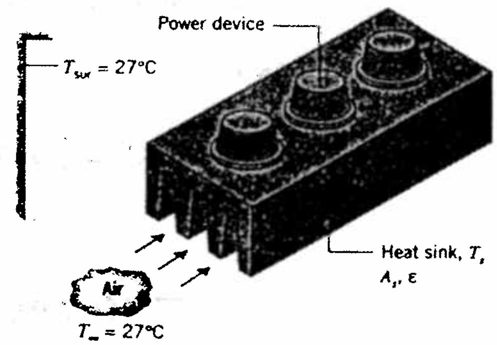
1.38 Electronic power devices are mounted to a heat sink having an exposed surface area of 0.045 m² and an emissivity of 0.80. When the devices dissipate a total power of 20 W and the air and surroundings are at 27°C, the average sink temperature is 42°C. What average temperature will the heat sink reach when the devices dissipate 30 W for the same environmental condition?



<u>ASSUME</u>: • STEADY-STATE

• SMALL OBJECT IN BIG ROOM (FOR RAD. EQN.)

① DEFINE SYSTEM: HEAT SINK

② CONS. OF ENERGY

$$\cancel{\frac{dU}{dt}}^{\,0\ \text{(steady state)}} = \dot{Q} + \dot{W} + \cancel{\text{mass flow terms}}^{\,0\ \text{(no mass flows)}}$$

③ $$0 = -\dot{Q}_{rad} - \dot{Q}_{con} + \dot{W}_{pd}$$

$$\dot{Q}_{rad} = A_s \epsilon \sigma (T_s^4 - T_{surr}^4)$$

$$\dot{Q}_{conv} = h A_s (T_s - T_\infty)$$

$\dot{W}_{pd}$ – power dissipated by power devices

<u>Case 1</u>: $T_s = 42°C + 273 = 315\ K$

$T_{sur} = T_\infty = 27°C + 273 = 300\ K$

$\dot{W}_{pd} = 20\ W$

$$0 = \dot{W}_{pd} - h A_s (T_s - T_\infty) - A_s \epsilon \sigma (T_s^4 - T_{sur}^4)$$

solve for h

$$h = \frac{1}{A_s (T_s - T_\infty)} \left[\dot{W}_{pd} - A_s \epsilon \sigma (T_s^4 - T_{sur}^4) \right]$$

$$h = \frac{1}{(0.045\ m^2)(315-300)K} \left\{ 20\ W - (0.045\ m^2)(0.8)\left(5.67\times10^{-8} \frac{W}{m^2 K^4}\right)(315^4 - 300^4)\ K^4 \right.$$

$$\boxed{h \simeq 24.35\ \frac{W}{m^2 \cdot K}}$$

CASE 2: Assume same surroundings and heat transfer coefficients when $\dot{W}_{pd} = 30\,W$.

Then

$$0 = 30\,W - \left(24.35\,\frac{W}{m^2 K}\right)\left(0.045\,m^2\right)\left(T_s - 300\right)K$$

$$- \left(0.045\,m^2\right)(0.8)\left(5.67\times10^{-8}\,\frac{W}{m^2 \cdot K^4}\right)\left(T_s^4 - 300^4\right)K^4$$

$$30 = 1.098\left(T_s - 300\right) + 2.041\times10^{-9}\left(T_s^4 - 300^4\right)$$

Solve using maple (see next page)

$$T \simeq 322.4K - 273 = 49.4^\circ C$$

$$\boxed{T = 49.4^\circ C}$$

How much $\dot{Q}$ was convection vs. radiation?

$$\dot{Q}_{conv} = 1.098\left(322.4 - 300\right) \simeq \boxed{24.5\,W}$$

$$\dot{Q}_{rad} = 2.041\times10^{-9}\left(322.4^4 - 300^4\right) = \boxed{5.5\,W}$$

Case 1:

```
> h:=1/(0.045*(315-300))*(20-0.045*0.8*5.67e-8*(315^4-300^4));
```

Case 2:

$$h := 24.35093333$$

```
> qconv:=h*0.045*(T-300);
```

$$qconv := 1.095792000\,T - 328.7376000$$

```
> qrad:=0.045*0.8*5.67e-8*(T^4-300^4);
```

$$qrad := 0.204120\;10^{-8}\,T^4 - 16.53372000$$

```
> eq1:=0=30-qconv-qrad;
```

$$eq1 := 0 = 375.2713200 - 1.095792000\,T - 0.204120\;10^{-8}\,T^4$$

```
> sol1:=solve(eq1,T);
```

no imaginary T values   no T<0   no imaginary T values

$$sol1 := 322.3525903,\ \cancel{291.0936870 + 738.7742580\,I},\ \cancel{-904.5399643},\ \cancel{291.0936870 - 738.7742580\,I}$$

```
> Tnum:=sol1[1];
```

$$\boxed{Tnum := 322.3525903 \text{ K}}$$

```
> subs(T=Tnum,qconv);
```

$$\boxed{24.4937896 \text{ W}} = \dot{Q}_{conv}$$

```
> subs(T=Tnum,qrad);
```

$$\boxed{5.50621032 \text{ W}} = \dot{Q}_{rad}$$

```
>
```