

MA/CSSE 473

Day 17



Exam Was Day 16

ConvexHull

**Matrix
Multiplication**

MA/CSSE 473 Day 17

- HW 7 is due tomorrow.
- **Student Questions**
- Convex Hull
- Matrix Multiplication



Convex Hull Problem

- Again, sort by x-coordinate, with tie going to larger y-coordinate.

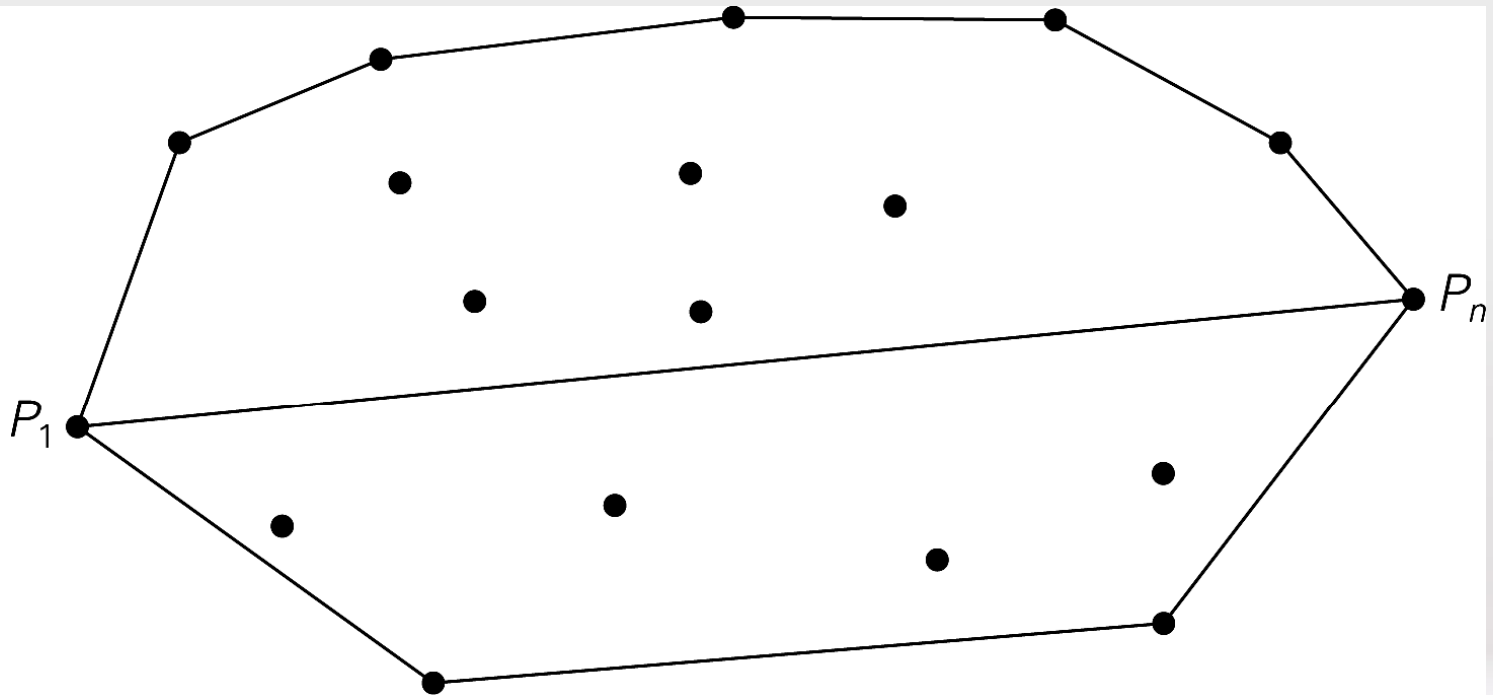


FIGURE 4.8 Upper and lower hulls of a set of points



Recursive calculation of Upper Hull

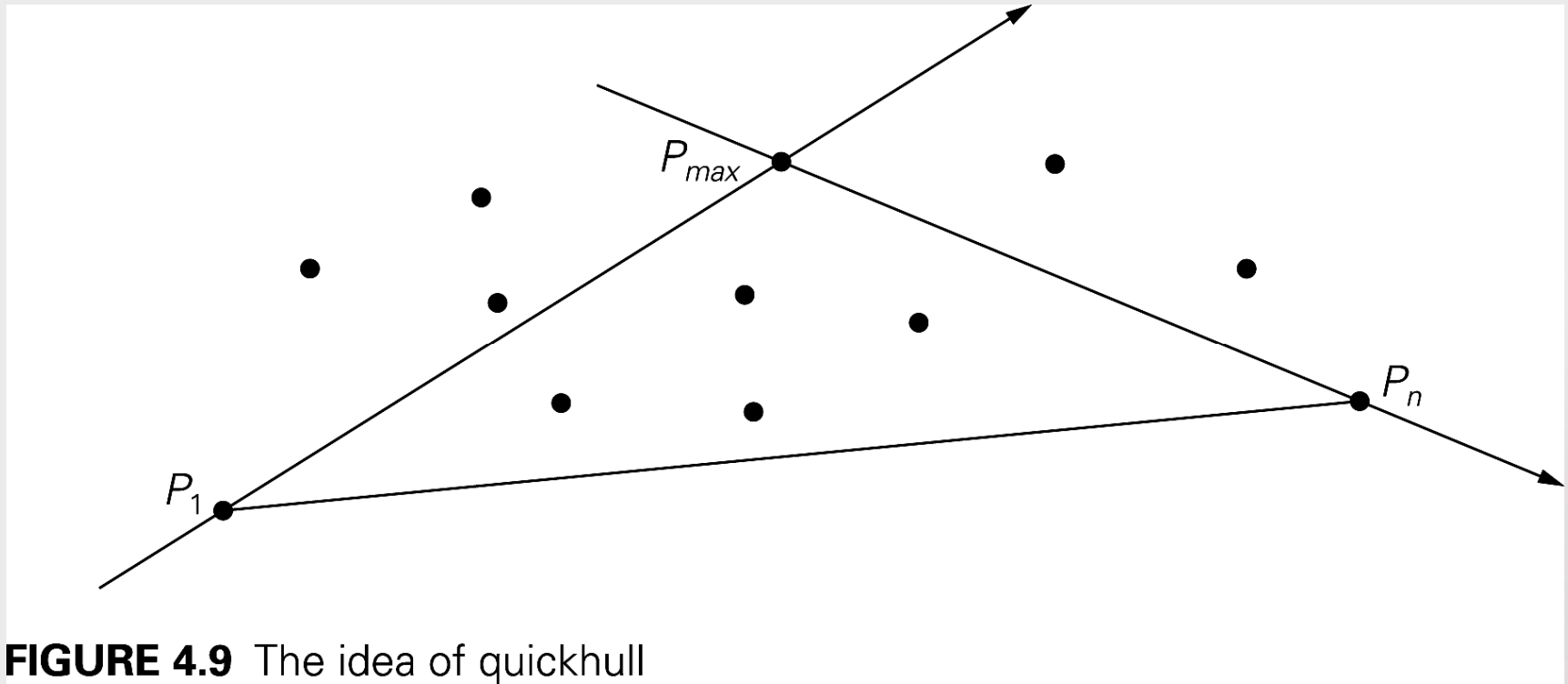


FIGURE 4.9 The idea of quickhull



Efficiency of quickhull algorithm

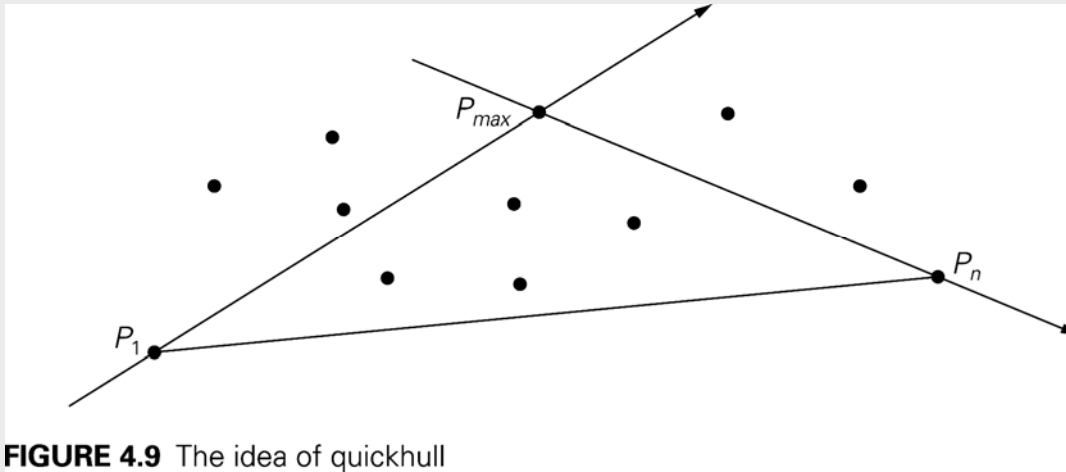


FIGURE 4.9 The idea of quickhull

- What arrangements of points is worst case?
- Average case is much better. Why?



Simplifying the Calculations

- The maximum distance of P from line P_1P_2
- Determining whether P is "to the left" of P_1P_2
 - The area of the triangle through $P_1=(x_1,y_1)$, $P_2=(x_2,y_2)$, and $P_3=(x_3,y_e)$ is $\frac{1}{2}$ of the absolute value of the determinant

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = x_1y_2 + x_3y_1 + x_2y_3 - x_3y_2 - x_2y_1 - x_1y_3$$

