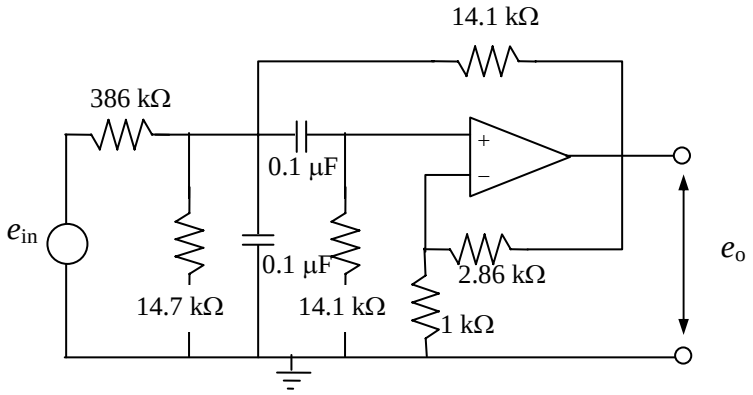
Lesson 07

Problem 7.1

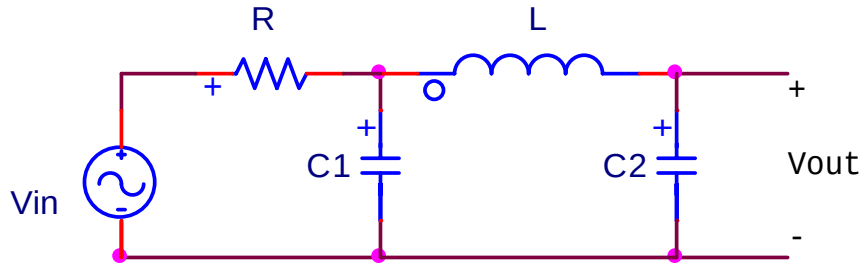
Sallen and Key developed many circuits based on positive feedback including low-pass, high-pass and band-pass filters. For the second-order band-pass filter shown below:



Determine the transfer function relating e_{in} to e_o.

Problem 7.2

For the third-order Butterworth low pass filter circuit below:



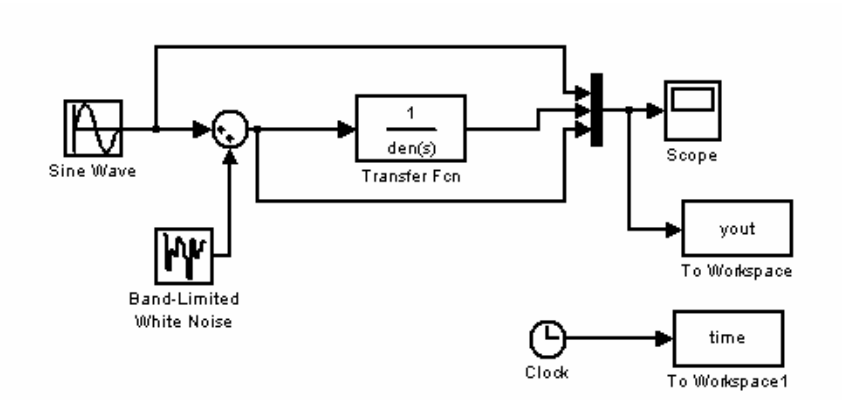
a) Write the two nodal equations needed to solve for the transfer function between V_{in} and V_{out}.
b) Solve the nodal equations (using appropriate software if you want) to show that the transfer function between V_{in} and V_{out} is given by

$$TF = \frac{1}{(RLC_1C_2)s^3 + (LC_2)s^2 + (RC_1 + RC_2)s + 1}$$

c) The TF in part (b) is more commonly written as follows, where ω_c is the *cutoff frequency* of the filter. Set the cutoff frequency ω_c = 5 rad/sec and determine the transfer function.

$$TF = \frac{1}{\left(\frac{s}{\omega_c}\right)^3 + 2\left(\frac{s}{\omega_c}\right)^2 + 2\left(\frac{s}{\omega_c}\right) + 1}$$

d) Implement this transfer function in the following simulation diagram in Simulink.

