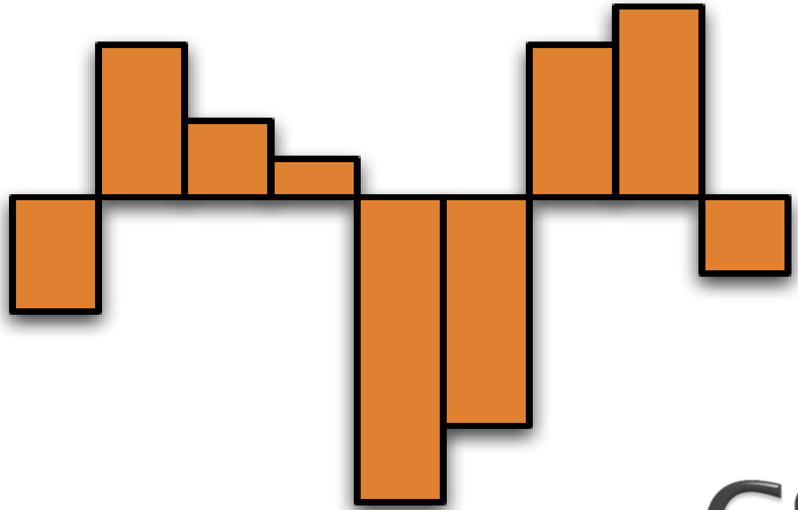


Q0

CSSE 230 Day 6

Maximum Contiguous Subsequence Sum

Check out from SVN: **MCSSRaces**

Reminder of good code style

- **Good comments:**
  - Javadoc comments for public fields and methods.
  - Explanations of anything else that is not obvious.
- **Good variable and method names:**
  - Eclipse has name completion (ALT /), so the "typing cost" of using long names is small
- **Use local variables and static methods** (instead of fields and non-static methods) where appropriate
  - "where appropriate" includes any place where you can't explicitly justify creating instance fields
- **No super-long lines of code**
- **No super-long methods:** use top down design
- **Consistent indentation** (ctrl-shift f)
- **Blank lines** between methods, **space** after punctuation

Q1

Recap: MCSS

Problem definition: Given a non-empty sequence of n (possibly negative) integers $A_1, A_2, \ldots, A_n$, find the maximum consecutive subsequence $S_{i,j} = \sum_{k=i}^{j} A_k$, and the corresponding values of i and j.

- In {−2, **11**, **−4**, **13**, −5, 2}, MCSS is $S_{2,4}$ = ?
- In {1, −3, 4, −2, −1, 6}, what is MCSS?

Recap: Eliminate the most obvious inefficiency, get $\Theta(N^2)$

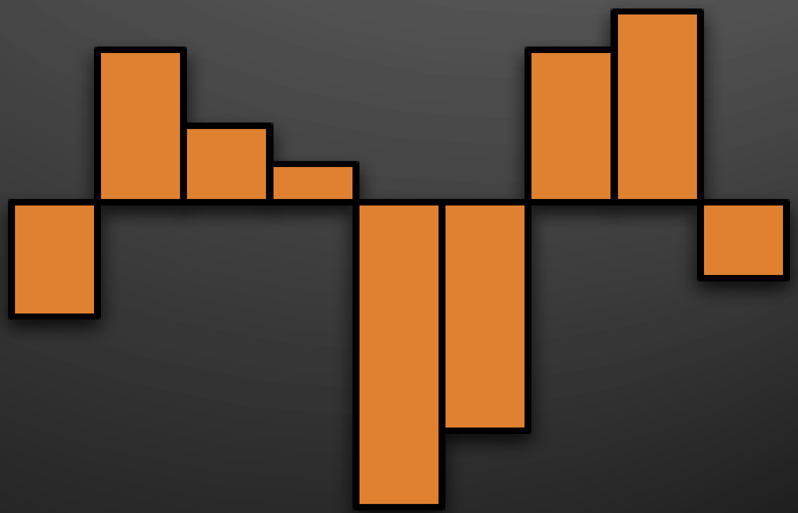
```
for( int i = 0; i < a.length; i++ ) {
    int thisSum = 0;
    for( int j = i; j < a.length; j++ ) {
        thisSum += a[ j ];

        if( thisSum > maxSum ) {
            maxSum = thisSum;
            seqStart = i;
            seqEnd   = j;
        }
    }
}
```

We can do even better than this!

