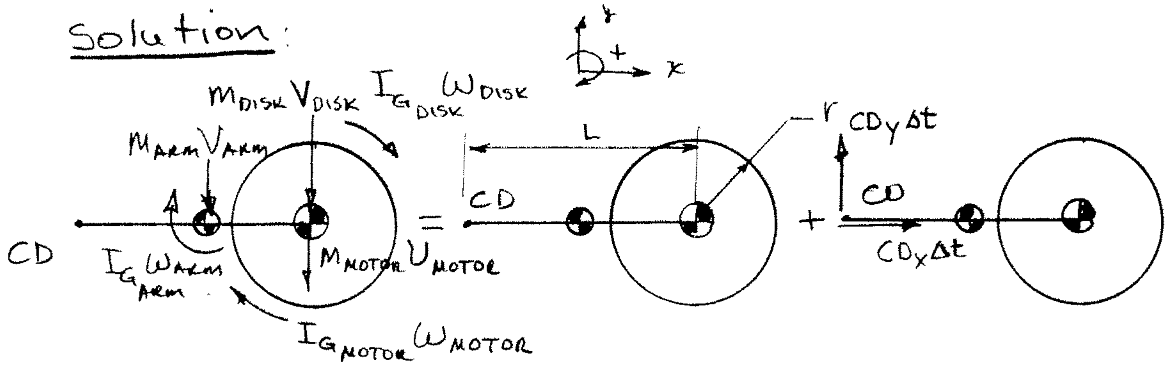
System: Arm/Motor/Disk

Strategy: COAM Finite, Rel. Vel., FAR Kinematics

Solution:



Final Momenta about CD = Intitial Momenta about CD + Impulses During about CD

$$M_{ARM}v_{ARM}\frac{L}{2} + I_{G_{ARM}}\omega_{ARM} + M_{MOTOR}v_{MOTOR}L + I_{G_{MOTOR}}\omega_{MOTOR}$$

$$+ M_{DISK}v_{DISK}L + I_{G_{DISK}}\omega_{DISK} = 0 + 0 \quad (1)$$

v_{ARM}, ω_{ARM}, v_{DISK}, ω_{DISK}, $v_{MOTOR}, \omega_{MOTOR}$

Rel. velocity ⇒ The velocites in eqn (1) are all absolute velocities. The normal operating speed of the motor is 360 RPM =

$\cancel{360}^{6}\frac{Rev}{min}\cdot\frac{2\pi\, rad}{Rev}\cdot\frac{1\, min}{\cancel{60}\, sec} = 12\pi$ rad/sec, So,

$$\omega_{MOTOR} = \omega_{ARM} + \omega_{MOTOR/ARM},$$ Now $\omega_{MOTOR/ARM} = 12\pi$ rad/s at normal op. speed.

$$\therefore \omega_{MOTOR} = \omega_{ARM} + 12\pi \quad (2)$$

Since the motor shaft is attached to the disk

$$\omega_{MOTOR} = \omega_{DISK} \quad (3)$$

From FAR Kinematics:

$$v_{DISK} = \omega_{ARM}L \quad (4) \qquad v_{MOTOR} = \omega_{ARM}L \quad (5)$$

(Note $v_{DISK} = v_{MOTOR}$)

$$v_{ARM} = \omega_{ARM}\frac{L}{2} \quad (6)$$

6 eqns / 6 unknowns

Sub (2) through (6) into eqn (1) to get a $f(\omega_{ARM})$

$$M_{ARM}\left(\frac{L}{2}\right)^2\omega_{ARM} + I_{G_{ARM}}\omega_{ARM} + M_{MOTOR}\,\omega_{ARM}L^2 + I_{G_{MOTOR}}(\omega_{ARM}+12\pi)$$

$$+ M_{DISK}\,\omega_{ARM}L^2 + I_{G_{DISK}}(\omega_{ARM}+12\pi) = 0$$

Solving for ω_{ARM}

$$\omega_{ARM}\left[\underbrace{M_{ARM}\left(\frac{L}{2}\right)^2 + I_{G_{ARM}}}_{} + \underbrace{M_{MOTOR}(L)^2 + I_{G_{MOTOR}}}_{} + M_{DISK}L^2 + I_{G_{DISK}}\right] =$$

$$-12\pi\left(I_{G_{MOTOR}} + I_{G_{DISK}}\right) \qquad (7)$$

Now $\quad I_{CD_{ARM}} = M_{ARM}\left(\frac{L}{2}\right)^2 + I_{G_{ARM}}$ from the parallel axis thm.

$$I_{CD_{MOTOR}} = M_{MOTOR}(L)^2 + I_{G_{MOTOR}}$$

and $\quad I_{CD_{ARM+MOTOR}} = I_{CD_{ARM}} + I_{CD_{MOTOR}}$

$$I_{G_{DISK}} = \frac{1}{2}M_{DISK}\,r^2$$

substituting these MOI's into (7)

$$\omega_{ARM} = -\frac{12\pi\left(I_{G_{MOTOR}} + \frac{1}{2}M_{DISK}r^2\right)}{I_{CD_{ARM+MOTOR}} + M_{DISK}L^2 + \frac{1}{2}M_{DISK}r^2}$$

If we assume $I_{G_{MOTOR}} = 0$ b/c of pt. mass assumption

then, $$\omega_{ARM} = -\frac{12\pi\left(0 + \frac{1}{2}\left(\frac{10}{32.2}\right)\left(\frac{5}{12}\right)^2\right)}{0.032 + \underbrace{\frac{10}{32.2}\left(\frac{6}{12}\right)^2}_{0.07764} + \underbrace{\frac{1}{2}\left(\frac{10}{32.2}\right)\left(\frac{5}{12}\right)^2}_{0.026958}} = \frac{-1.016302}{0.136598}$$

$\omega_{ARM} = -7.4401$ rad/s $= -71.048$ RPM , From (2) & (3)

$\omega_{DISK} = -71.048 + 360 = 288.95$ RPM

$$\therefore \boxed{\bar{\omega}_{ARM} = 71 \text{ RPM} \circlearrowleft \; ; \quad \bar{\omega}_{DISK} = 289 \text{ RPM} \circlearrowright}$$

if $I_{G_{MOTOR}} \neq$ zero, then ω_{ARM} will $\uparrow$ and ω_{DISK} $\downarrow$.