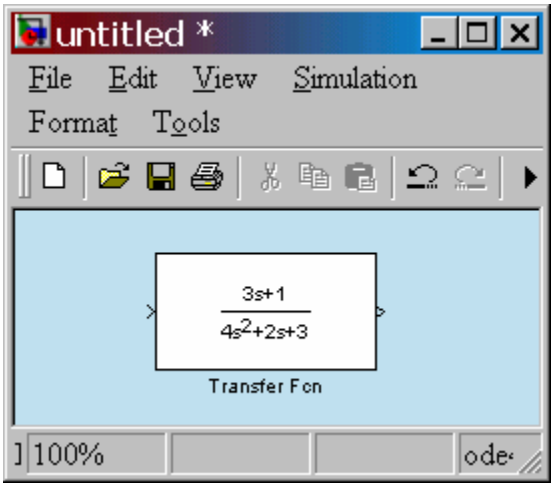


Be sure to indicate that the *To Workspace* variables are output as arrays (in the block parameters), rather than as a structure. In the transfer function parameter block, set the numerator to [1], and the denominator to [0.008 0.08 0.4 1] to implement the transfer function. Be sure to include the brackets [] ! (See the example on the last page.)

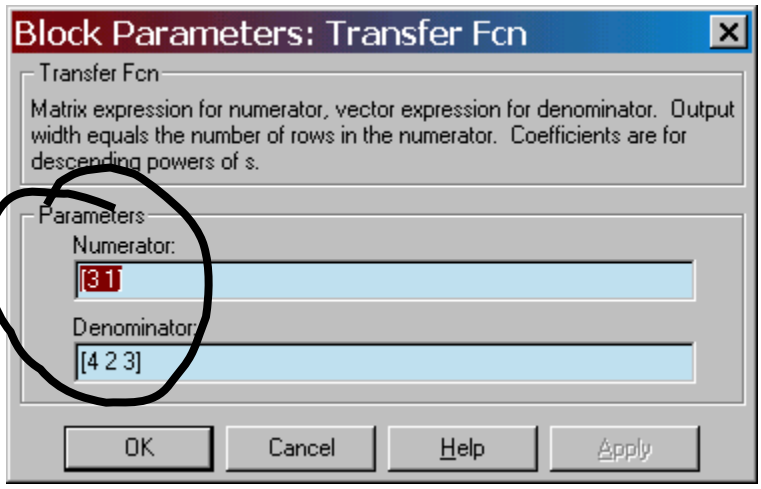
e) Set the band-limited white noise power (in the mask parameters) to zero, and the sample rate to 0.05. (We will not use the noise in this part.) In Matlab, write a script that will generate and plot the output of this circuit when the input is sin(ωt), where ω = 1, 3, 10, and 15 rad/sec. Use the *subplot* command to plot all four plots on one page. Each subplot should have a title and show both the input sine wave and the output sine wave (two plots on the same figure). Your plots should show that as the frequencies of the input sine wave get higher, the magnitude of the of the output sine wave decreases. This is what is meant by a "low pass" filter, i.e., it lets signals with "low" frequencies pass and attenuates signals with "high" frequencies. You should observe that the filter introduces a phase change between the input and the output.

f) Low-pass filters are often used to "smooth out" noisy signals, since noisy signals have a lot of high frequency content. To show this, first set the input sine wave frequency to 3 rad/sec and keep it fixed at this frequency for all of this part. From Matlab write a script that will generate and plot the output of this circuit when the input is a noisy sine wave, with noise power 0.0 (no noise), 0.001, 0.01 and 0.1 (with the noise sample rate at 0.05). Use the subplot command to plot all four plots on one page. Each subplot should show both the input sine wave (getting progressively noisier) and the output (smoothed) sine wave.

Simulink Notes



This is a transfer function block, found under the *Continuous* library.



Transfer functions in Simulink are denoted as a ratio of polynomial coefficients. Thus the numerator of the transfer function shown in the window is denoted [3 1] and the denominator is denoted [4 2 3]. You set these values by double-clicking on the transfer function icon in the model.