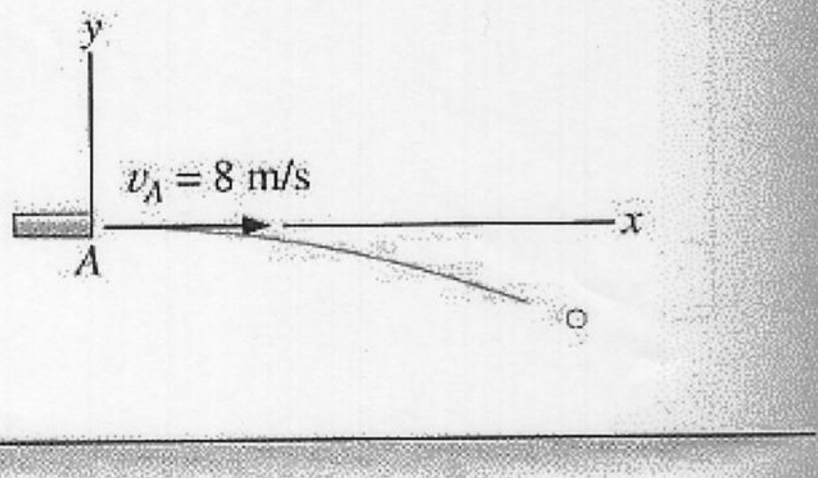
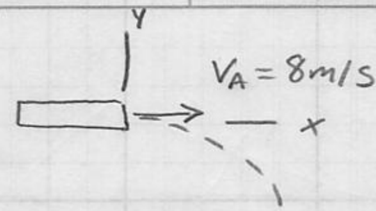
12-122. The ball is ejected horizontally from the tube with a speed of 8 m/s. Find the equation of the path, $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = 0.25$ s.



Prob. 12–122

PR 12-122



GIVEN:

REQUIRED: $y = f(x)$

SOLUTION: $x = 8t$

$$y = -\tfrac{1}{2} g t^2 = -4.905 t^2$$

$$\underline{\underline{y}} = -4.905 \left(\frac{t}{8}\right)^2 = \underline{\underline{-0.0766 x^2}}$$

$$\vec{v} = 8\hat{\imath} - 9.81t\ \hat{\jmath}$$

$$\vec{v}(0.25) = 8\hat{\imath} - \tfrac{1}{4} g\ \hat{\jmath} \qquad \underline{\underline{v(0.25)}} = \underline{\underline{8.37\ m/s}}$$

$$a(0.25) = -9.81\hat{\jmath}$$

$$a_t = \vec{a} \cdot \frac{\vec{v}}{v} \qquad a_n^2 = a^2 - a_t^2$$

$$= 9.81^2 - 2.875^2$$

$$\underline{\underline{a_t = 2.875\ m/s^2}} \qquad \underline{\underline{a_n = 9.38\ m/s^2}}$$

$$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n}$$

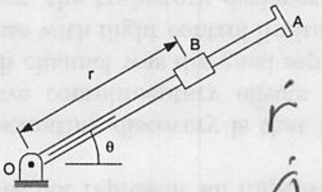
$$\rho = \frac{8.37^2}{9.38} = 7.469\,m$$

Name 30 pts

ES204 Examination I **Problem 1** Dec. 18, 1998

1.1) The rod OA shown in the figure is rotating in the x-y plane such that at any instant $\theta = t$ rad., where t is time measured in seconds. At the same time, Collar B is sliding outward along OA so that $r = 110\ t$ mm. (Again t is time measured in seconds). Determine the magnitude of the collar velocity when $t = 1$ s.

