MA/CSSE 473 Day 23

Transform and Conquer

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- quickhull implementation due now
- HW 10 due Wednesday at noon
  - Bring hard copy to my office, or leave it in my mailbox in F-233
- HW 11 will be due Friday, Oct 24
- Exam 2, Friday, Oct 31
- **Student Questions**
- Transform and conquer
  - Most examples should be review.

Transform and Conquer Algorithms

- Transform a problem to a simpler instance of the same problem – **instance simplification**
- Transformation to a different representation of the same instance – **representation change**
- Transformation to an instance of a different problem that we know how to solve – **problem reduction**

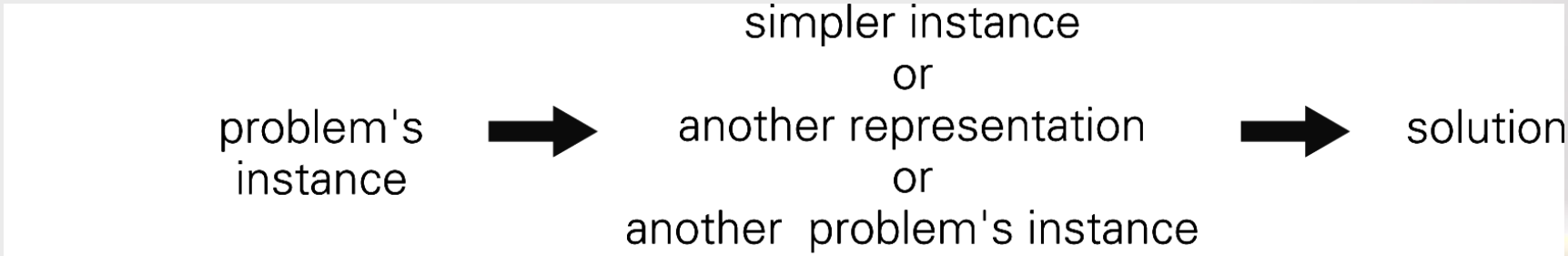

FIGURE 6.1 Transform-and-conquer strategy

Presorting an Array

- The following problems are simplified by pre-sorting the array:
  - Search (can do Binary or Interpolation search)
  - Determine whether the array contains duplicates
  - Find the **mode** of the elements of the array
    - The most frequently-occurring element
  - A related problem: Anagrams
    - In a large collection of words, find words that are anagrams of each other
    - How can pre-sorting help?
    - Sort the letters of each word
  - Interval union problem from early part of PLC

Gaussian Elimination (review of MA221)

- Solve a system of n linear equations in n unknowns
  - Represent the system by an augmented coefficient matrix
  - Transform the matrix to triangular matrix by a combination of the following solution-preserving elementary operations:
    - exchange two rows
    - multiply a row by a nonzero constant
    - replace a row by that row plus or minus a constant multiple of a different row
  - Look at the algorithm and analysis on pp 207-208; if you can't understand them, ask at some point
  - $\Theta(n^3)$

Other Applications of G.E.

- Matrix inverse
  - Augment a square matrix by the identity matrix
  - Perform elementary operations until the original matrix is the identity.
  - The "augmented part" will be the inverse
  - More details and an example at http://en.wikipedia.org/wiki/Gauss-Jordan_elimination

Other Applications of G.E.

- Determinant calculation
  - Calculation of the determinant of a triangular matrix is easy
- What effect does each of the elementary operations have on the determinant?
  - exchange two rows
  - multiply a row by a nonzero constant
  - replace a row by that row plus or minus a constant multiple of a different row
- Do these operations until you get a triangular matrix
- Keep track of the operations' cumulative effect on the determinant

LU Decomposition

- This can speed up all three applications of Gaussian Elimination
- Write the matrix A as a product of a Lower Triangular matrix L and an upper Triangular matrix U.
- Example:

$$[A]=\begin{bmatrix}25 & 5 & 1\\ 64 & 8 & 1\\ 144 & 12 & 1\end{bmatrix}$$

$$[L]=\begin{bmatrix}1 & 0 & 0\\ 2.56 & 1 & 0\\ 5.76 & 3.5 & 1\end{bmatrix}\quad [U]=\begin{bmatrix}25 & 5 & 1\\ 0 & -4.8 & -1.56\\ 0 & 0 & 0.7\end{bmatrix}$$

Balanced Search Trees

- Why do we introduce the idea of Binary Search Trees?
  - i.e., why are array lists or linked lists not good enough?
- What is the problem if we don't make sure Binary Search Trees are kept balanced after insert or delete?
- AVL Trees (height balanced)
- Review of why we know that the max height of an AVL tree is $\Theta(\log N)$.
- Review the balancing algorithms after insertion, deletion.