Lab 5. First-order system identification

Determining the time constant of an RC circuit.

Agenda

Time estimates	Item
30 min	Determining the time constant using the log-incomplete response method
15 min	Organize for using the apparatuses
145 min	Lab tasks 1. Modeling 2. Acquiring experimental response data 3. Analysis 4. Reporting

Summary of learning objectives

By the end of the lab period, students should be able to:

1. Estimate the time constant of a first-order system using several methods.
2. Explain the methods and their differences.
3. Evaluate which method may be superior and explain why.

Summary of deliverables

The deliverables for this experiment consist of a completed lab worksheet, plus figures and a discussion described later in the handout. Your worksheet and figures will be included as attachments to a team memo that contains a discussion of your lab results.

Your team memo is due at the beginning of Lab 7.

Log-incomplete response

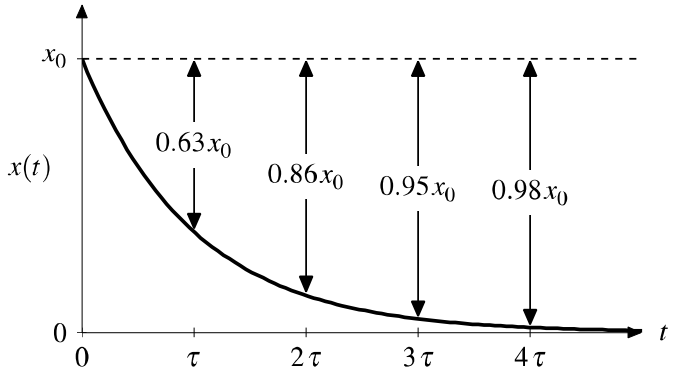
The standard form of a first-order differential with a step input of amplitude A is given by

$$\tau \frac{\mathrm{d}x}{\mathrm{d}t} + x = KA \tag{1}$$

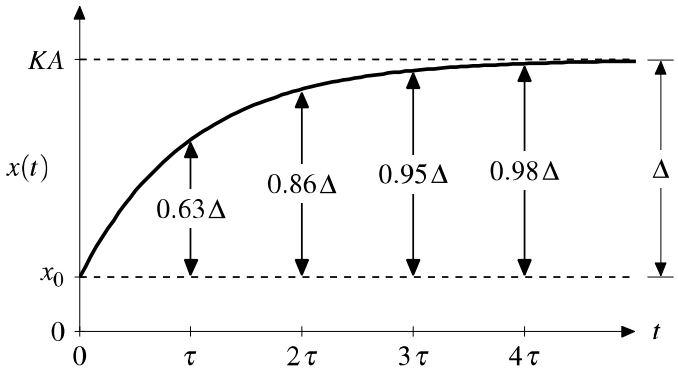
So far, what techniques do we have for finding the system parameters, in particular the time constant τ?

Method 1: Graphical inspection

We can deduce τ from plots of the free or step responses of a first-order system.



Free response



Step response

Method 2: Minimizing a standard error estimate (SEE)

As you did in Lab 5, you could start with a nominal value of τ from using Method 1 and then adjust this value until you minimize the SEE to achieve a better quantitative fit between response data and simulation results.

Method 3: Using the log-incomplete response

The solution to the first-order differential equation (1) with initial condition x_0 is

$$x(t) = x_{\mathrm{ss}} + (x_0 - x_{\mathrm{ss}})e^{-\frac{t}{\tau}} \tag{2}$$

where the steady-state value $x_{\mathrm{ss}} = KA$. Rearranging Eq. (2),

$$\frac{x(t) - x_{\mathrm{ss}}}{x_0 - x_{\mathrm{ss}}} = e^{-\frac{t}{\tau}} \tag{3}$$