(a) 110 mm/s (b) 155 mm/s (c) 220 mm/s (d) 12,100 mm/s (e) none of these



$$\cancel{\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{d\,110t^2}{dt} = 220t}$$

$r = 110$

$\dot{\theta} = 1$

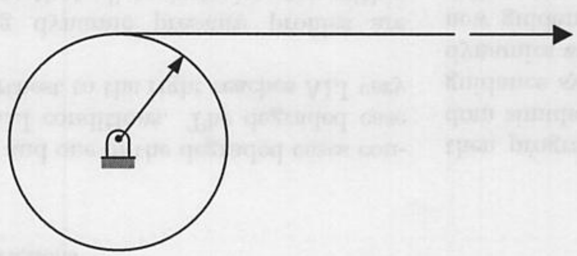
$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$= 110\hat{e}_r + (110)(1)e_\theta$

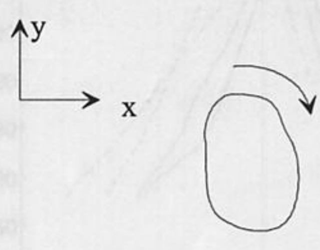
$|\vec{v}| = \sqrt{2}\,110$

$= 155$ mm/s

1.2) The disk shown below has a radius of 0.5 ft and the rope is being pulled with a velocity of 2 ft/s. Determine the magnitude of the angular velocity of the disk.



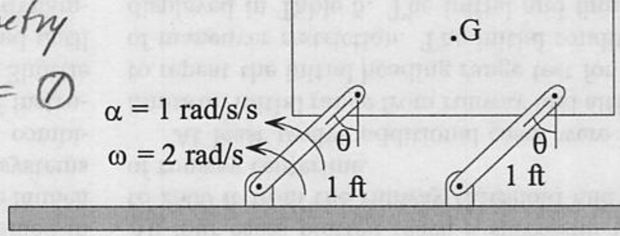
1.3) An rigid body is rotating at 3 rad/s clockwise. Express the angular velocity as a vector in terms of it's $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ components.



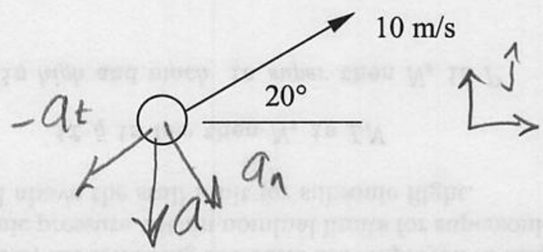
$\omega = -3\hat{k}$ r/s

1.4) A rectangular plate swings from arms of equal length as shown below. Determine the angular velocity of the plate.

Due to symmetry $\omega = 0$



1.5) A baseball is thrown as shown below. Neglecting air resistance, what is the radius of curvature of the path immediately after the ball is released?



$$v = 10\,m/s\,(\cos 20°\,\hat{i} + \sin 20°\,\hat{j})$$

$$a = -9.81\,m/s\,\hat{j}$$

$$a_t = \vec{a} \cdot \frac{\vec{v}}{|v|}$$

$$= -9.81\,\hat{j} \cdot \{9.39\,\hat{i}\ \ 3.42\,\hat{j}\}/10$$

$$a_t = -3.355$$

$$a_n = \sqrt{9.81^2 - 3.355^2} = 9.218$$

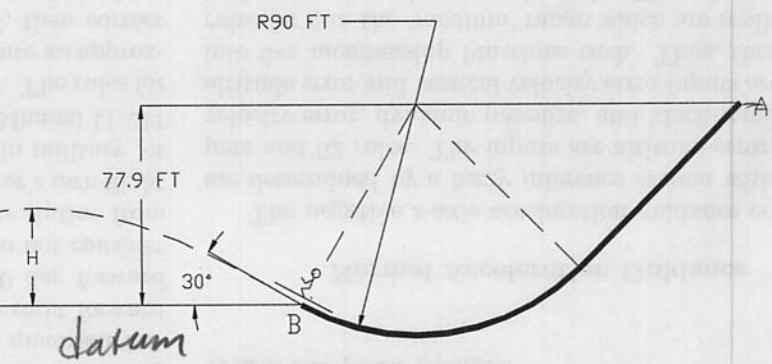
$$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n} = \frac{100}{9.218}$$

$$\boxed{\rho = 10.19\,m}$$

Name ______________________ 35 pts

ES204 Examination I **Problem 2** Dec. 18, 1998

A 120 pound ski jumper begins from rest at point A. a) Ignoring friction and wind resistance, determine her speed as she passes through point B at the takeoff position. b) Determine the normal force, N, exerted by the snow on her skis, just as she reaches point B. c) Determine the maximum height. H. she will reach.



