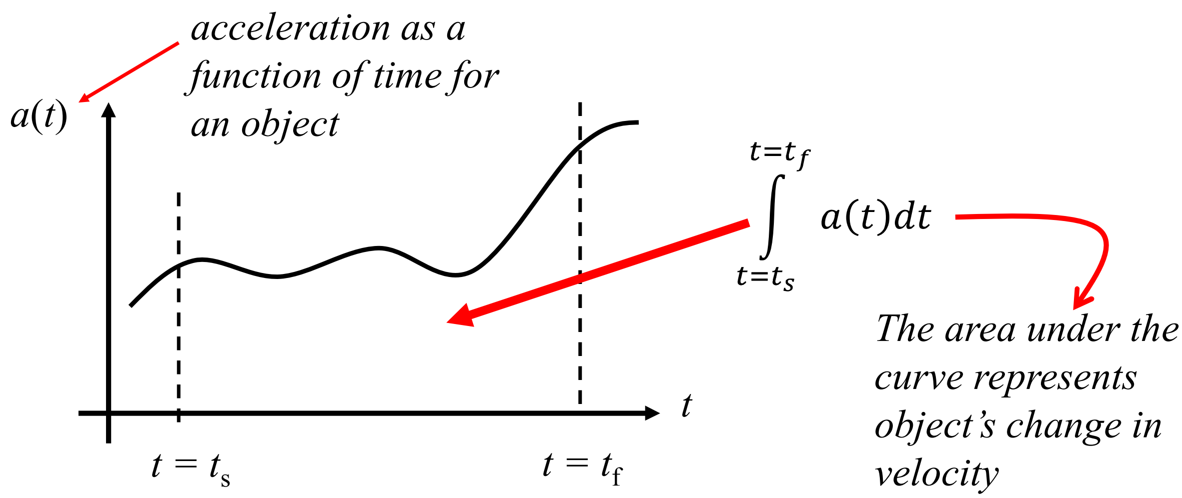
Day 6

- (Concept Question)
- Numerical integration
- Exercises

Numerical integration

- The integral of acceleration is velocity.
- The area under an acceleration curve is the change in velocity.



Numerical integration

Why would anyone find change in velocity by integrating the area under an acceleration curve?

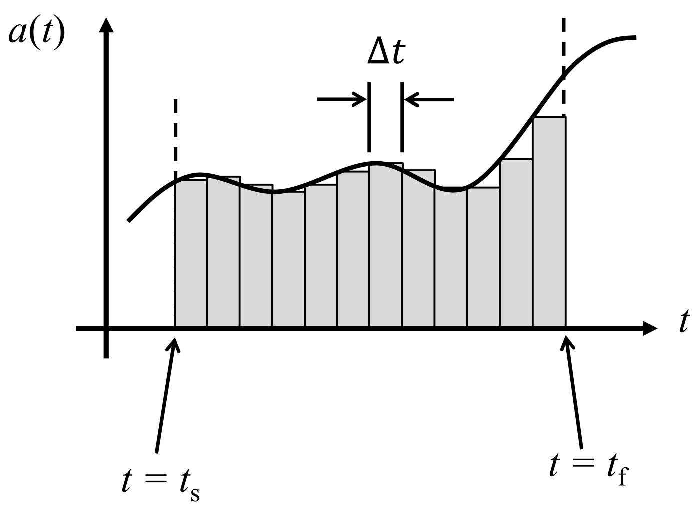
Missiles and rockets often use something called an "inertial guidance system."

This system uses accelerometers on the missile to measure acceleration and then integrates the acceleration to get velocity.

(It then also integrates velocity to get position.)

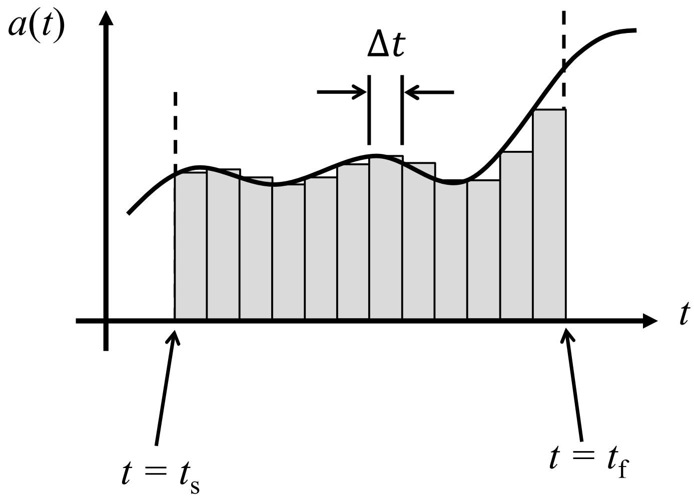
Numerical integration

We can approximate the integral of the acceleration curve by breaking it up into rectangles:



