

CSSE 220  
Day 21  
Recursion, Efficiency, and  
the Time-Space Trade Off;  
Mutual Recursion

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# CSSE 220 Day 21

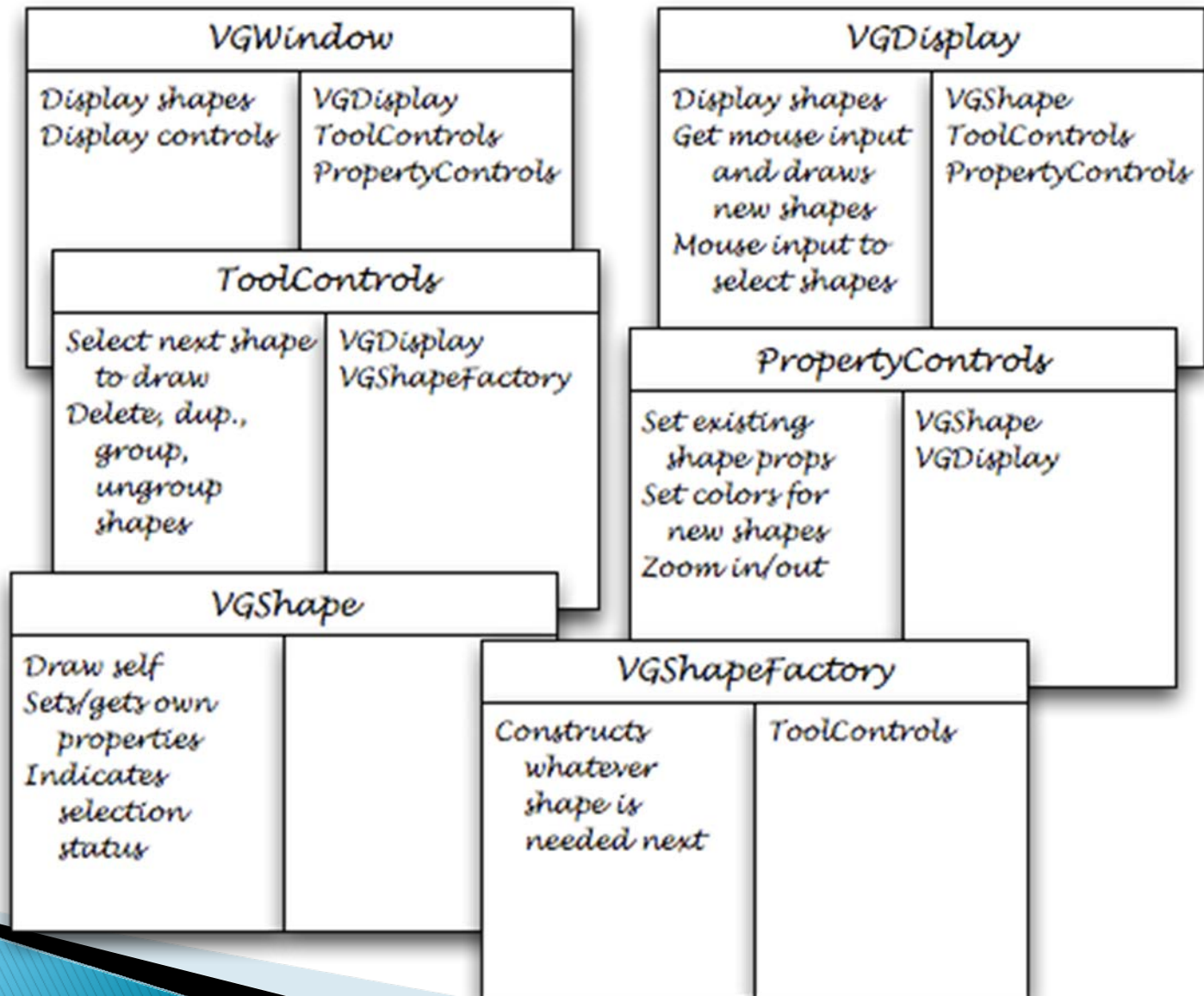
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Checkout *Recursion2* project from SVN

# No late day for VectorGraphics

- ▶ Why? Because the exam is 24 hours after the due time, and you ought to focus on the exam rather than the project at that point.