Energy A to B:

$$E_{sysB} - E_{sysA} = W^{nc} \overset{0}{}$$

$$\frac{1}{2} m V_B^2 - mg(77.9 ft) = 0$$

$$V_B = \sqrt{2gh} = \sqrt{(2)(32.2)(77.9)} = 70.83 ft/s$$

B normal force:

FBD KD

mg, N, $30°$; $m\frac{v^2}{r}$, ma_t

$$\sum F_{nFBD} = \sum F_{nKD}$$

$$N - mg\cos\theta = m\frac{v^2}{r}$$

$$N = m\frac{v^2}{r} + mg\cos\theta$$

$$= \frac{120}{32.2}\frac{70.83^2}{90} + 120\cos 30°$$

$$\boxed{N = 311.7\ lbf}$$

After B

$$mg = ma$$

$$a = -g\hat{j} \quad \text{const accel}$$

$$V = V_{0_y} + at$$

$$t = \frac{V - V_0}{a} = \frac{0 - 70.83\sin 30^\circ \text{ ft/s}}{32.2 \text{ ft/s}^2}$$

$$= 1.1 \text{ s}$$

$$H = \cancel{h_0}^{\,0} + V_{0_y}t + \frac{1}{2}a_y t^2$$

$$= (70.83\sin 30^\circ)(1.1) - \left(\frac{1}{2}\right)(32.2)(1.1)^2$$

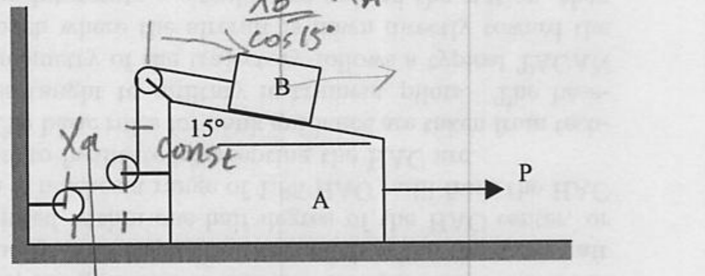
$$\boxed{H = 19.48 \text{ ft}}$$

p 1-16 of course notes

Name ______________________________ 35 pts

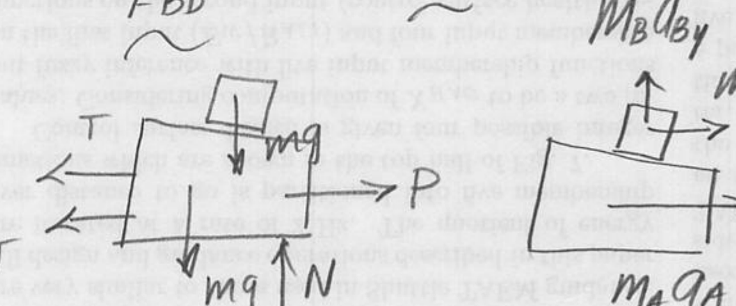
ES204 Examination I **Problem 3** Dec. 18, 1998

For the system shown the mass of A is m_A and the mass of B is m_B and the applied force is P. Assume the friction between all surfaces is negligible. Derive the equations necessary to solve for the tension in the cable and the accelerations of the two blocks but **DO NOT SOLVE THESE EQUATIONS**. Your final answer should be a list of unknowns and a list of equation numbers that could be used to solve for the unknowns.