- The sign of the determinant is positive if the order of the three points is clockwise, and negative if it is counter-clockwise
 - For a proof of this property, see <http://mathforum.org/library/drmath/view/55063.html>
- Clockwise means that P_3 is "to the left" of directed line segment P_1P_2
- What about max distance?

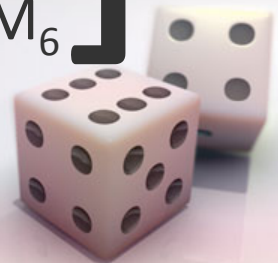


Strassen's Matrix Multiplication

Strassen observed [1969] that the product of two matrices can be computed as follows:

$$\begin{bmatrix} C_{00} & C_{01} \\ C_{10} & C_{11} \end{bmatrix} = \begin{bmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{bmatrix} * \begin{bmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{bmatrix}$$
$$= \begin{bmatrix} M_1 + M_4 - M_5 + M_7 & M_3 + M_5 \\ M_2 + M_4 & M_1 + M_3 - M_2 + M_6 \end{bmatrix}$$

Values of M_1, M_2, \dots, M_7 are on the next slide



Formulas for Strassen's Algorithm

$$M_1 = (A_{00} + A_{11}) * (B_{00} + B_{11})$$

$$M_2 = (A_{10} + A_{11}) * B_{00}$$

