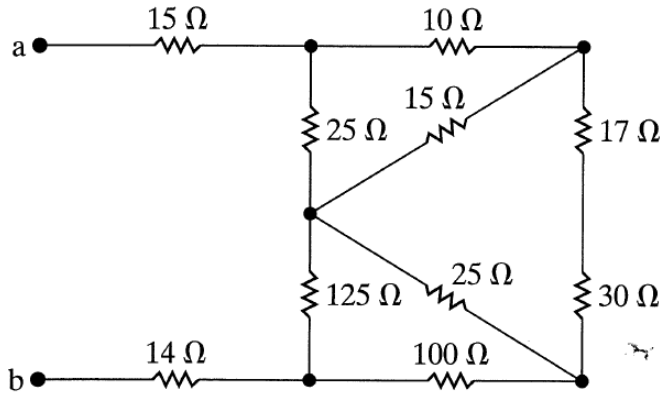
Homework Set #9
DUE Tuesday, March 28, 2017

1. Determine the resistance R_{ab} in the following circuit:

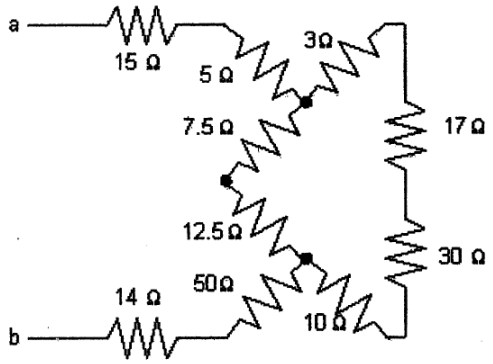


Replace the upper and lower deltas with the equivalent wyes:

$$R_{1U} = \frac{(25)(10)}{50} = 5\,\Omega; R_{2U} = \frac{(10)(15)}{50} = 3\,\Omega; R_{3U} = \frac{(25)(15)}{50} = 7.5\,\Omega$$

$$R_{1L} = \frac{(125)(25)}{250} = 12.5\,\Omega; R_{2L} = \frac{(25)(100)}{250} = 10\,\Omega; R_{3L} = \frac{(125)(100)}{250} = 50\,\Omega$$

The resulting circuit is shown below:

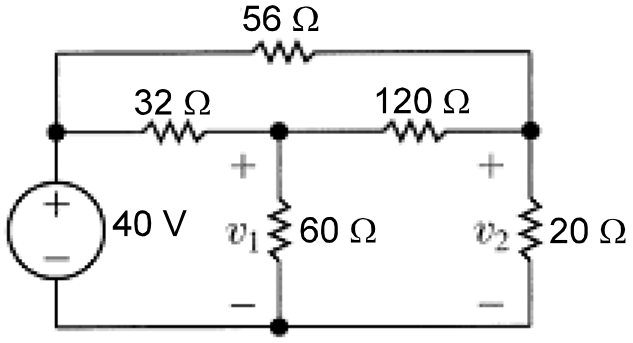


Now make series and parallel combinations of the resistors:

$$(7.5 + 12.5) \| (3 + 17 + 30 + 10) = 20 \| 60 = 15\,\Omega$$

$$R_{ab} = 15 + 5 + 15 + 50 + 14 = 99\,\Omega$$

2. a) Use Y – Δ Transformations (or Δ – Y) to obtain v_1 & v_2 in the following circuit.
   b) Check your result using Nodal Analysis.

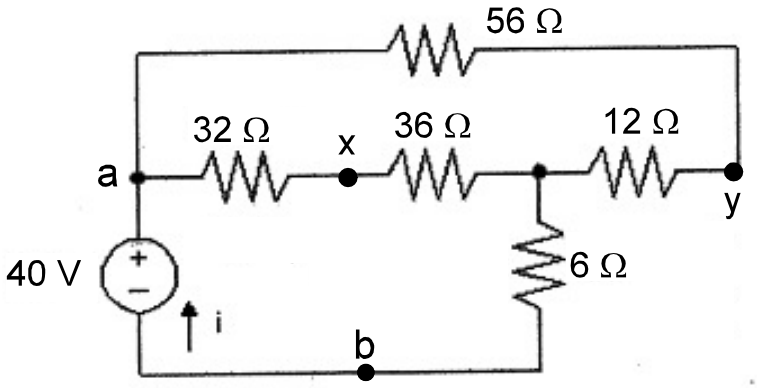


(HINT: Begin by transforming the Δ-connected 60Ω – 120Ω – 20Ω resistors into Y and then reduce the circuit to a single resistor and get the source current.)

Take the delta made-up of R_c = 120 Ω, R_b = 60 Ω, and R_a = 20 Ω and convert to wye as follows:

$R_1 = \frac{(60)(120)}{20 + 60 + 120} = 36\ \Omega$, $R_2 = \frac{(120)(20)}{20 + 60 + 120} = 12\ \Omega$, $R_3 = \frac{(60)(20)}{20 + 60 + 120} = 6\ \Omega$

This conversion leads to the following circuit:



The 56 Ω and 12 Ω are in series to give 68 Ω, while the 32 Ω and 36 Ω are also in series to give 68 Ω. Both 68 Ω resistors are in parallel to give 34 Ω, which is in series with the 6 Ω, so R_{ab} = 40 Ω.