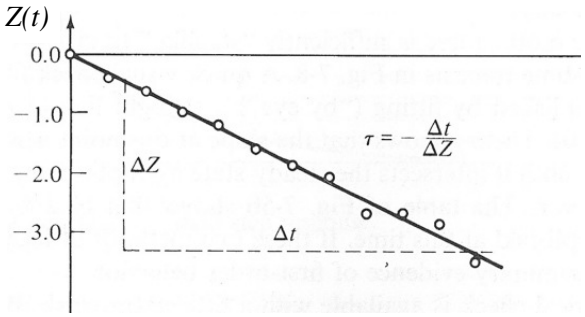
Taking the natural log of both sides of Eq. (3) gives

$$Z(t) = \ln\left(\frac{x(t) - x_{\mathrm{ss}}}{x_0 - x_{\mathrm{ss}}}\right) = -\frac{t}{\tau} \tag{4}$$

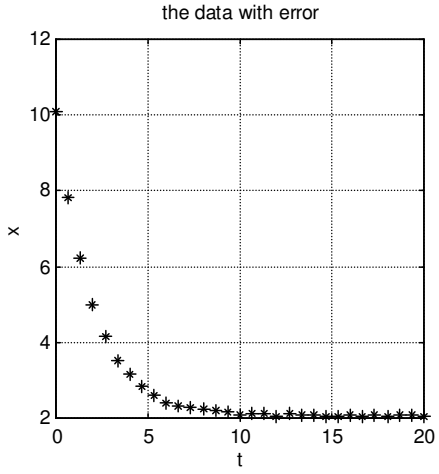
where $Z(t)$ is called the *logarithm of the incomplete response*, or the log-incomplete response. Note that in a plot of $Z(t)$ against time t, if the plotted line is straight, then the slope is given by $-1/\tau$.

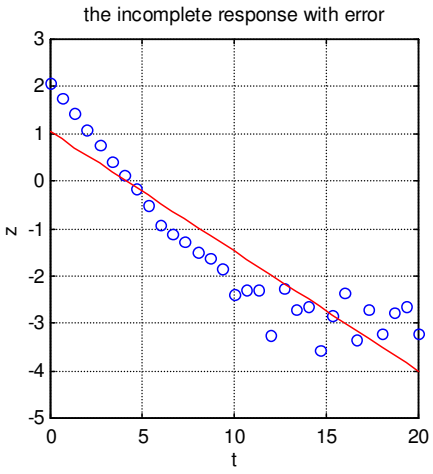
Given data that appears to be first-order, the log-incomplete response method may be used to determine the time constant τ through the following procedure:

1. Estimate the time constant from the 63% point on the free or step response plot from data.
2. Extract the data points up to approximately 4τ (more on this in a bit).
3. Calculate $Z(t)$.
4. Plot $Z(t)$ against time t. If the resulting plot is linear, then the system is likely first-order.
5. Perform a linear curve fit, forcing the line to go through the origin, if possible.
6. Determine the time constant τ from the slope of the linear curve fit.



So why only look at the data from $t = 0$ to $t = 4\tau$? As the data approaches the steady-state value, noise in the data masks the behavior of the response, making it difficult to obtain a reliable value for the time constant.





Exercise

For the step response data distributed to you in an Excel file (with time t in sec in the first column, and displacement x in cm in the second column),

1. Estimate the time constant from the 63% point.
2. Compute and plot the log-incomplete response $Z(t)$.
3. Plot a linear curve fit and determine the time constant.
4. Can you conclude that the system is first-order?

Introduction to the lab

Identification is a process in which experimental measurements are used to draw inferences about the characteristics of a system. In this experiment, a capacitor is subjected to a *step input* in voltage. The response is measured, and analysis yields an estimate of the system's time constant. The *time constant* is the system characteristic being *identified*.

Nomenclature

C	capacitance, F	V_0	initial capacitor voltage , V
R	resistance, Ω	V_{ss}	steady-state capacitor voltage, V
V	capacitor voltage, V	τ	system time constant, s

Experimental system

The system in this experiment is a resistor-capacitor (RC) circuit, as illustrated in Figure 1. The capacitor in the circuit is charged when it is connected to a power supply at point *A* and discharges when the power is turned off (*i.e.*, when it is connected to point *B*). To avoid inconsistencies in our data due to internal power supply dynamics, we will use a switch in the circuit to control the input voltage while the power supply is on. The voltage across the capacitor is the quantity we will measure and is the dependent variable in the mathematical model of the system.

