Lesson 12

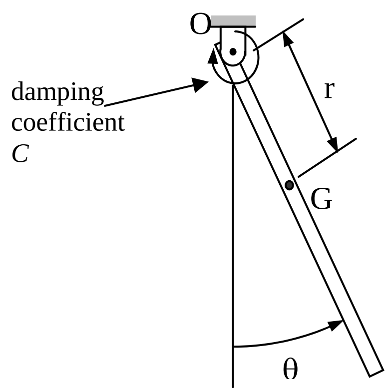
Problem 12.1

A sled weighing 64 lb, starting from rest, is pulled across a frozen pond by a force P = 10 lb. Coefficients of static and sliding friction between the sled and the ice are negligible. The air-resistance force (not negligible) is proportional to the velocity, v, and the proportionality constant, C, is 2 lb/ft/s.

a) Draw the FBD and obtain the DE model of the system.
b) Solve the model analytically (by hand) using the classical (homogeneous and particular) solution technique. Verify your solution using Maple and show your Maple result.
c) Plot the analytical solution $v(t)$ using a MATLAB file (not Simulink). Select a time vector such that the steady-state value is obtained.
d) From the graph of $v(t)$, estimate the *velocity* at t = 1 s, at 5 s, and at terminal velocity. Indicate on the graph how these quantities are found. Compare your three estimates from the graph to exact solutions obtained using the function $v(t)$.
e) From the graph of $v(t)$, estimate the *acceleration* at t = 1 s, at 5 s, and at terminal velocity. Indicate on the graph how these quantities are found. Compare your three estimates from the graph to exact solutions obtained using the function $v(t)$. (Used how? Explain.)

Problem 12.2

Assume the bar shown below has a mass m, mass moment of inertia about point O, I_o, and a center of gravity located a distance r from the hinge.



a) Show that the equation of motion (EOM) is

$$I_o\ddot{\theta} + C\dot{\theta} + mgr\sin\theta = 0\,, \qquad (1)$$

and that for small angles, the EOM is approximately given by:

$$I_o\ddot{\theta} + C\dot{\theta} + mgr\theta = 0\,. \qquad (2)$$

b) *Comparing linear and nonlinear numerical solutions*. Implement a simulation model in Simulink to solve both (1) and (2). Run simulations for I_o = 2 kg m^2, mgr = 30 N-m, C = 2, and using initial conditions:

case 1: $\theta(0)$ = 5°, $\dot{\theta}(0) = 0$	case 4: $\theta(0)$ = 90°, $\dot{\theta}(0) = 0$
case 2: $\theta(0)$ = 15°, $\dot{\theta}(0) = 0$	case 5: $\theta(0)$ = 5°, $\dot{\theta}(0) = 7.8$ rad/s
case 3: $\theta(0)$ = 40°, $\dot{\theta}(0) = 0$	*Note: convert degrees to radians.*

These runs can all be made at once if you use a vector for the initial conditions in the Simulink integration blocks. Output your results to the MATLAB workspace and plot each case comparing the linear and the non-linear responses in the same subplot. Use a 5×1 array of subplots so that all five cases are on one figure. Discuss your comparisons and the small angle assumption.

c) *Comparing linear analytical and numerical solutions*. For the linear (small-angle assumption) model, determine the analytical solution for Case 1. You can start with the known analytical forms for systems of this type. Apply the ICs to obtain the final solution, then use MATLAB to plot this function in a new figure (use a line). In the same figure plot the Case 1 linear numerical solution from part (b) using a symbol but no line. Comment on the comparison between the two solutions.