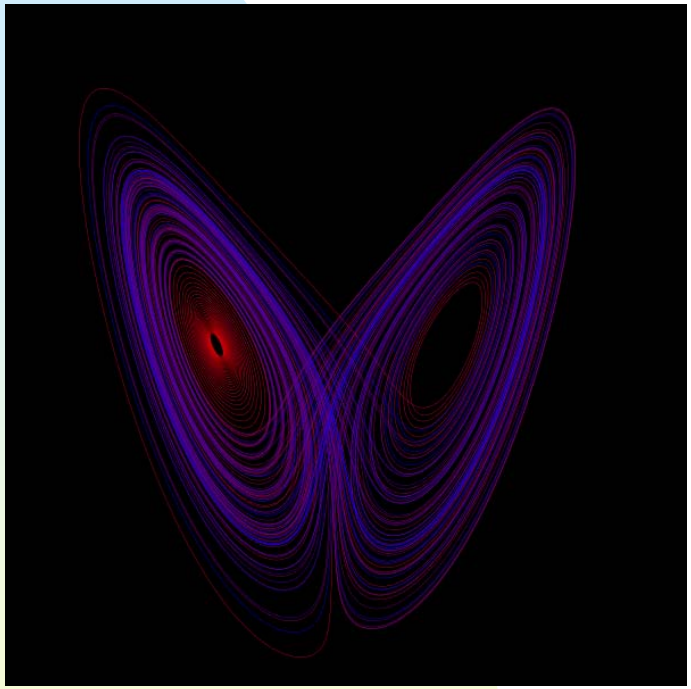
Session overview



- Lyapunov exponents
- Quizzes coming back:
  - Points on Day23-24 will be doubled.
- Please turn in HW4 now
- HW5 due Monday.
- Math talk today:
  10th Hour
  G-219
  Kim Montgomery, Univ. Utah
  Hair Cells and Hopf Bifurcations

Other attractors

- Rossler attractor
- Defined by 3 differential equations (w.r.t. time):

$$x' = -(y + z)$$

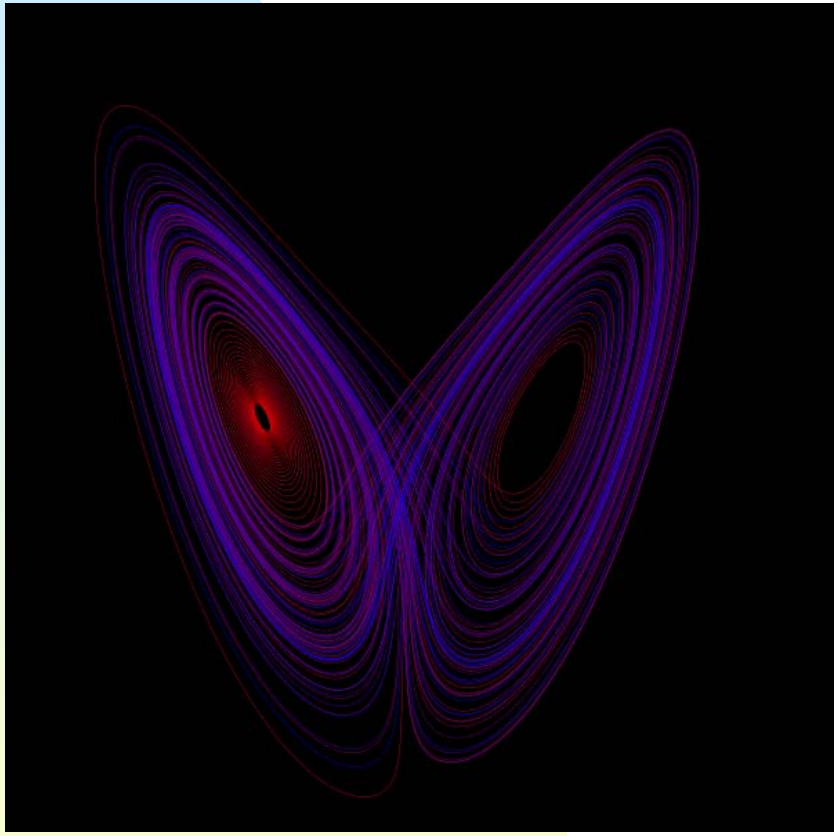
$$y' = x + ay$$

$$z' = b + xz - cz$$

- Pics and animation:

http://chaos.phy.ohiou.edu/~thomas/chaos/ode.html

Other attractors



Wikipedia Commons

- Lorenz attractor
- Defined by 3 differential equations (w.r.t. time):

$$x' = \sigma(y - x)$$

$$y' = Rx - y - xz$$

$$z' = -Bz + xy$$

$$\sigma = 10, B = \frac{8}{3}, R = 28$$

Properties of strange attractors

- Let $T(x, y)$ be a transformation in the plane
- A bounded subset A of the plane is a chaotic and strange attractor for T if there exists a set R with the following properties:
  - Attractor - R is a neighborhood of A. R is a trapping region. Each orbit in R remains in R for all iterations. Moreover, the orbit becomes close to A and stays as close to it as we desire. Thus, A is an *attractor*.

Properties of strange attractors (cont.)

- Sensitivity - Orbits started in *R* exhibit sensitive dependence on initial conditions. This makes *A* a *chaotic attractor*.
- Fractal - The attractor has a fractal structure and is therefore called a *strange attractor*.
- Mixing - *A* cannot be split into two different attractors. There are initial points in *R* with orbits that get arbitrarily close to any point of *A*

Sensitivity to initial conditions

- How do we measure this?
- The largest Lyapunov exponent, λ, gives the rate of separation of two close trajectories.
  - They diverge with rate c^t, where $c=e^{\lambda}$
  - A positive exponent usually means the system is chaotic.
- Discussion and derivation on board

Error propagation

- See PJS Table 10.10

Examples

- Henon attractor: $\lambda = 0.419217$
- Lorenz attractor: $\lambda = 0.90563$ (for the parameters given earlier)
- Rossler attractor: $\lambda = 0.13$ (for a=0.15, b=0.2, c-10