$$M_3 = A_{00} * (B_{01} - B_{11})$$

$$M_4 = A_{11} * (B_{10} - B_{00})$$

$$M_5 = (A_{00} + A_{01}) * B_{11}$$

$$M_6 = (A_{10} - A_{00}) * (B_{00} + B_{01})$$

$$M_7 = (A_{01} - A_{11}) * (B_{10} + B_{11})$$

How many additions and multiplications?

- If we do "normal matrix multiplication" recursively?

- If we use Strassen's formulas?



Analysis of Strassen's Algorithm

If N is not a power of 2, matrices can be padded with zeros.

Number of multiplications:

$$M(N) = 7M(N/2), \quad M(1) = 1$$

Solution: $M(N) = 7^{\log_2 N} = N^{\log_2 7} \approx N^{2.807}$ vs. n^3 of brute-force algorithm.

What if we also count the additions?

Algorithms with better asymptotic efficiency are known but they are even more complex.



General Divide-and-Conquer Recurrence

$$T(n) = aT(n/b) + f(n) \quad \text{where } f(n) \in \Theta(n^d), \quad d \geq 0$$

Master Theorem:

- If $a < b^d$, $T(n) \in \Theta(n^d)$
- If $a = b^d$, $T(n) \in \Theta(n^d \log n)$
- If $a > b^d$, $T(n) \in \Theta(n^{\log_b a})$

Note: The same results hold with O instead of Θ .