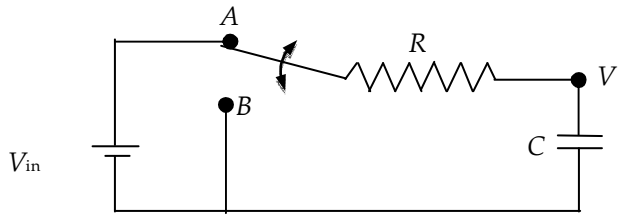


Figure 1: Resistor-capacitor circuit.

System modeling

To model the system, we assume that the conductors used are perfect and have no resistance or capacitance. The voltage across the capacitor is V.

> *Question 1.* Model the system (by hand – a pencil-and-paper derivation is fine). Express the model in standard first-order form. On the Lab 7 worksheet, write your final model in the space for equation (1). Include your modeling work as an attachment to the lab memo.

The physical system will be subjected to a step input in voltage.

> *Question 2*. Obtain a solution to the DE model using the Laplace transform approach with initial condition $V(0) = V_0 \neq 0$ and a step input of magnitude V_{in}. When you have the exact solution for $V(t)$, write it on the worksheet in the space for equation (2). Include your analysis as an attachment to the lab memo.

We will be using the log-incomplete-response function $Z(t)$ to determine the circuit's time constant.

> *Question 3*. Derive the expression for $Z(t)$ using your solution to Question 2. When you have the expression for $Z(t)$, write it on the worksheet in the space for equation (3). Include your analysis as an attachment to the lab memo.

Experimental procedure

The system is subjected to a step input in voltage by flipping the switch on or off, and the resulting voltage response across the capacitor is measured. To determine if the direction of the step has any effect on the system response (i.e., the time constant), the system is subjected to two step inputs in sequence: a step from low to high voltage (charging the capacitor), followed by a step from high to low voltage (discharging the capacitor). Theoretically, do you expect a difference?

First set up the circuit illustrated in Figure 1 on a breadboard. Every station has the same capacitor, but you are free to choose the resistance. Make note of these values. Terminal A of the switch is connected to the positive terminal of the power supply (you can choose the supplied voltage), while terminal B is connected to ground. Flipping the switch controls the step input of voltage to the circuit. If you need help setting up your circuit, feel free to ask me.

The capacitor voltage V is measured using a TDS 1002 Tektronix oscilloscope, as shown in Figure 2. The scope is set up for real-time viewing of the capacitor voltage and can be set to record both time and voltage onto an SD memory card. The time base of the scope should be adjusted to display an output that changes slowly enough to clearly show the capacitor charging and discharging in the display window.

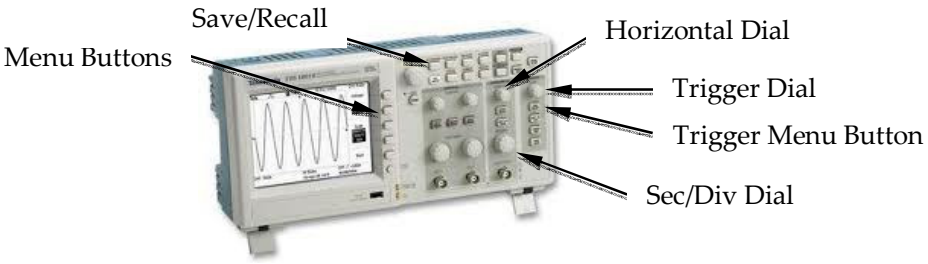


Figure 2: TDS 1002 Tektronix oscilloscope.

Use the following procedure to acquire and record the capacitor voltage data:

1. Press the Save/Recall button on the scope. Press the button next to "Action" until the "Save All" option is selected. Press the button next to "Print Button" until the "Saves All To Files" option is selected.
2. I highly recommend using the single sequence ("Single Seq") button to collect data because it will make it easier to capture the entire voltage response in the scope's display window. Instructions for recording data by this method are given below for charging and discharging the capacitor.

