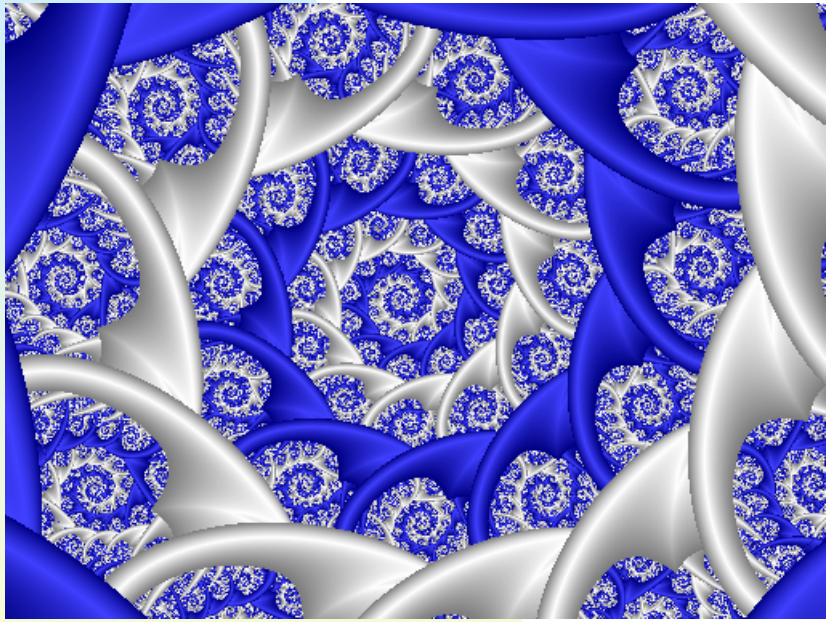
# Session overview



- One-dimensional Brownian motion
- Announcements:
  - Project 2 due now
  - Daily quiz includes take-home portion due tomorrow.
  - Project 3 due Monday

# Brownian motion

- Small particles of solid matter suspended in a liquid can be seen under a microscope to move about in an irregular and erratic way
- Simplest case - one particle being hit on the left or right by the other particles
- Each hit results in a unit displacement left or right
- Can we predict the total displacement after *n* hits?

# Mean square displacement

- Expected value of total displacement is 0, since $\pm 1$ equally likely
- So, look at the square of the displacement
- Average of the squared displacements, called the ***mean square displacement***, tells how much the particles spread out in a given number of steps (time units)

# Calculating the MSD

$$MSD = E(d_1 + d_2 + \ldots + d_n)^2 = E(\sum_{i=1}^{n}\sum_{j=1}^{n} d_i d_j)$$

- $d_i = \pm 1$
- $d_i$ independent of $d_j$ for $i \neq j$
- Table of possible outcomes:

| $d_i$ | $d_j$ | $d_i d_j$ | Probability |
|---|---|---|---|
| 1 | 1 | 1 | 0.25 |
| 1 | -1 | -1 | 0.25 |
| -1 | 1 | -1 | 0.25 |
| -1 | -1 | 1 | 0.25 |

- $d_i d_j = \pm 1$ equally likely for $i \neq j$
- For $i = j$, $d_i d_i = 1$ always
- Therefore, MSD = *n*, the number of time units

# Changing the step size

- Suppose the step size is ½
- Table of possible outcomes:

| $d_i$ | $d_j$ | $d_i d_j$ | Probability |
|---|---|---|---|
| ½ | ½ | ¼ | 0.25 |
| ½ | -½ | -¼ | 0.25 |
| -½ | ½ | -¼ | 0.25 |
| -½ | -½ | ¼ | 0.25 |

- $d_i d_j = \pm ¼$ equally likely for $i \neq j$
- For $i = j$, $d_i d_i = ¼$ always
- Therefore, the MSD = $¼n$

# Generalizing

- The MSD $\propto \Delta t$, the time difference, with the proportionality factor depending upon:
  - the number of steps, *n*
  - the length, L, of the individual displacements
  - MSD = $L^2t$

# Finish Quiz

- Simulate Brownian motion using individual displacements
- Use to experimentally confirm theoretical MSD results