Maximum Contiguous Subsequence Sum

A linear algorithm.

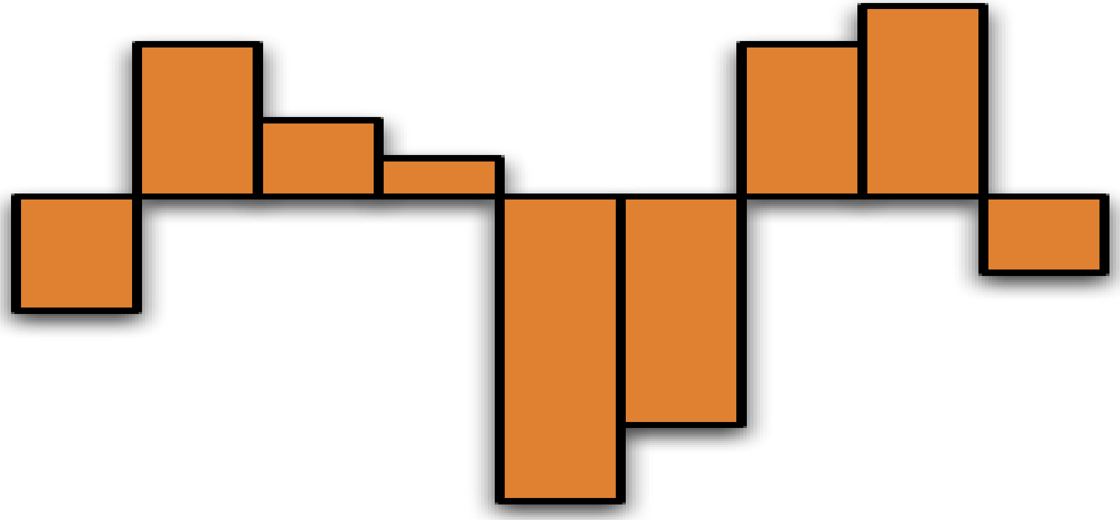
{−3, 4, 2, 1, −8, −6, 4, 5, −2}



Q2

Observations?

- Consider {−3, 4, 2, 1, −8, −6, 4, 5, −2}



- Any subsequences you can safely ignore?
  - Discuss with another student (2 minutes)

Q3

Observation 1

- We noted that a max-sum sequence $A_{i,j}$ cannot begin with a negative number.
- Generalizing this, it cannot begin with a prefix ($A_{i,k}$ with $k<j$) whose sum is negative.
  - **Proof: If $S_{i,k}$ is negative, then $S_{k+1,j} > S_{i,j}$, so $A_{i,j}$ would not be a sequence that produces the maximum sum.**

Observation 2

- All contiguous subsequences that border the maximum contiguous subsequence must have negative (or zero) sums.
  - **Proof:** If one of them had a positive sum, we could simply append (or “prepend”) it to get a sum that is larger than the maximum. Impossible!

Observation 3

For any i, let $j \geq i$ be the smallest number such that $S_{i,j} < 0$.

Then for any p and q such that $i \leq p \leq j$ and $p \leq q$:

- either $A_{p,q}$ is not a MCS, or
- $S_{p,q}$ is less than or equal to a sum already seen (i.e., one with subscripts less than i and j respectively).

Proof of Observation 3

Proof: Note that $S_{i,q} = S_{i,p-1} + S_{p,q}$. By assumption, $S_{i,p-1} \geq 0$, since $p-1 < j$, and $S_{i,p-1} \geq 0$ implies $S_{i,q} \geq S_{p,q}$. Consider cases:

- Suppose $q > j$, then $A_{i,j}$ is part of $A_{i,q}$ and (by Obs. 1) $A_{i,q}$ is not a MCS. But $S_{i,q} \geq S_{p,q}$, so $A_{p,q}$ is not a MCS either.
- Suppose $q \leq j$, then $S_{i,q}$ is a "sum already seen". Since $S_{p,q} \leq S_{i,q}$ the claim holds.

Q6

So What!?

- If we find that $S_{i,j}$ is negative, we can skip all sums that begin with any of A_i, A_{i+1}, …, A_j.
- There is no new MCS that starts anywhere between A_i and A_j.
- So we can "skip i ahead" to be j+1.