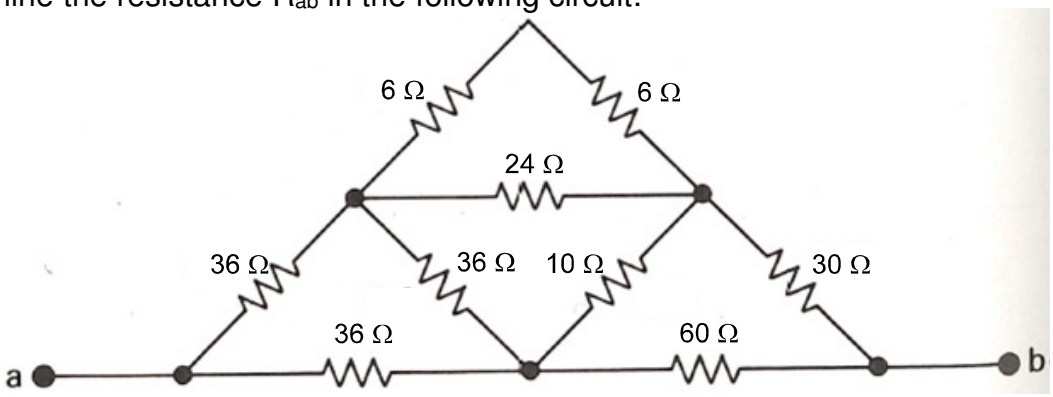
The current drawn from the source is: $i = \frac{40}{40} = 1\ A$

Use current division to obtain the currents flowing in the 56 Ω and the 32 Ω resistors:

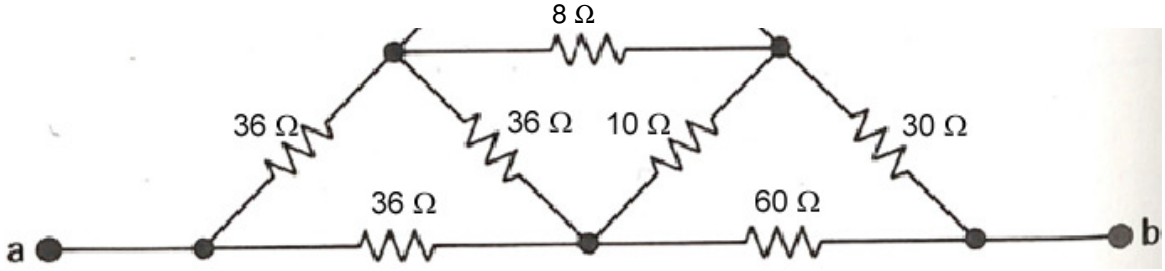
i_{56} = 0.5 A, i_{32} = 0.5 A (This is coincidence)

$v_1 = v_{xb} = 40 - 32 \times 0.5 = 24$ V $\quad v_2 = v_{yb} = 40 - 56 \times 0.5 = 12$ V

3. Determine the resistance R_{ab} in the following circuit:

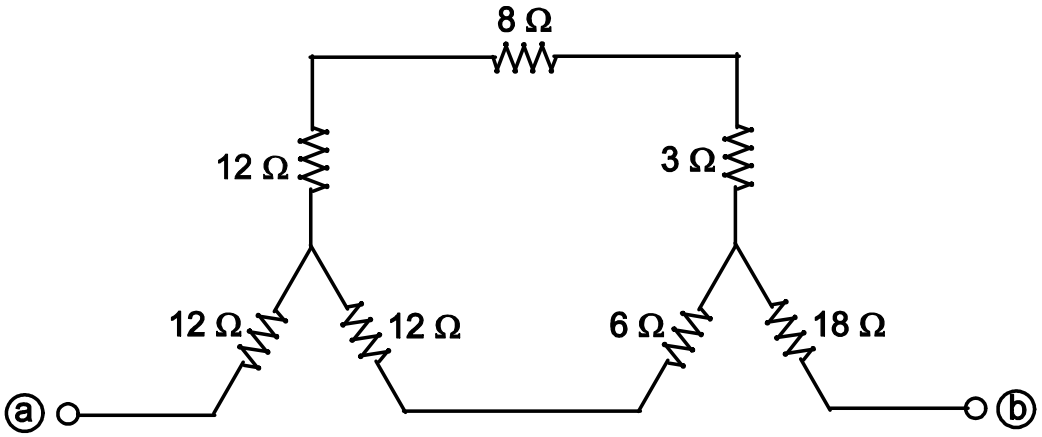


The top two 6 Ω resistors are in series and the resulting 12 Ω is in parallel with the 24 Ω below. This means the top three resistors can be replaced by a single 8 Ω resistor to give:

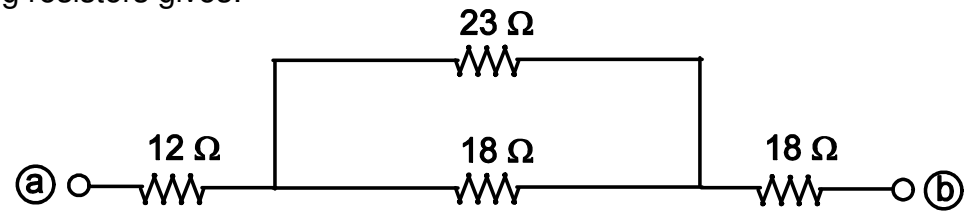


The lower triangles are deltas (upside-down) they can be replaced by upside-down wyes.

$$R_1 = \frac{(30)(60)}{10+30+60} = 18\,\Omega; \quad R_2 = \frac{(60)(10)}{10+30+60} = 6\,\Omega; \quad R_3 = \frac{(30)(10)}{10+30+60} = 3\,\Omega$$



Combining resistors gives:



The 23 Ω and 18 Ω are in parallel to give 10.1 Ω, which is in series with the 12 Ω and the 18Ω resulting in:

R_{ab} = 40.1 Ω