# Sample CRC Cards



# Session 20 Leftovers



# Recursive Functions

- ▶ Factorial:

$$n! = \begin{cases} 1 & \text{if } n \leq 1 \\ n * (n - 1)! & \text{otherwise} \end{cases}$$

Base Case

Recursive step

- ▶ Ackermann function:

$$A(m, n) = \begin{cases} n + 1 & \text{if } m = 0 \\ A(m - 1, 1) & \text{if } m > 0 \text{ and } n = 0 \\ A(m - 1, A(m, n - 1)) & \text{otherwise} \end{cases}$$

# Recursive Helpers

- ▶ Our `isPalindrome()` makes lots of new Sentence objects
- ▶ We can make it better with a “recursive helper method”
- ▶ 

```
public boolean isPalindrome() {  
    return isPalindrome(0, this.text.length() - 1);  
}
```

# Key Rules to Using Recursion

- ▶ Always have a **base case** that **doesn't recurse**
- ▶ Make sure recursive case always makes **progress**, by **solving a smaller problem**
- ▶ **You gotta believe**
  - Trust in the recursive solution
  - Just consider one step at a time

# Another Definition of Recursion

- ▶ “If you already know what recursion is, just remember the answer. Otherwise, find someone who is standing closer to Douglas Hofstadter than you are; then ask him or her what recursion is.”

