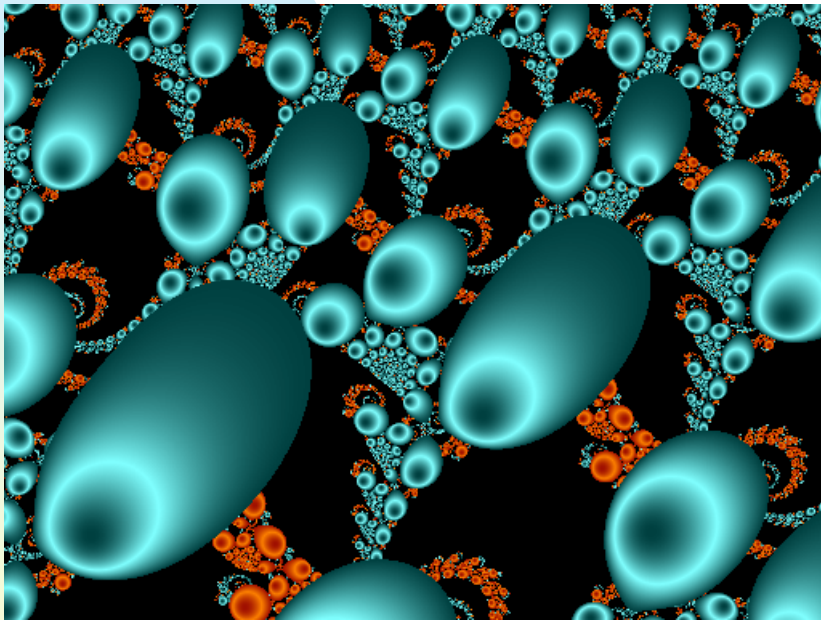
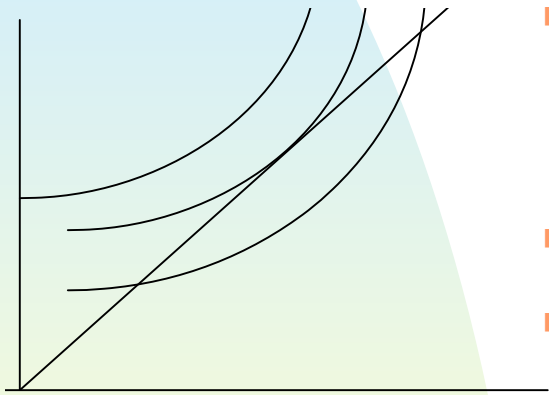


Session overview



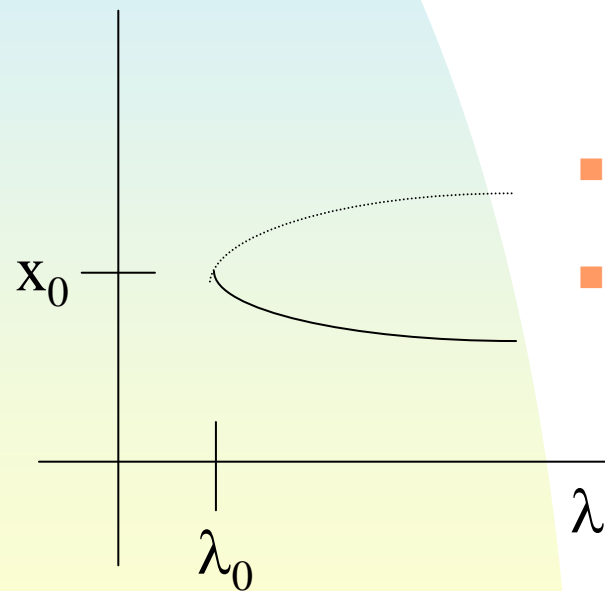
- Bifurcations
- Edward Lorenz, father of chaos theory (and the “butterfly effect”), died yesterday at age 90
- <http://xkcd.com/378/>
- Pass in HW3 now

Tangent bifurcation



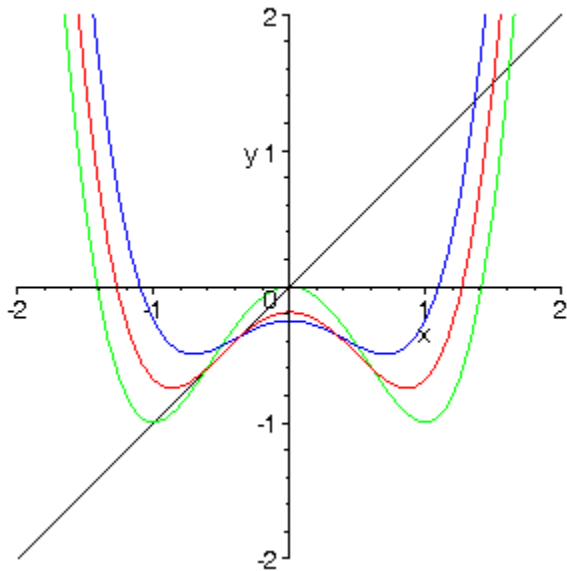
- Consider a family of curves $F_\lambda(x)$
- Note that the center curve is tangent to the replacement line at one point, and thus has one fixed point
- The curve to the left has no fixed points
- The curve to the right has two fixed points
- The value of λ for which the curve is tangent to the replacement line at one point is called the *bifurcation value* (λ_0)
- The ordered pair (λ_0, x_0) is called the *bifurcation point*

Bifurcation diagram

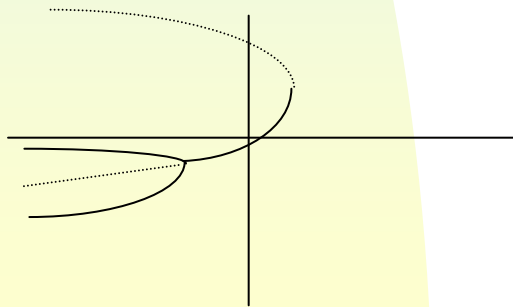


- A graph of the limiting state values (fixed points, periodic points, etc.) as a function of the parameter
- Solid lines indicate attraction
- Dashed lines indicate repulsion

Period-doubling bifurcation



- Consider the system $Q_c = x^2 + c$
- The plots to the left are of Q_c^2 for values of c around $-3/4$
 - ◆ blue: $c = -1/2$
 - ◆ red: $c = -3/4$
 - ◆ green: $c = -1$
- Note that the bifurcation point is $(-3/4, -1/2)$
- The slope of Q_c at the bifurcation point is -1



Quiz

- The quiz reviews logistic map bifurcations and their relationship to the Feigenbaum diagram