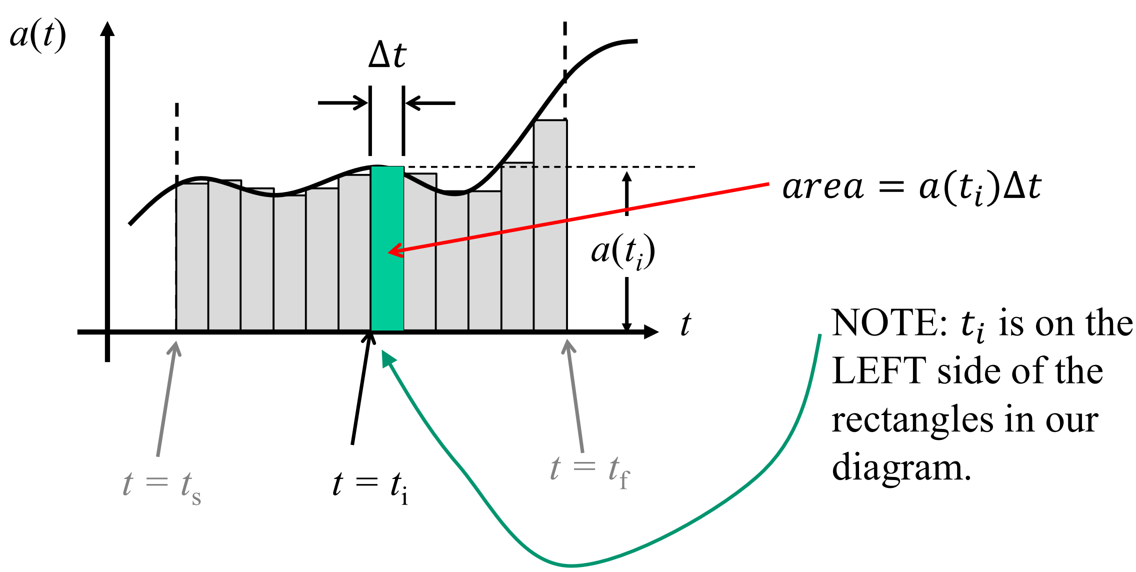
Numerical integration

The integral (area under the curve) is the sum of all of the small rectangles. This is “numerical integration”.



Numerical integration

For an arbitrary rectangle, we can write the area as



Numerical integration

Using the rectangles, the area under the curve can be approximated by a summation:

$$v(t) = \int_{t=t_s}^{t=t_f} a(t)dt \approx \sum_{i=1}^{N} a(t_i)\Delta t$$

Where

$t_i = t_s + (i-1)\Delta t$

$\Delta t = (t_f - t_s)/N$

(N is the number of rectangles used to slice up the interval between $t = t_s$ and $t = t_f$)

Numerical integration

Use For loops and recursive assignments to calculate the sum.

$$v(t) = \sum_{i=1}^{N} a(t_i)\Delta t$$

No new Matlab concepts– we are practicing what we learned yesterday on an engineering application.

Numerical integration

Note: Numerical integration is only approximate.

- Poor approximation for small N
- Better approximation for large N

Exercises

Exercise 1. Write a MATLAB program that approximates the area under this increasing acceleration curve:

$$a(t) = 0.2\exp(2.1t) \text{ m/s}^2$$

Integrate from $t = 0$ seconds to $t = 5$ seconds using 5 rectangles. Print your solution nicely (including units) to a file. (Ans: 1013.5 m/s)

Exercises

Exercise 2. Now make your program more general:

Set the number of rectangles as a variable near the beginning of your code. Integrate from t=0 seconds to t=5 seconds using 100 rectangles. Print your solution nicely (including units) to a file. (Ans: 3280.1 m/s)

Don't forget to change the value of Δt in your code so that the integral still goes from $t = 0$ seconds to $t = 5$ seconds. You can do this by writing a line in your code that calculates Δt in terms of N (the number of rectangles).

Exercises

Exercise 3. Since we chose a known function, we can figure out the analytical value of the change in velocity from t=0 to t=5 seconds, and it is 3458.5 m/s.

Keep changing N by factors of 10 in your script from Exercise 2 until the result matches the analytical value (to all 5 significant digits).

For this problem, just turn in your '.txt' file since the script is basically the same as for Exercise 2 and we have been killing a lot of trees.