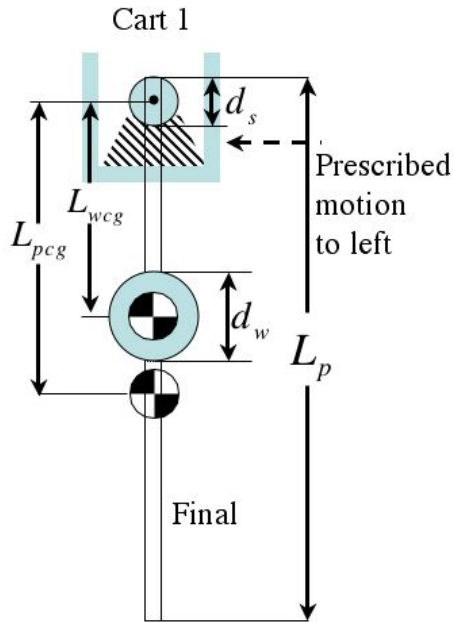


## Problem CP-1 – Analysis of an Overhead Crane



The illustrated pendulum/cart system is used to model the motion of an overhead crane. If the cart is given a specified acceleration profile

$$b(t) = \frac{3597e^x(1 - e^x)}{(1 + e^x)^3} + \frac{3597e^y(1 - e^y)}{(1 + e^y)^3} \text{ cm/s}^2$$

where

$$x = -43.5668t + 9.38$$

$$y = -43.5668t + 31.1633$$

and  $0 \text{ s} < t < 0.9 \text{ s}$ , determine the best location of the moveable weight's mass center  $L_{wcg}$  such that the pendulum's angular velocity is zero at  $t = 0.9 \text{ s}$ . Note that there will be an unknown force acting in the direction of the cart motion so that the cart experiences the specified acceleration.

**Figure 1: Crane system schematic.**

Values for the various system parameters are provided in Table 1.

**Table 1: Crane system parameters and their values.**

Parameter	Value	Units
Pendulum mass, $m_p$	68.5	g
Moveable weight's mass, $m_{add}$	88	g
Pendulum length, $L_p$	43.2	cm
Sensor diameter, $d_s$	2.5	cm
Moveable weight's diameter, $d_w$	5	cm

### Notes:

- Draw the system in a displaced orientation to obtain the governing differential equation. (This is the most important part of the analysis!).
- Write your resulting non-linear differential equation in the following format:

$$[\text{something}]\theta + [\text{something else}] \sin(\theta) = [\text{something different}]b(t)\cos(\theta)$$

When implementing this equation in Maple, it should look something like this:

**diff\_eq:=(IGp+IGw+mp\*LGp^2+mw\*Lw^2)\*diff(theta(t),t\$2)+(mp\*LGp+mw\*Lw)\*g\*sin(theta(t))= ...**  
**(right-hand side missing on purpose – we don't want to give you the whole answer!)**

- Maple can numerically solve the non-linear differential equation of motion using the following syntax:  
 $\text{> soln} := \text{dsolve}(\{\text{diff\_eq}, \text{theta}(0) = 0, \text{D}(\text{theta})(0) = 0\}, \text{theta}(t), \text{numeric});$

To plot your solution, use the syntax

$\text{> odeplot}(\text{soln}, [t, \text{D}(\text{theta})(t)], 0..2, \text{numpoints} = 300);$

Be sure to include  $\text{> with}(\text{plots})$ : at the beginning of your Maple worksheet. Once your Maple worksheet is working correctly for a particular value of  $L_{wcg}$ , you can then vary  $L_{wcg}$  to find the location where the angular velocity is zero at the end of the acceleration at  $t = 0.9 \text{ s}$ .