Observation 3 again:

For any i, let $j \geq i$ be the smallest number such that $S_{i,j} < 0$.

Then for any p and q such that $i \leq p \leq j$ and $p \leq q$:

- either $A_{p,q}$ is not a MCS, or
- $S_{p,q}$ is less than or equal to a sum already seen (i.e., one with subscripts less than i and j respectively).

New, improved code!

Q7

```java
public static Result mcssLinear(int[] seq) {
    Result result = new Result();
    result.sum = 0;
    int thisSum = 0;

    int i = 0;
    for (int j = 0; j < seq.length; j++) {
        thisSum += seq[j];

        if (thisSum > result.sum) {
            result.sum = thisSum;
            result.startIndex = i;
            result.endIndex = j;
        } else if (thisSum < 0) {
            // advances start to where end
            // will be on NEXT iteration
            i = j + 1;
            thisSum = 0;
        }
    }
    return result;
}
```

$S_{i,j}$ is negative. So, skip ahead per Observation 3

Running time is is Θ (?)
How do we know?

Time Trials!

- From SVN, checkout *MCSSRaces*
- Study code in *MCSS.main()*
- For each algorithm, how large a sequence can you process on your machine in less than 1 second?

Q9-10

MCSS Conclusions

- The first algorithm we think of may be a lot worse than the best one for a problem
- Sometimes we need clever ideas to improve it
- Showing that the faster code is correct can require some serious thinking
- Programming is more about careful consideration than fast typing!

Pair programming

A cheezy, helpful video

http://www.youtube.com/watch?v=rG_U12uqRhE&feature=plcp

Finite State Machines

Also known as
Deterministic Finite Automata

A Finite State Machine (FSM)

- A finite set of **states**,
  - One is the **start state**
  - Some are **final**, a.k.a **accepting**,states
- A finite **alphabet** (input symbols)
- A **transition function**
- How it works:
  - Begin in start state
  - Read an input symbol
  - Go to the next state according to transition function
  - More input?
    - Yes, then repeat
    - No, then if in accept state, return true, else return false.

Example

- Draw a FSM to determine whether a lowercase sequence of characters contains each of the 5 regular vowels once in order
  - Example: facetious

- In some versions of FSMs, each transition generates output.

Another FSM Example

A

C

B

D

Draw state diagrams for these FSMs

- Indicate the Start State and final (accepting) states
- FSM1:
  - Input alphabet {0, 1}
  - Accepts (ends in an accepting state) all input strings that do NOT contain 010 as a substring
- FSM2: (only if you get the first one done quickly)
  - Input alphabet {0, 1}
  - Accepts (ends in an accepting state) all input strings that are binary representations of numbers that are divisible by 3

Hints: Use 4 states, a start state plus 1 state each for x%3==0, x%3==1, and x%3==2.
What does the arrival of a 0 do to the current value? (doubles it) What about a 1?

x	binary	x	binary
0	0	7	111
1	1	8	1000
2	10	9	1001
3	11	10	1010
4	100	11	1011
5	101	12	1100
6	110	13	1101

Colorize

- A pair programming assignment.
- Due (along with Hardy, Part 2) on Class Day 10.

Colorize program assignment

- Input: legal Java source code
- Output: colorized HTML
  - Keywords in blue, strings in red, comments in green, everything else in black
  - Layout just like original Java input file

```
// Opening comment.  Note that a "string" is ignored here.
class /* Bad name */  Stupid {
 int x;
 String t = "A string with a /* in it";
 String p = "A string with a \" in it";
 boolean b = t.compareTo(p) < 0;

 public static void main(String [] args) {
   System.out.println("" + t + " " + p);
   System.out.println("Can you think of other interesting cases that your
 }
 /* Notice that comments /* do not "nest" in Java //  */
}
```

We can use an FSM for this!

More About Colorize

FSM representations

Q8-10

Possible Representations of the Finite State Machine

Diagrams on the whiteboard

- 2-Dimensional array:
  - Rows indexed by state, Columns by input character.
  - Each array entry is a pair object (as in DS Section 3.7):
    - [next state, what to print]
- Monolithic controller with nested switch statements
- The first choice may be more efficient and have shorter code
- The second choice is probably easier to write and modify
  - Can be made more modular by having a method for each state