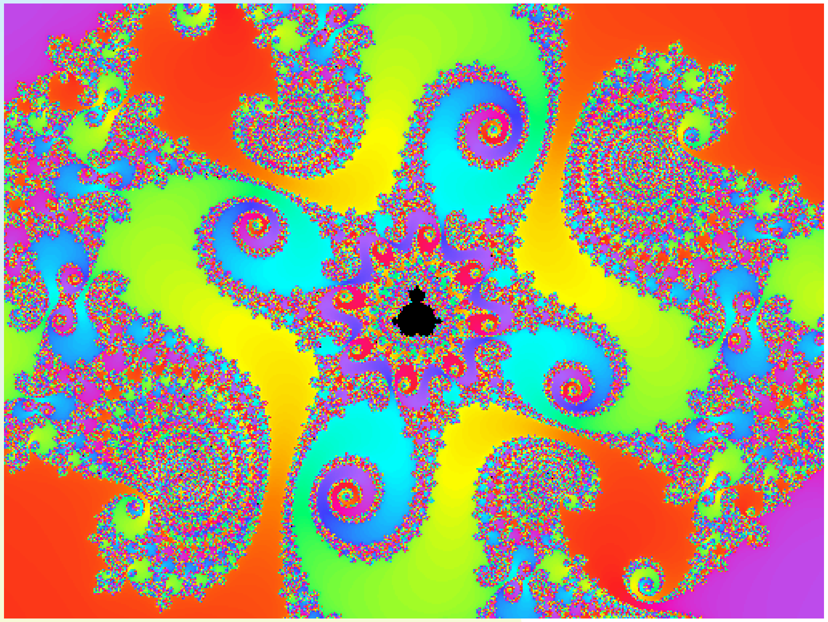
Session overview



- Feigenbaum diagrams
- Announcements:
  - HW3 posted, due Thursday

Logistic map analysis

- Finish discussion of solutions to the logistic map analysis worksheet

Logistic map orbits

- Consider F(x) = $ax(1 - x)$
- It is interesting to look at the long term behavior of orbits for various values of a
- Program `logistState.cpp` plots such orbits
- Let's look at plots for a = 1, 1.1, 1.5, 2, 2.5, 2.9, 3, 3.1, 3.4, 3.5, 3.56, 3.569, 3.6, 4

Analysis of orbits

- For some values of *a* the orbit converged to a fixed point
- For some values of *a* the orbit exhibited period-2 behavior
- For some values of *a* the orbit exhibited period-4 behavior
- This progression is known as *period doubling*

Feigenbaum diagrams

- It is interesting to observe a plot of *a* versus the fixed points of the orbits
- The resulting image is known as a Feigenbaum diagram
  - It is a fractal
- Program `logistFeigenbaum.cpp` generates the image

Feigenbaum point

- Note that there are two areas to the Feigenbaum diagram:
  - the period-doubling area on the left
  - the chaotic area on the right
- The value of *a* which separates the two areas (3.5699456…) is known as the *Feigenbaum point*

Feigenbaum constant

- The branches of the period doubling area of the Feigenbaum diagram are not equal in length
- They, in fact, grow shorter as we approach the Feigenbaum point
- If we take the ratio of the length (along the *a*-axis) of one period doubling area to the next, we get the constant 4.6692…, the *Feigenbaum constant* (δ)
- δ is a constant of chaos

Project #5

- Generate Feigenbaum diagrams for several different maps
- Due next Monday