Charging the capacitor:

3. Move the arrow cursor at the top of the display window close to the left edge of the screen using the dial under "Horizontal". Also move the arrow cursor on the right edge of the window to just above the initial capacitor voltage using the dial under "Trigger". Your arrow cursor positions should be similar to what is shown in Figure 3. Press the "Trig Menu" button and make sure that "Slope" is set to the "Rising" option.

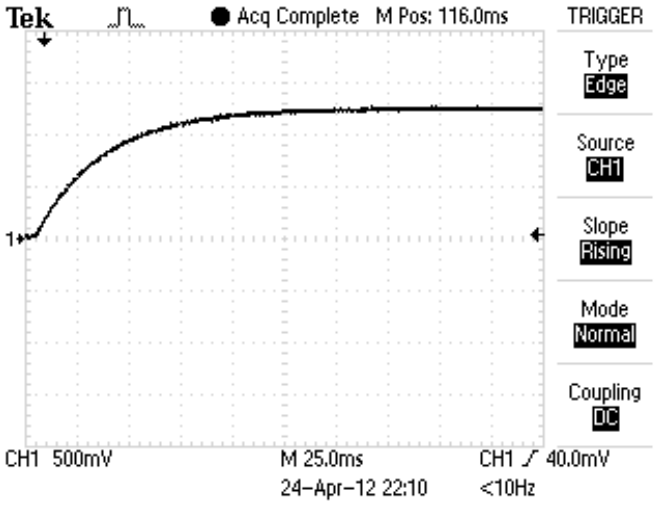


Figure 3: Acquiring data while charging the capacitor.

4. When you're ready to collect data, press the "Single Seq" button in the upper right corner of the scope, and then flip the circuit switch on. When data has been acquired, you should see the capacitor voltage response on the screen, with "Acq Complete" displayed at the top (see Figure 3). Press the "Print" button to save the data – be sure to note where the data is being saved! Once the data has been saved (it takes a few seconds), press the "Run/Stop" button until the screen is cleared.

Discharging the capacitor:

5. Keep the arrow cursor at the top of the display window close to the left edge of the screen, but move the arrow cursor on the right edge of the window to just below the initial capacitor voltage, similar to what is shown in Figure 4. Under the "Trigger" menu, press the button next to "Slope" until the "Falling" option is selected.

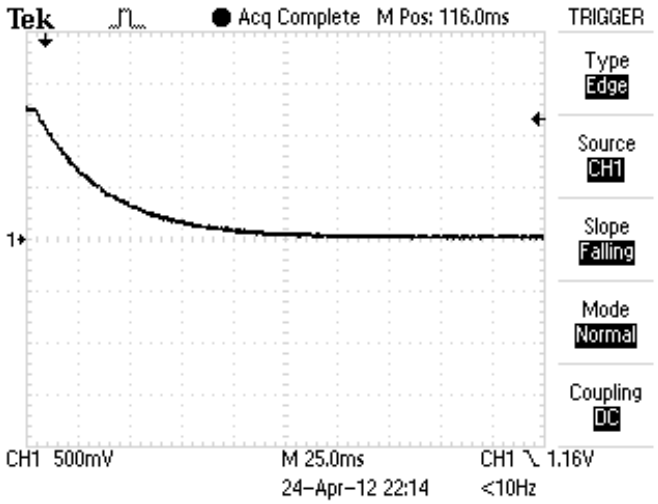


Figure 4: Acquiring data while discharging the capacitor.

6. When you're ready to collect data, press the "Single Seq" and flip the circuit switch off to discharge the capacitor. Once the voltage response is on the screen with confirmation that data has been acquired (see Figure 4), press the "Print" button to save the data, and then clear the display window.

Obtaining and retrieving data for multiple trials:

7. Repeat Steps 3 though 6 two more times so that you have a total of 3 complete trials each for charging and discharging the capacitor.
8. Once all data has been recorded, retrieve the memory card and use the USB interface provided to transfer the data from the memory card to your laptop for analysis. Once the files have been transferred, delete them from the memory card. The data files generated by the scope's data acquisition system are in comma-separated-value (csv) format.

