Day 33

- (Concept Question)
- Euler Background
- Simple Euler Example
- Review Euler Process
- More Complicated Euler Example
- Euler Convergence
- (Exercises)

Euler Background

Euler's method is used to solve first order differential equations.

Given: $\frac{dx}{dt} = f(x,t) \quad x(0) = x_o$

Find: $x(t)$

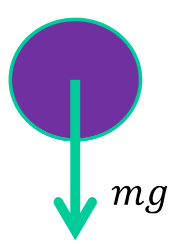
Euler's method is often used for time (t) integration

Euler Background

- Euler's method does not involve new MATLAB programming concepts – just applications of the concepts we've already learned
- Euler's method is useful for solving equations that can't be integrated analytically

Simple Euler Example

Suppose we have a ball falling:



$$m\frac{dV}{dt} = -mg$$

$$\frac{dV}{dt} = -g$$

Note that we can solve this one analytically, but we're using it to *practice*.

Simple Euler Example

Given: $$\frac{dV}{dt} = -g$$

$$V(0) = 0$$

Find: $V(t)$

Simple Euler Example

Replace the differentials with small changes

$$\frac{dV}{dt} = -g$$

$$\frac{\Delta V}{\Delta t} = -g$$

Simple Euler Example

Replace the small change in V with a difference in V

$$\frac{\Delta V}{\Delta t} = -g$$

$$\frac{V_{i+1} - V_i}{\Delta t} = -g$$

V_i is the velocity at time $(i-1)(\Delta t)$

V_{i+1} is the velocity a time (Δt) later

Simple Euler Example

Isolate V_{i+1}

$$\frac{V_{i+1} - V_i}{\Delta t} = -g$$

$$V_{i+1} - V_i = (-g)(\Delta t)$$

$$V_{i+1} = V_i + (-g)(\Delta t)$$

Simple Euler Example

Choose $\Delta t = 0.1$s and put in g=9.8 m/s^2

$$V_1 = 0$$

Recall that the initial condition was V(0)=0

$$V_{i+1} = V_i + (-9.8)(0.1)$$

Simple Euler Example

Now start “time marching” from the initial value

i	t(sec)	V(m/s)
1	0.0	0.00
2	0.1	-0.98
3	0.2	-1.96
4	0.3	-2.94
5	0.4	-3.92
6	0.5	-4.90

$$V_1 - g(\Delta t) = V_2$$

$$V_2 - g(\Delta t) = V_3$$

Simple Euler Example

```
clc
clear variables
close all
%
dt=0.1;   % delta t, in seconds
g=9.8;   % gravity, m/s^2
%
fprintf('   i   t(sec)   V(m/s)   \n');
fprintf('-----------------------\n');
i=1;
V(1)=0;   % initial velocity
t(1)=0;   % initial time
for i=1:6
    fprintf('   %1.0f    %3.1f    %5.2f   \n',i,t(i),V(i))
    V(i+1)=V(i)-g*dt;
    t(i+1)=(i)*dt;
end
```

Euler is simple to implement

Review Euler Process

1. Replace differentials with small differences

$$\frac{dx}{dt} = f(x,t)$$

$$\frac{\Delta x}{\Delta t} = f(x,t)$$

$$\frac{x_{i+1} - x_i}{\Delta t} = f(x,t)$$

Review Euler Process

2. Evaluate rhs at time i

$$\frac{x_{i+1} - x_i}{\Delta t} = f(x, t)$$

$$\frac{x_{i+1} - x_i}{\Delta t} = f(x_i, t_i)$$

Review Euler Process

3. Isolate x_{i+1}

$$\frac{x_{i+1} - x_i}{\Delta t} = f(x_i, t_i)$$

$$x_{i+1} - x_i = (\Delta t)\, f(x_i, t_i)$$

$$x_{i+1} = x_i + (\Delta t)\, f(x_i, t_i)$$

Review Euler Process

4. March in time starting from initial condition

$$x_1 = x(0)$$

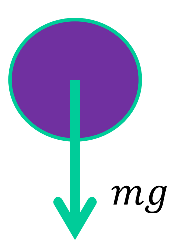
$$x_2 = x_1 + (\Delta t)\, f(x_1, t_1)$$

$$x_3 = x_2 + (\Delta t) f(x_2, t_2)$$

$$x_{i+1} = x_i + (\Delta t)\, f(x_i, t_i)$$

More Complicated Euler Example

Suppose we still have a ball falling:



$$\frac{dV}{dt} = -g \qquad V(0) = 0$$

$$\frac{dy}{dt} = V \qquad y(0) = 0$$

Note that we can still solve this one analytically. You will work with these equations in the exercises.

More Complicated Euler Example

Euler implementation:

$$\frac{dV}{dt} = -g \qquad V(0) = 0 \quad \Rightarrow \quad V_{i+1} = V_i - g\,(\Delta t) \qquad V_1 = 0$$

$$\frac{dy}{dt} = V \qquad y(0) = 0 \quad \Rightarrow \quad y_{i+1} = y_i + V_i(\Delta t) \qquad y_1 = 0$$

Note subscript!

Note that we can still solve this one analytically

Euler Convergence

- Euler gives you an approximate answer to the equations
- The smaller Δt is, the closer the answer is to the correct solution
- When you don't know the correct solution, just keep making Δt smaller until the answer doesn't change much anymore