CSSE463: Image Recognition Day 14

- Lab due Weds.
  - These solutions assume that you don't threshold the shapes.ppt image: Shape1: elongation = 1.632636, C1 = 19.2531, C2 = 5.0393
- This week:
  - Tuesday: Support Vector Machine (SVM) Introduction and derivation
  - Thursday: Project info, SVM demo
  - Friday: SVM lab

Feedback on feedback

Delta

- Want to see more code
- Math examples caught off guard, but OK now.
- Tough if labs build on each other b/c no feedback until lab returned.
- Project + lab in same week is slightly tough
- Include more examples
- Application in MATLAB takes time.

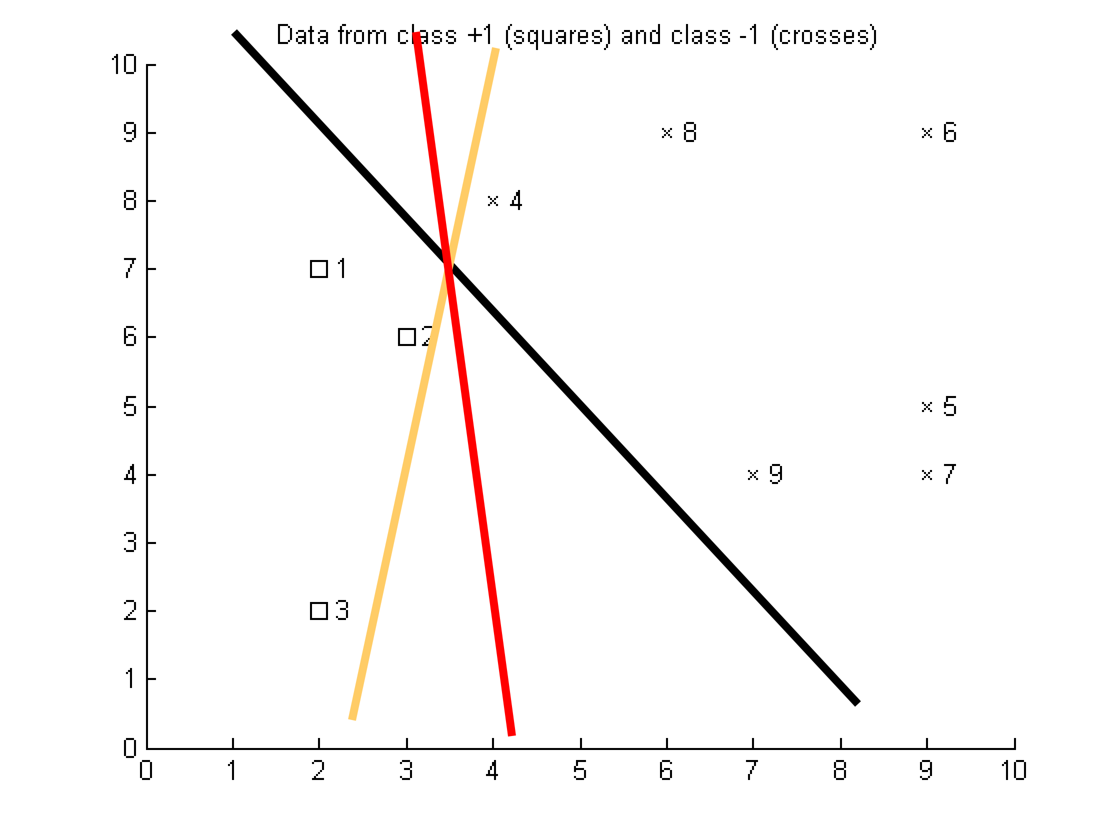
Plus

- Really like the material (lots)
- Covering lots of ground
- Labs!
- Quizzes 2
- Challenging and interesting
- Enthusiasm
- Slides
- Groupwork
- Want to learn more

Pace:

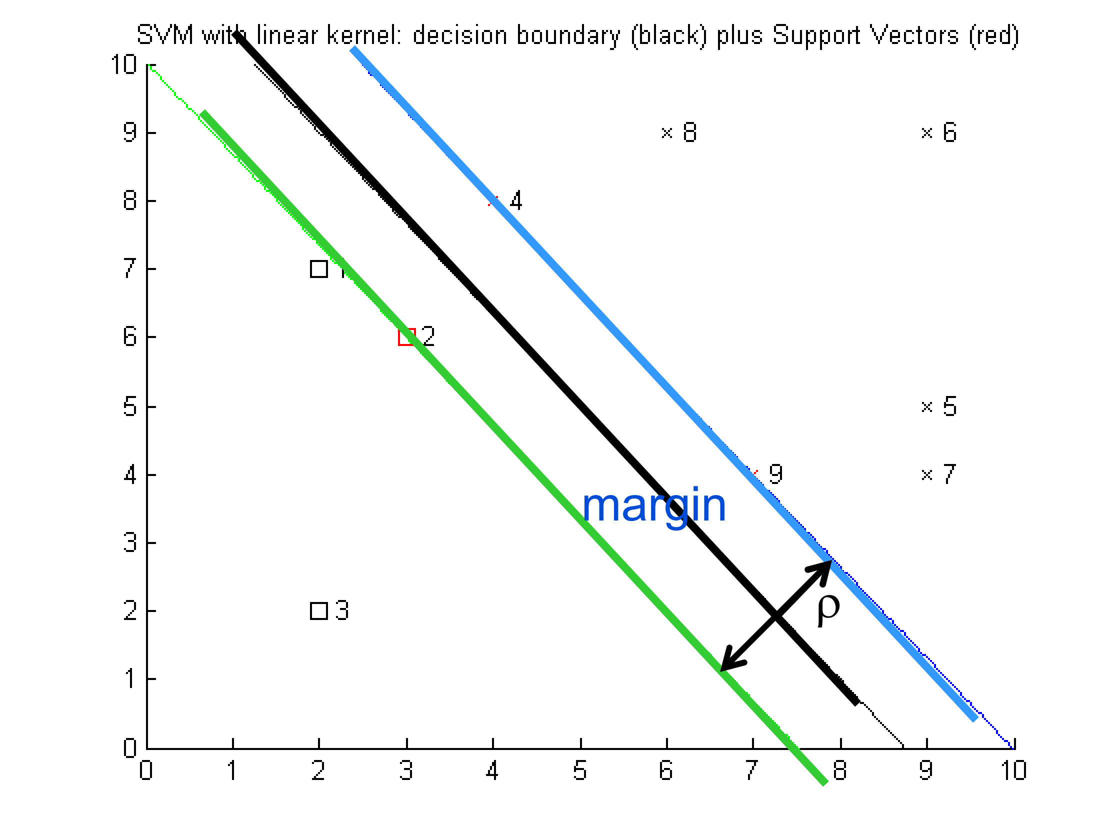
Lectures and assignments: OK – slightly fast

