

MA/CSSE 473

Day 22

Sequence number →
permutation
Fake coin
Josephus
Median Finding



MA/CSSE 473 Day 22

- HW 9 duenow.;
- quickhull implementation due Monday
- HW 10 due Wednesday . Available soon
- Due to break, there will be no HW due on Tuesday, Oct 21.
- HW 11 will be due Friday, Oct 24
- Exam 2, Friday, Oct 31
- **Student Questions**
- Permutations by lexicographic order number
- Fake coin
- Josephus
- Median Finding



Permutations and order

number	permutation	number	permutation
0	0123	12	2013
1	0132	13	2031
2	0213	14	2103
3	0231	15	2130
4	0312	16	2301
5	0321	17	2310
6	1023	18	3012
7	1032	19	3021
8	1203	20	3102
9	1230	21	3120
10	1302	22	3201
11	1320	23	3210

- Given a permutation of $0, 1, \dots, n-1$, can we directly find the next permutation in the lexicographic sequence?
- Given a permutation of $0..n-1$, can we determine its permutation sequence number?

- Given n and k , can we directly generate the k^{th} permutation of $0, \dots, n-1$?



Generate k^{th} permutation of 0..n-1

```
def kthPermutation(s, k):  
    """return the kth lexocographic permutation  
    of the characters in list s"""  
    s = list(s)  
    result = []  
    factTable = factorialTable(len (s))  
    for divisor in factTable:  
        multiple = k // divisor  
        k = k % divisor  
        element = s[multiple]  
        result.append(element)  
        s.remove(element)  
    return result
```



Quote of the Day

- There are 10^{11} stars in the galaxy. That used to be a huge number. But it's only a hundred billion. It's less than the national deficit! We used to call them astronomical numbers. Now we should call them economical numbers.
- Richard Feynman



Reduce by a Constant Factor

- **Examples that we have already seen:**
 - Binary Search
 - Exponentiation (ordinary and modular) by repeated squaring
 - Multiplication à la Russe (The Dasgupta book that I followed for the first part of the course called it "European" instead of "Russian")

- Example

11	13
5	26
2	52
1	104
	<hr/>
	143

Then strike out any rows whose first number is even, and add up the remaining numbers in the second column.



Fake Coin Problem



- We have n coins
- All but one have the same weight
- One is lighter
- We have a balance scale with two pan.
- All it will tell us is whether the two sides have equal weight, or which side is heavier
- What is the minimum number of weighings that will guarantee that we find the fake coin?
- Decrease by factor of two.



Josephus problem - background

- Flavius Josephus was a Jewish general and historian who lived and wrote in the 1st century AD
- Much of what we know about 1st century life in Israel (and the beginning of Christianity) before and after the Roman destruction of the Jewish temple in 70 AD comes from his writings
- The "Josephus problem" is based on an odd suicide pact that he describes
 - He and his men stood in a circle and counted off
 - Every other person (or every third person, accounts vary) was killed
 - The last person was supposed to kill himself
 - He must have been the next-to-last person!
 - When it got down to two people, he persuaded the other person that they should surrender instead
- <http://en.wikipedia.org/wiki/Josephus>



Josephus Problem

- n people, numbered 1- n , are in a circle
- Count starts with 1
- Every 2nd person is eliminated
- The last person left, $J(n)$, is the winner
- Examples: $n=8$, $n=7$
- $J(1) = 1$
- Solution if n is even
- Solution if n is odd
- Use it to find $J(2) \dots J(8)$
- Clever solution: cyclic bit shift left



Decrease by a variable amount

- Search in a Binary Search Tree

- Interpolation Search

- A CSSE 220/230 topic
- See Levitin, pp190-191
- Also Weiss, pp 187-188
- performance when searching

- Social Security numbers of US students
- Phone book
- CSSE department employees, 1985-2008

adkins	curry	merkle
ardis	defoe	mohan
azhar	dalkolic	mouck
bagert	degler	mutchler
bohner	jeschke	oexmann
bowman	kaczmarczyk	sengupta
boutell	kinley	sullivan
chenoweth	laxer	surendran
chidanandan	mcleish	wollowski
clifton	mellor	young
criss		
15/31 A-D	24/31 A-D, K-M	28/31 A-D, J-M, S



Median finding

- Find the k^{th} element of an (unordered) list of n elements
- Start with quicksort's partition method
- Best case analysis



General Divide-and-Conquer Recurrence

$$T(n) = aT(n/b) + f(n) \quad \text{where } f(n) \in \Theta(n^d), \quad d \geq 0$$

Master Theorem: If $a < b^d$, $T(n) \in \Theta(n^d)$

If $a = b^d$, $T(n) \in \Theta(n^d \log n)$

If $a > b^d$, $T(n) \in \Theta(n^{\log_b a})$

Note that the same results also hold with O instead of Θ .

