

$$(a) \quad L_a \frac{di_a}{dt} + R_a i_a + K_e \omega = e_a(t)$$

$$J \frac{d\omega}{dt} + b\omega = K_t i_a$$

$$(b) \quad \frac{\Omega}{E_a}(s) = \frac{K_t}{JL_a s^2 + (bL_a + JR_a)s + K_t K_e + bR_a}$$

$$\frac{I_a}{E_a}(s) = \frac{Js + b}{JL_a s^2 + (bL_a + JR_a)s + K_t K_e + bR_a}$$

$$(c) \quad \omega_n = \sqrt{\frac{K_t K_e + bR_a}{JL_a}}$$
$$\zeta = \frac{bL_a + JR_a}{2\sqrt{JL_a(K_t K_e + bR_a)}}$$

$$\frac{\Omega}{E_a} \text{ static gain } G_1 = \frac{K_t}{K_t K_e + bR_a}$$

$$\frac{I_a}{E_a} \text{ static gain } G_2 = \frac{b}{K_t K_e + bR_a}$$