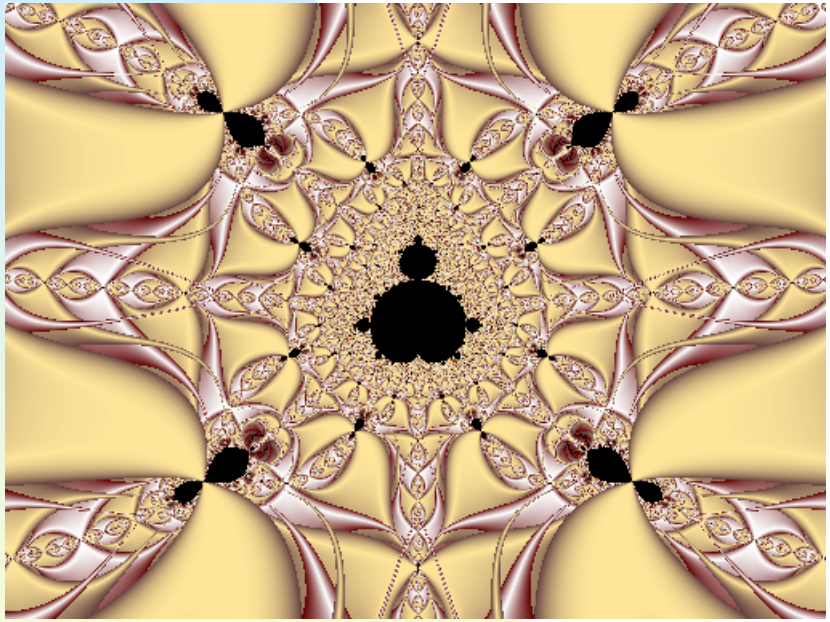
Session overview



- L-systems

Chaos Game extension

- What if, instead of 3 vertices arranged in a circular pattern, you had n?
- Demo n = 6, n = 5, n = 4
- Thanks to Adam for suggesting this extension

- Questions on project?

L-systems

- Model ***growth*** of a fractal
- Developed in 1968 by the biologist Aristid Lindenmayer
- Also known as parallel rewriting systems

Context-free L-systems

- Defined by:
  - an alphabet, $V = \{ a_1, a_2, \ldots, a_n \}$
  - the production map

    $P: V \rightarrow V^*$

    $a \rightarrow P(a)$

    where V^* is the set of all strings formed by symbols from V
  - an axiom $\alpha^{(0)} \in V^*$, the initial string

Rewriting

- Note that for all symbols of the alphabet $a \in V$ there is exactly one production (or rewriting) rule P(a)
- Starting with the axiom $\alpha^{(0)}$, the L-system generates a sequence of strings: $\alpha^{(0)}$, $\alpha^{(1)}$, $\alpha^{(2)}$, …
- The string $\alpha^{(i+1)}$ is obtained from the preceding string $\alpha^{(i)}$ by applying the production rules to all symbols $\alpha_1^{(i)}$, …, $\alpha_m^{(i)}$ of the string simultaneously:

$$\alpha^{(i+1)} = P(\alpha_1^{(i)})P(\alpha_2^{(i)})P(\alpha_3^{(i)})\ldots P(\alpha_m^{(i)})$$

Example 1

- Alphabet V = { A, B }
- Production rules:
  - A $\rightarrow$ AB
  - B $\rightarrow$ B
- Axiom: A
- Successive strings are:
  - A
  - AB
  - ABB
  - ABBB
  - ABBBB

Example 2

- Alphabet V = { F, f }
- Production rules:
  - F $\rightarrow$ FfF
  - f $\rightarrow$ fff
- Axiom: F
- Successive strings are:
  - F
  - FfF
  - FfFfffFfF
  - FfFfffFfFfffffffffFfFfffFfF
- This is the L-system description of the Cantor set

An alphabet with which to work

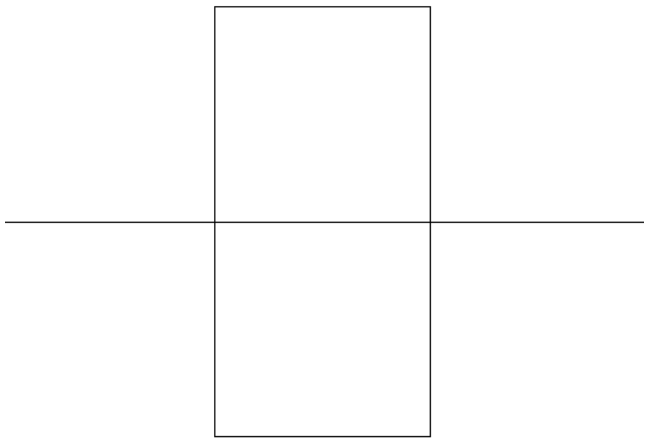
- Used in *turtle graphics*
- Let's define a working alphabet as the following:
  - F: move forward a fixed length and draw a line along the way
  - f: move forward a fixed length but don't draw a line
  - +: turn left (CC) by angle δ
  - -: turn right (C) by angle δ

The Koch curve L-system

- Production rules:
  - F $\rightarrow$ F-F++F-F
  - + $\rightarrow$ +
  - - $\rightarrow$ -
- Axiom: F
- Parameter: δ = 60º
- Generated strings:
  - F
  - F-F++F-F
  - F-F++F-F-F-F++F-F++F-F++F-F-F-F++F-F
- Assumed that the correct scale factor is applied when drawing a given level

The Peano curve

- Start with a line segment
- At the 1/3 and 2/3 points draw a square above and below the line
- Develop an L-system to represent this curve



Example program

- L-systems are implemented via recursive routines
- Code is in file `Lsystems.cpp`