Day 34

- (Concept Question)
- Adding Air Drag
- Review Euler Process
- Trajectory with Drag
- Euler Convergence
- (Exercises)

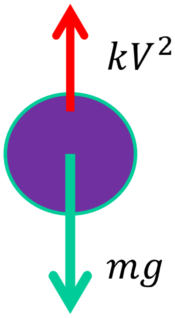
Adding Air Drag

- Yesterday we used Euler's method to solve equations for which we knew the exact solution
- We only did that for practice

- Today we will solve equations which have no exact solution

Adding Air Drag

Suppose we have a ball falling:

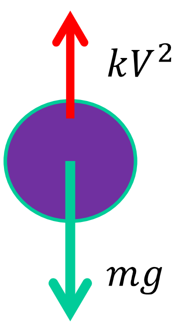


$$m\frac{dV}{dt} = -mg + kV^2$$

$$\frac{dV}{dt} = -g + \frac{k}{m}V^2$$

This one is much harder to solve analytically.

Adding Air Drag



This is the only new term!

$$\frac{dV}{dt} = -g + \frac{k}{m}V^2 \qquad V(0) = 0$$

$$\frac{dy}{dt} = V \qquad y(0) = 0$$

You will work with these equations in the exercises.

Review Euler Process

1. Replace differentials with small differences

$$\frac{dx}{dt} = f(x,t)$$

$$\frac{\Delta x}{\Delta t} = f(x,t)$$

$$\frac{x_{i+1} - x_i}{\Delta t} = f(x,t)$$

Review Euler Process

2. Evaluate rhs at time i

$$\frac{x_{i+1} - x_i}{\Delta t} = f(x,t)$$

$$\frac{x_{i+1} - x_i}{\Delta t} = f(x_i,t_i)$$

Review Euler Process

3. Isolate x_{i+1}

$$\frac{x_{i+1} - x_i}{\Delta t} = f(x_i, t_i)$$

$$x_{i+1} - x_i = (\Delta t)\, f(x_i, t_i)$$

$$x_{i+1} = x_i + (\Delta t)\, f(x_i, t_i)$$

Review Euler Process

4. March in time starting from initial condition

$$x_1 = x(0)$$

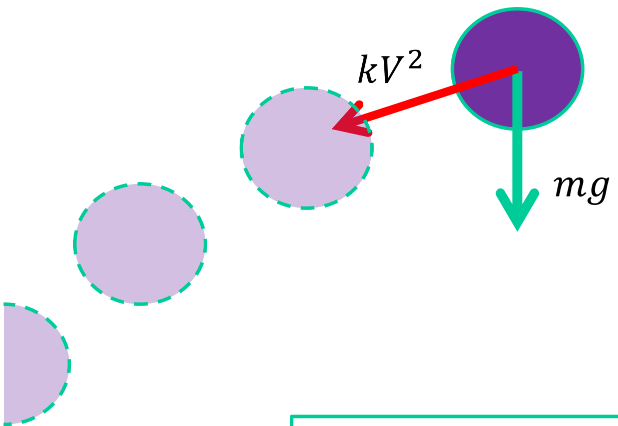
$$x_2 = x_1 + (\Delta t)\, f(x_1, t_1)$$

$$x_3 = x_2 + (\Delta t) f(x_2, t_2)$$

$$x_{i+1} = x_i + (\Delta t)\, f(x_i, t_i)$$

Trajectory with Drag

Launch a projectile with air drag:



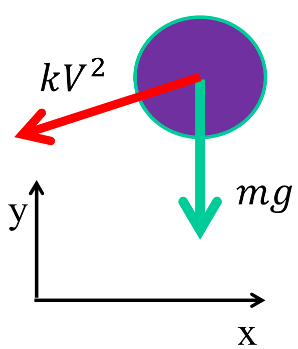
The air drag always acts to oppose the motion of the projectile.

Its magnitude depends on the square of the magnitude of the velocity.

This one has NO exact solution.
It MUST be solved numerically.

Trajectory with Drag

Launch a projectile with air drag:



$$m\frac{dV_x}{dt} = -k\frac{V_x}{|V|}V^2 \qquad V_x(0) = V_{launch}\cos\theta$$

$$m\frac{dV_y}{dt} = -k\frac{V_y}{|V|}V^2 - mg \qquad V_y(0) = V_{launch}\sin\theta$$

$$\frac{dx}{dt} = V_x \qquad x(0) = 0$$

$$\frac{dy}{dt} = V_y \qquad y(0) = 0$$

You will work with these equations in the Exercises.

Euler Convergence

- Euler gives you an approximate answer to the equations
- The smaller Δt is, the closer the answer is to the correct solution
- When you don't know the correct solution, just keep making Δt smaller until the answer doesn't change much anymore