

(b) The DE for disk  $J_1$  is given by

$$J_1 \ddot{\theta}_1 + c \dot{\theta}_1 - c \dot{\theta}_2 + (k_1 + k_2) \theta_1 - k_2 \theta_2 = T_1(t).$$

(c) The matrices of the 2<sup>nd</sup>-order form:  $\mathbf{M} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}$ ,  $\mathbf{C} = \begin{bmatrix} c & -c \\ -c & c \end{bmatrix}$ ,  $\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$ .

(d) 
$$\frac{\theta_2}{T_2}(s) = \frac{J_1 s^2 + cs + k_1 + k_2}{J_1 J_2 s^4 + (J_1 + J_2) cs^3 + [k_2 J_1 + (k_1 + k_2) J_2] s^2 + k_1 cs + k_1 k_2}.$$

(e) One possible solution is given by

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -(k_1 + k_2)/J_1 & k_2/J_1 & -c/J_1 & c/J_1 \\ k_2/J_2 & -k_2/J_2 & c/J_2 & -c/J_2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/J_1 & 0 \\ 0 & 1/J_2 \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

The exact arrangement of these elements depends on your selection of state variables.