SVMs: “Best” decision boundary



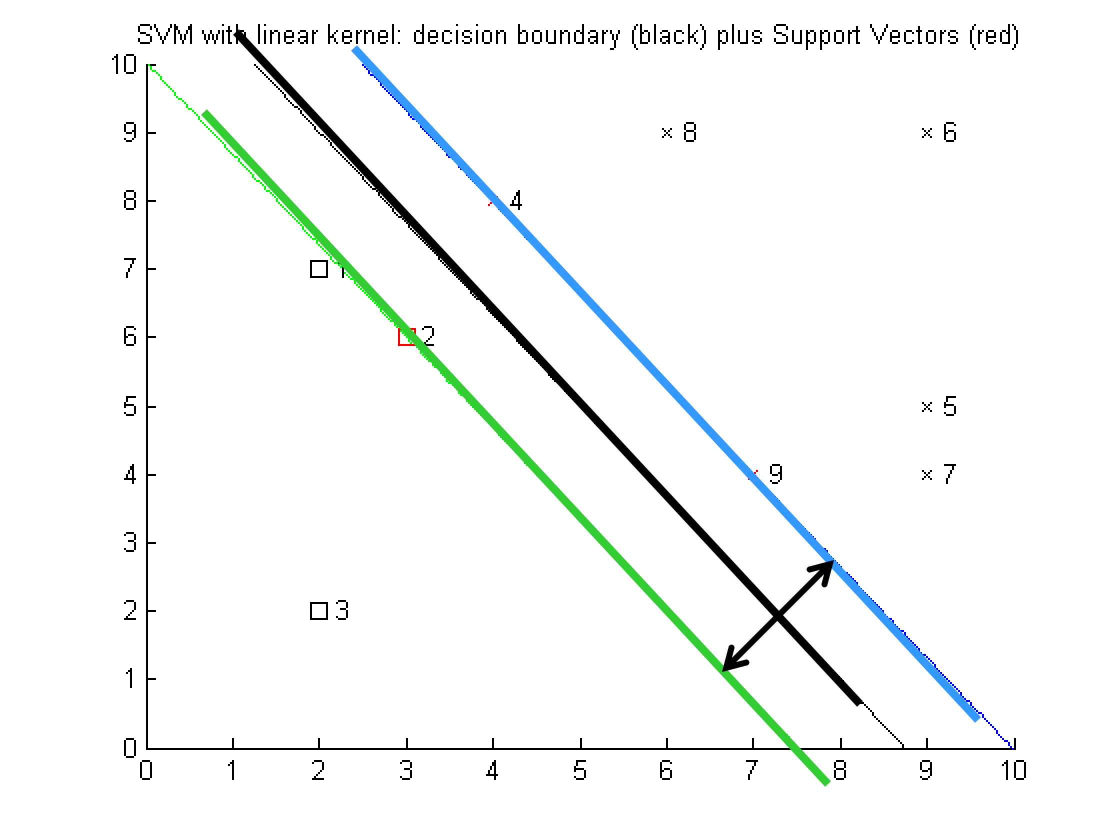
- Consider a 2-class problem
- Start by assuming each class is linearly separable
- There are many separating hyperplanes…
- Which would you choose?

SVMs: "Best" decision boundary



- The "best" hyperplane is the one that *maximizes the margin, ρ,* between the classes.
- Some training points will always lie on the margin
  - These are called *"support vectors"*
  - #2,4,9 to the left
- Why does this name make sense intuitively?

Support vectors



- The support vectors are the toughest to classify
- What would happen to the decision boundary if we moved one of them, say #4?
- A different margin would have maximal width!

Problem

- Maximize the margin width
- while classifying all the data points correctly…

Mathematical formulation of the hyperplane

- On paper
- Key ideas:
  - Optimum separating hyperplane: $w_0{}^T x + b_0$
  - Distance to margin: $g(x) = w_0{}^T x + b_0$
  - Can show the margin width = $\rho = \frac{2}{\|w_0\|}$
  - Want to maximize margin

Finding the optimal hyperplane

- We need to find w and b that satisfy the system of inequalities:

