac{3E-3}{(-4E9)} e^{\underbrace{-4E9(t-6E-9)}_{u}} \underbrace{(-4E9)dt}_{du}$

$= -750E-15\, e^{-4E9(t-6E-9)}$

So, $$v_c(t) = \frac{1}{5E-12}\left(\left((-750E-15)e^{-4E9(t-6E-9)} - \left[(-750E-15)e^{-4E9(6E-9 - 6-E9)}\right]\right) + v_c(6\,ns)\right)$$

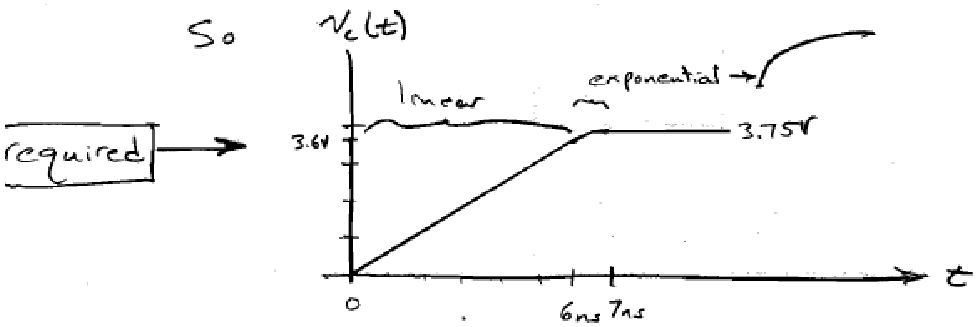
$$v_c(t) = \frac{1}{5E-12}\left(750E-15\left(1-e^{-4E9(t-6E-9)}\right)\right) + 3.6\ V$$

$$v_c(t) = 0.15\left(1 - e^{-4E9(t-6E-9)}\right) + 3.6V$$

for $6ns \leq t < 7ns$

So, the voltage increase linearly from 0 to 6ns, and the increases with an exponential decay from 6ns to 7ns. Again, the time constant is $\frac{1}{4}$ ns.

So



b) the power absorbed by the capacitor at $t_1 = 6.5ns$ is given by $p_{abs}(t_1) = v(t_1)\, i(t_1)$.

$$v(t_1) = 0.15\left(1 - e^{-4E9(0.5E-9)}\right) + 3.6V = 3.723\ V$$

$$i(t_1) = (3E-3)\left(e^{-4E+9(0.5E-9)}\right) = 406\ \mu A$$

$$\boxed{p(t_1) = 1.514\ mW}$$

c) $$w = \frac{1}{2}Cv^2 = \frac{1}{2}(5E-12)\left(600E6\,(2E-9)\right)^2$$

$$= 3.6E-12 = \boxed{3.6\ pJ}$$