If the modelers are finished before the experimentalists

If data for your team is not yet acquired, the modelers can begin setting their calculations by using *sample* data distributed in csv format. When your team's *experimental* data is ready, simply import it into MATLAB using the m-file you have made already.

Preliminary data reduction

In a professional experimental analysis, we would use the results from all three trials to allow the random uncertainty to be estimated. Today, to save time, you will use only one trial. Plot all three trials and select the one that you think best represents the response of your system. In all calculations to come, use just this one trial. Before launching into your detailed analysis, do the following:

1. For the capacitor charging data:
    a. Average the recorded data points prior to flipping the switch on to estimate the initial condition V_0.
    b. Estimate the steady-state voltage V_{ss} by choosing the *largest* voltage value toward the end of the charging data.
    c. Record V_0 and V_{ss} in the appropriate sections of the worksheet.
2. For the capacitor discharging data:
    a. Average the recorded data points prior to flipping the switch off to estimate the initial condition V_0.
    b. Estimate the steady-state voltage V_{ss} by choosing the *smallest* voltage value toward the end of the discharging data.
    c. Record V_0 and V_{ss} in the appropriate sections of the worksheet.
3. Delete data prior to the voltage input so that the first measurement is at the beginning of the step.
4. Adjust the measured time values so that the voltage response time starts at $t = 0$ s.

Detailed analysis

Method 1: Time constant from the charging step response plot

1. Plot the capacitor voltage as a function of time.
2. Estimate the time constant from the plot using the 63% point. Show your work on the graph (handwritten is fine). Record this time constant estimate in the lab worksheet. Repeat your analysis by using the 86% point (when $t = 2\tau$) and the 95% point (when $t = 3\tau$). Your best estimate of the time constant using this graphical method is the average of your three values.

Method 2: Time constant from the log-incomplete response plot for the charging capacitor

1. Recall that we only want to use data for $t \leq 4\tau$ when calculating the log-incomplete response $Z(t)$, so you will want to eliminate data for $t > 4\tau$.
2. Compute and plot $Z(t)$, and use your plot to estimate the time constant. Record this value on your worksheet.

Method 3: Time constant from the log-incomplete response plot for the discharging capacitor
Use Method 1 to estimate the time constant for the discharging capacitor voltage response, and then repeat the procedure in Method 2 to determine the time constant from the log-incomplete response.

Average the time constant values from the three methods to determine your best estimate of τ. Using this overall best estimate and your known capacitance and resistor resistance values, estimate the total resistance R_{total} (resistance and circuit resistance) and the unmodeled circuit resistance R_{circuit}. Record these values in your worksheet.

Reporting

In a team memo, discuss the differences and similarities among the values you obtained for the time constant. Refer to your figures to support your observations. The discussion should be thoughtful, thorough, and, wherever possible, quantitative. Explain the methods for finding the time constant and their differences. Evaluate which method may be superior and explain why. Use precise technical vocabulary in your discussion.

Attach your filled-in worksheet, properly formatted figures, handwritten derivations, and a copy of your m-files. Check that answers that are supposed to have units, do!

You should have a minimum of five figures. Arrange the figures in an organized way with potentially more than one figure on a page. For some of the following figures, it may be beneficial to use subplots. Don't forget to properly format your plots (axis labels, a figure caption, etc.)!

Figure 1: Experimental step response, $V_{\text{exp}}(t)$, for the charging capacitor showing the 1τ, 2τ, and 3τ estimates of the time constant.
Figure 2: Log-incomplete response $Z(t)$ for the charging capacitor with the linear curve fit. Indicate the slope of the curve fit.
Figure 3: Log-incomplete response $Z(t)$ for the discharging capacitor with the linear curve fit. Indicate the slope of the curve fit.
Figure 4: A plot comparing the experimental step response, $V_{\text{exp}}(t)$, to the modeled responses using the three time constants determine from the three analysis methods.
Figure 5: The curves in Figure 4 will look very similar. To highlight the differences, plot the *difference* between the experimental data and the three predicted response.