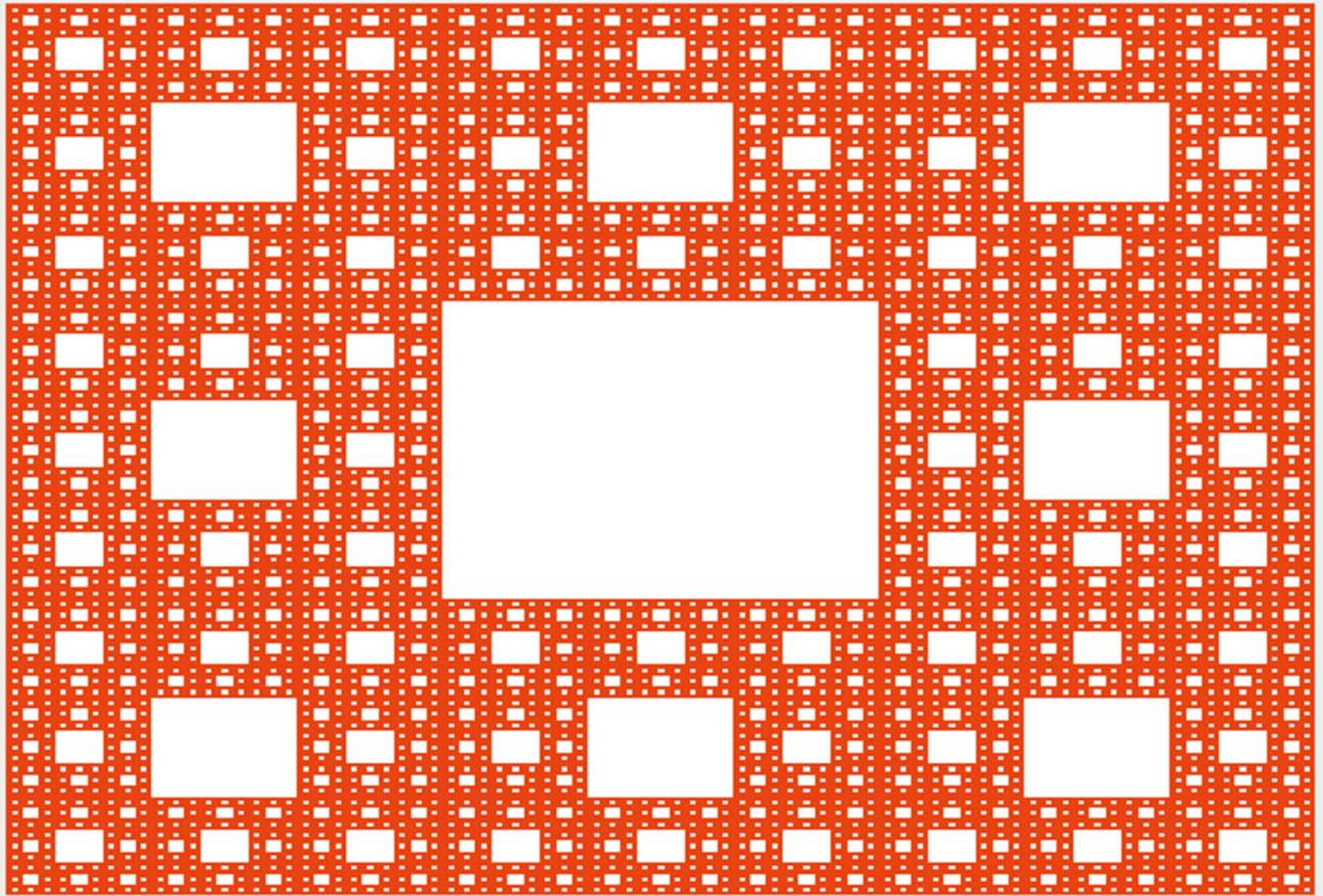
—Andrew Plotkin



# Towers of Hanoi

- ▶ Demo
- ▶ Code (on the board)
- ▶ Analysis

Sierpinski Carpet

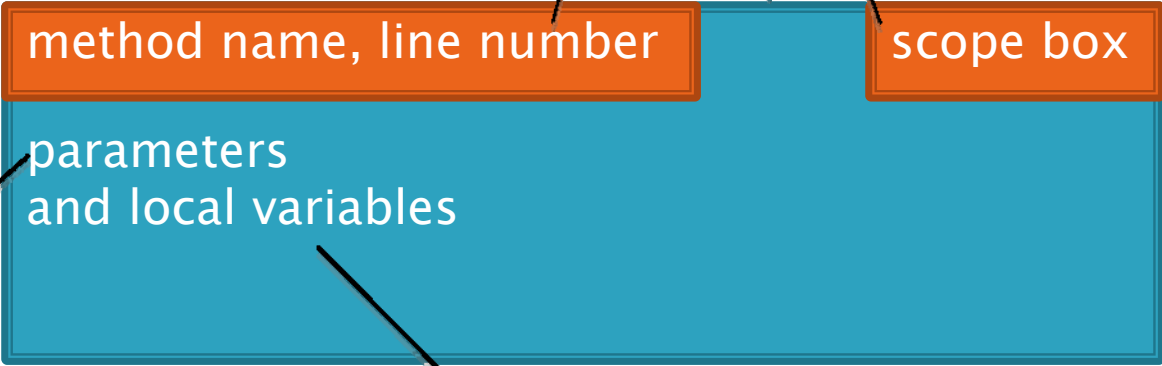


# Frames for Tracing Recursive Code

1. Draw box when method starts

2. Fill in name and first line no.

3. Write class name (for static method) or draw reference to object (for non-static method)



4. List every parameter and its argument value.

5. List every local variable declared in the method, but no values yet

6. Step through the method, update the line number and variable values, draw new frame for new calls

7. "Erase" the frame when the method is done.