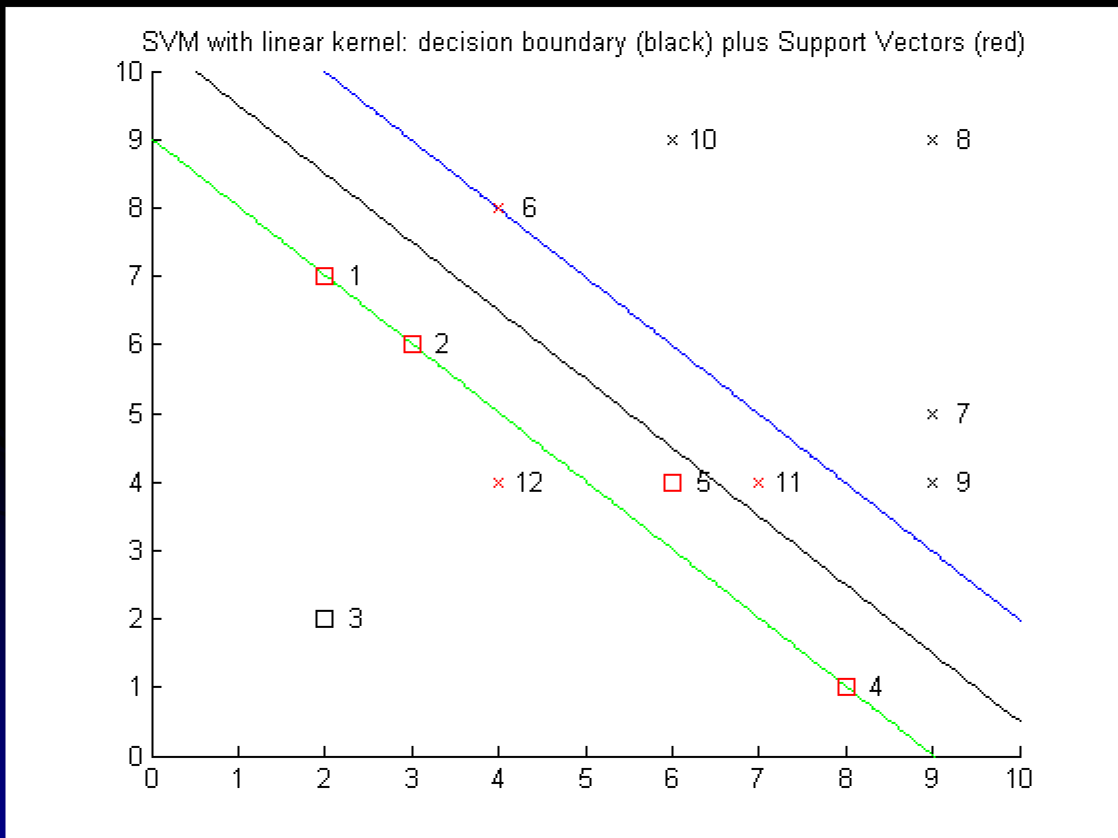
$$d_i(w^T x_i + b) \geq 1 \ for \ i = 1,2,\ldots.N$$

- where w minimizes the cost function:

$$\phi(w) = \frac{1}{2} w^T w$$

- (Recall that we want to minimize $||w_0||$, which is equivalent to minimizing $||w_o||^2 = w^T w$)
- Quadratic programming problem
  - Use Lagrange multipliers
  - Switch to the dual of the problem

Non-separable data



- Allow data points to be misclassifed
- But assign a cost to each misclassified point.
- The cost is bounded by the parameter C (which you can set)
- You can set different bounds for each class. Why?
  - Can weigh false positives and false negatives differently

Can we do better?

- Cover's Theorem from information theory says that we can map nonseparable data in the input space to a feature space where the data is separable, with high probability, if:
  - The mapping is nonlinear
  - The feature space has a higher dimension
- The mapping is called a *kernel function*.
- Lots of math would follow here

Most common kernel functions

- Polynomial
- Gaussian Radial-basis function (RBF)
- Two-layer perceptron

$$K(x, x_i) = (x^T x_i + 1)^p$$

$$K(x, x_i) = \exp\left(-\frac{1}{2\sigma^2}\left\|x - x_i\right\|^2\right)$$

$$K(x, x_i) = \tanh\left(\beta_0 x^T x_i + \beta_1\right)$$

- You choose p, σ, or β_i
- My experience with real data: **use Gaussian RBF!**

Easy — Difficulty of problem — Hard

p=1, p=2, — higher p — RBF

Q5