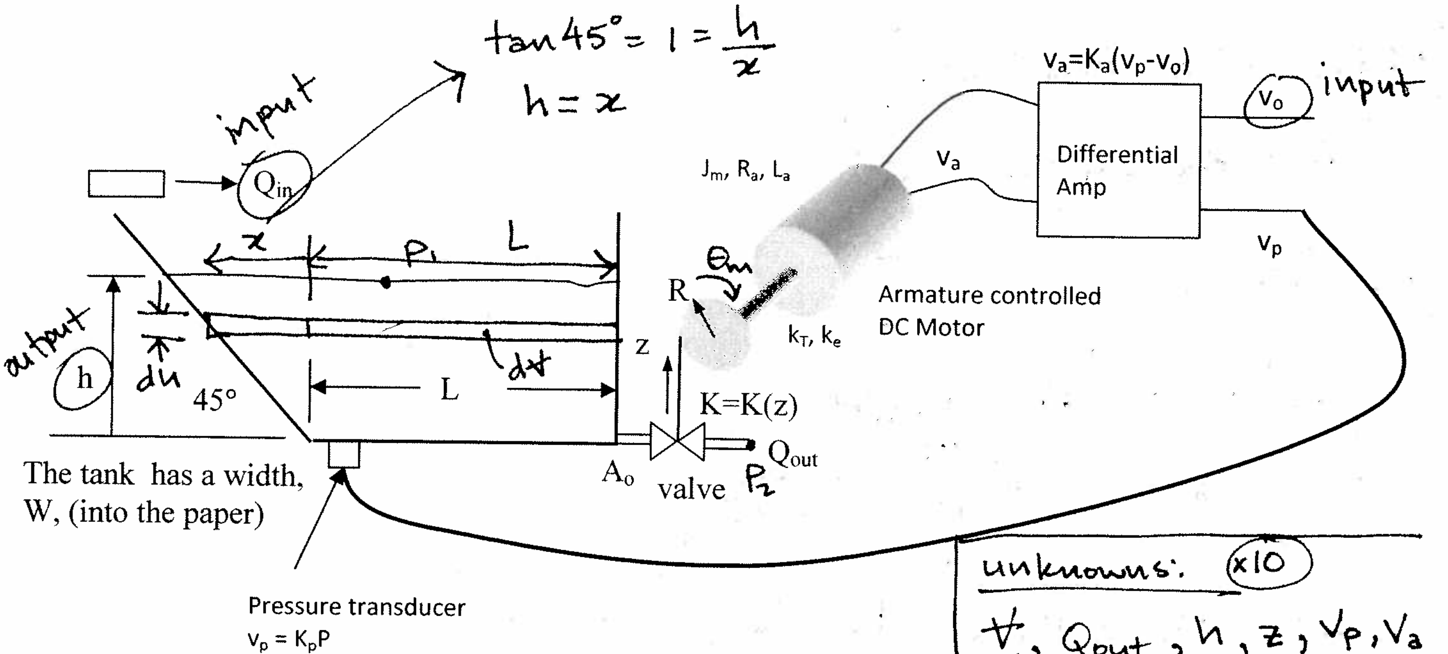
DAY 16 - IN-CLASS EXAMPLE

Consider the liquid level-control system shown below. The tank is to be maintained with a fluid level equal to H_o. At this height of fluid $v_p = v_0$. When fluid is added to the tank, Q_{in}, the pressure transducer voltage is greater than the input voltage, v_0, and the DC motor will open the valve to let out water. Assume the valve has a loss coefficient that is a function of the valve opening, z. You can neglect major losses in the short section of pipe. If the fluid level drops below H_o, the motor will close the valve to reduce the flow out of the tank. Determine the equations that govern the behavior of this system. Clearly label your equations and keep track of your unknowns.



unknowns: (x10)

$\forall$, Q_{out}, h, z, V_p, V_a, i_a, e_b, T, θ_m

- **Tank – Cons. of Mass**

$$\frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

$\rho = \text{const.}$

$$\frac{d(\rho \forall)}{dt} = \rho Q_{in} - \rho Q_{out}$$

$$(1) \quad \frac{d\forall}{dt} = Q_{in} - Q_{out}$$

- **VALVE – MODIFIED BERNOULLI**

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2 + g h_L$$

$P_1 = P_2 = P_{atm}$

$V_1 \ll V_2$, $V_1 \simeq 0$

$z_1 = h$, $z_2 = 0$

$$g h_L = K(z) \frac{V_2^2}{2}$$

$$g h = \frac{V_2^2}{2} + K(z) \frac{V_2^2}{2}$$

$$= \frac{V_2^2}{2} (1 + K(z))$$

$$V_2 = \frac{1}{\sqrt{1 + K(z)}} \sqrt{2gh}$$

$$Q_{out} = A_o V_2$$

$$Q_{out} = \frac{A_o}{\sqrt{1 + K(z)}} \sqrt{2gh} \quad (2)$$

- $d\forall/dt$ – differential volume change

$$\frac{d\forall}{dt} = LW \cdot \frac{dh}{dt} + Lx \frac{dh}{dt}$$

↳

$$\frac{d\forall}{dt} = (Lw + hw)\frac{dh}{dt} \quad (3)$$

- Pressure transducer

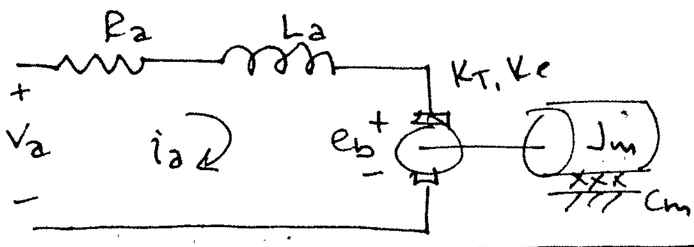
$$V_p = K_p \cdot P = K_p(P_{atm} + \rho g h)$$

$$V_p = K_p(P_{atm} + \rho g h) \quad (4)$$

- Differential Amplifier

$$V_a = K_a(V_p - V_o) \quad (5)$$

- DC Motor

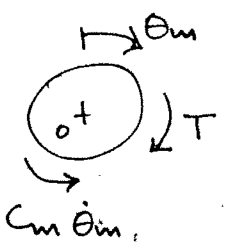


Loop: $-V_a + R_a i_a + L_a \frac{di_a}{dt} + e_b = 0 \quad (6)$

Back EMF: $e_b = K_e \dot{\theta}_m \quad (7)$

Torque: $T = K_T i_a \quad (8)$

- MOTOR ARMATURE



- Am Rate

$$\frac{d\vec{L}_{sys}}{dt} = \sum \vec{M}_o$$

$$J_m \ddot{\theta}_m = T - C_m \dot{\theta}_m \quad (9)$$

- Kinematics

$$R\theta_m = z \quad (10)$$

10 unknowns – 10 equs. ✓