Thanks for David Gries for this technique

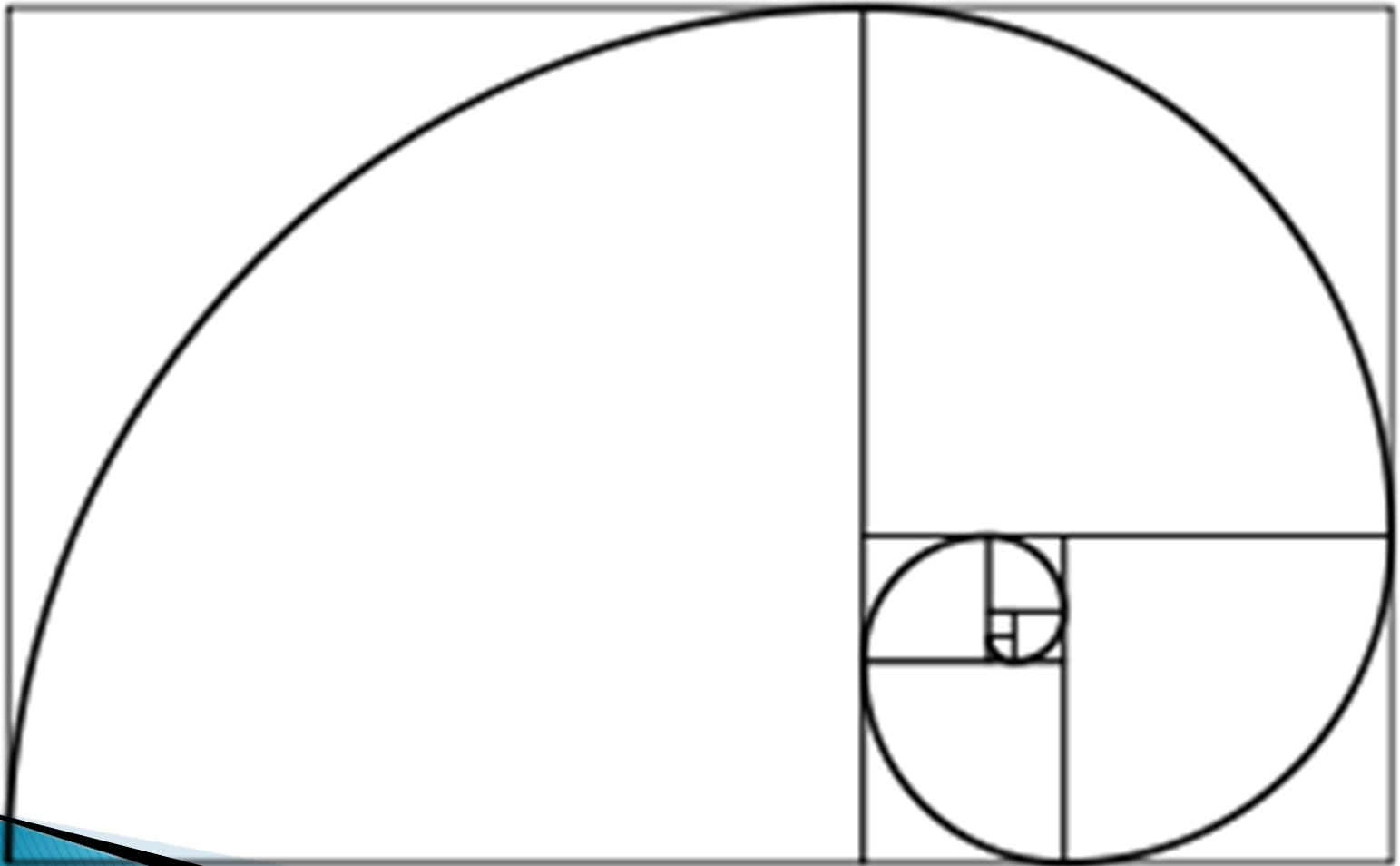
# That's Slow!

- ▶ Why does recursive Fibonacci take so long?!?
- ▶ Can we fix it?

# Classic Time–Space Trade Off

- ▶ A deep discovery of computer science
- ▶ In a wide variety of problems we can tune the solution by varying the amount of storage space used and the amount of computation performed
- ▶ Studied by “Complexity Theorists”
- ▶ Used everyday by software engineers

# Fibonacci Sequence in Squares



# Mutual Recursion

- ▶ Two or more methods that call each other repeated

# Example

- ▶ Hofstadter Female and Male Sequences:

$$F(n) = \begin{cases} 1 & \text{if } n = 0 \\ n - M(F(n - 1)) & \text{if } n > 0 \end{cases}$$

$$M(n) = \begin{cases} 0 & \text{if } n = 0 \\ n - F(M(n - 1)) & \text{if } n > 0 \end{cases}$$

- ▶ Questions:

- How often are the sequences different in the first 50 positions? first 500? first 5,000? first 5,000,000?



# Vector Graphics Work Time

- »» Should have completed Status Report for Cycle 1 and listed User Stories for Cycle 2