

**Homework Set #21**  
**DUE Friday, May 5, 2017**

1. Design an inverting summing amplifier so that

$$v_o = -(3v_a + 5v_b + 4v_c + 2v_d).$$

If the feedback resistor ( $R_f$ ) is chosen to be  $60\text{ k}\Omega$ , draw a circuit diagram of the amplifier and specify the values of  $R_a$ ,  $R_b$ ,  $R_c$ , and  $R_d$ .

We want the following expression for the output voltage:

$$v_o = -(3v_a + 5v_b + 4v_c + 2v_d)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage:

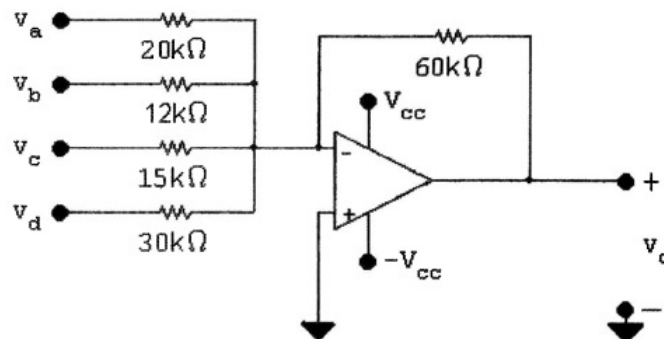
$$v_o = - \left[ \frac{60\text{k}}{R_a} v_a + \frac{60\text{k}}{R_b} v_b + \frac{60\text{k}}{R_c} v_c + \frac{60\text{k}}{R_d} v_d \right]$$

Solve for each input resistance value to yield the desired gain:

$$\therefore R_a = 60,000/3 = 20\text{ k}\Omega \quad R_c = 60,000/4 = 15\text{ k}\Omega$$

$$R_b = 60,000/5 = 12\text{ k}\Omega \quad R_d = 60,000/2 = 30\text{ k}\Omega$$

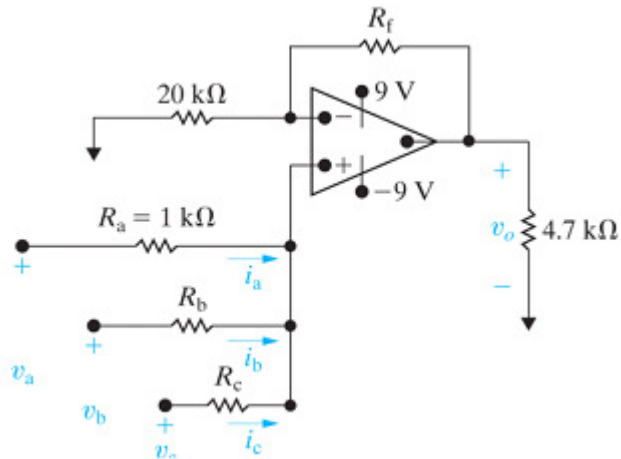
The final circuit is shown here:



2. The following circuit has to have an output voltage given by:

$$V_0 = 4v_a + v_b + 2v_c$$

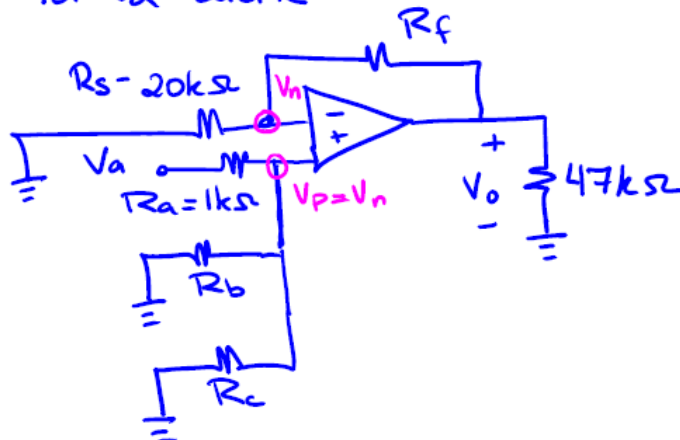
- Specify the numerical values of  $R_b$ ,  $R_c$ , and  $R_f$ .
- Using the values found in part (a) for  $R_f$ ,  $R_b$ , and  $R_c$ , calculate (in microamperes)  $i_a$ ,  $i_b$ , and  $i_c$  when  $v_a = 0.75$  V,  $v_b = 1.0$  V, and  $v_c = 1.5$  V.



(HINT: Use superposition to get an expression for  $V_0$  in terms of the unknown resistors. This gives you the ratios and  $R_a$  is known.)

analysis: using superposition

for  $v_a$  alone



$$\text{KCL } \left. \begin{matrix} v_n \\ v_n \end{matrix} \right) \quad \frac{v_n}{R_s} + \frac{v_n - v_o}{R_f} = 0$$

$$\Rightarrow v_n = \frac{v_o}{R_f} \left( \frac{1}{R_s} + \frac{1}{R_f} \right)^{-1}$$

$$= \frac{v_o}{R_f} \frac{R_s R_f}{R_s + R_f}$$

$$\text{KCL } \left. \begin{matrix} v_p \\ v_p \end{matrix} \right) \quad \frac{v_p - v_a}{R_a} + \frac{v_p}{R_b} + \frac{v_p}{R_c} = 0$$

$$\Rightarrow v_p = \frac{v_a}{R_a} \left( \frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c} \right)^{-1}$$

let  $R_{eq} = R_a \parallel R_b \parallel R_c$

set  $V_n = V_o$

$$V_o \left( \frac{R_s}{R_s + R_f} \right) = \frac{V_a R_{eq}}{R_a}$$

$$V_o = \left( 1 + \frac{R_f}{R_s} \right) \frac{R_{eq}}{R_a} V_a$$

using superposition

$$V_o = \left( 1 + \frac{R_f}{R_s} \right) R_{eq} \left( \frac{V_a}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c} \right)$$

Design eqns

$$\textcircled{1} \quad 4 = \left( 1 + \frac{R_f}{R_s} \right) \frac{R_{eq}}{R_a}$$

$$\textcircled{2} \quad 1 = \left( 1 + \frac{R_f}{R_s} \right) \frac{R_{eq}}{R_b}$$

$$\textcircled{3} \quad 2 = \left( 1 + \frac{R_f}{R_s} \right) \frac{R_{eq}}{R_c}$$

$$\textcircled{1} - \textcircled{3} \quad 4R_a = R_b = 2R_c$$

$$R_a = 1k\Omega$$

$$R_b = 4k\Omega$$

$$R_c = 2k\Omega$$

$$R_{eq} = R_a // R_b // R_c = 571.4\Omega$$

$$R_s = 20k\Omega$$