Mom rate form: Both

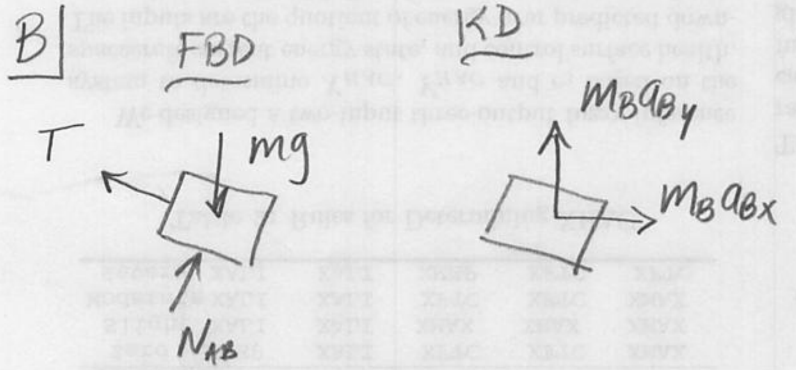
FBD = KD



$$\xrightarrow{+} \Sigma F_{xFBD} = \Sigma F_{xKD}$$

$$P - 2T = M_A a_A + M_B a_{Bx} \quad (1)$$

B FBD = KD



$$\Sigma F_{xFBD} = \Sigma F_{xKD}$$

$$(2) \quad T\cos 15° - N_{AB}\sin 15° = -M_B a_{Bx}$$

$$\uparrow + \ \Sigma F_{yFBD} = \Sigma F_{yKD} \quad 7$$

Unknowns	Equation Number
P	1
T	2
a_A	3
a_{Bx}	4
N_{AB}	5
a_{By}	6
$a_{B/A}$	

$$\uparrow + \ \Sigma F_{yFBD} = \Sigma F_{yKD} \quad (3)$$

$$-mg + N_{AB}\cos 15° + T\sin 15° = M_B a_{By}$$

Dep motion

$$L = 2X_A + \frac{X_B}{\cos 15°} - X_A + \text{const.}$$

$$0 = a_A + \frac{a_B}{\cos 15°} \quad (4)$$

Rel accel

$$a_B = a_A + a_{B/A}$$

$$\begin{matrix}(5)\\(6)\end{matrix} \begin{Bmatrix} a_{Bx} \\ a_{By} \end{Bmatrix} = \begin{Bmatrix} a_A \\ 0 \end{Bmatrix} + a_{B/A} \begin{Bmatrix} \cos 15° \\ -\sin 15° \end{Bmatrix}$$

Name ______________________________ 24 pts

ES204 Examination I **Problem 1** Dec. 14, 2001

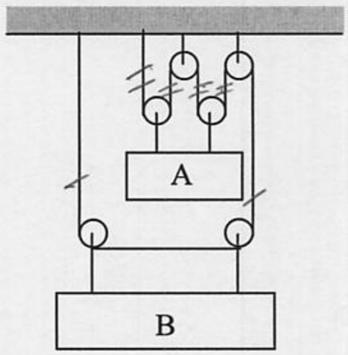
1.1) If B is moving at a constant 2 ft/s downwards what is the velocity of A? (3 pts)

a) 0.25 ft/s up
b) 0.4 ft/s up
c) 0.5 ft/s up
d) 1 ft/s up
e) 2 ft/s up
f) 2.5 ft/s up
g) 4 ft/s up
h) 10 ft/s up

$L = 4x_A + 2x_B$

diff: $0 = 4V_A + 2V_B$

$V_A = -\frac{1}{2} V_B$



$= \left(-\frac{1}{2}\right)(2) = -1 = 1$ ft/s UP

1.2) Object A has a velocity with respect to object B of $3\hat{i} + 4\hat{j}$ ft/s. If the actual velocity of A is $2\hat{i} + 2\hat{j}$ ft/s determine the velocity of object B. (3 pts)

a) $2\hat{i} + 2\hat{j}$ ft/s
b) $-\hat{i} - 2\hat{j}$ ft/s
c) $\hat{i} + 2\hat{j}$ ft/s
d) 0 ft/s
e) $5\hat{i} + 6\hat{j}$ ft/s

$V_{A/B} = 3\hat{i} + 4\hat{j}$

$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$

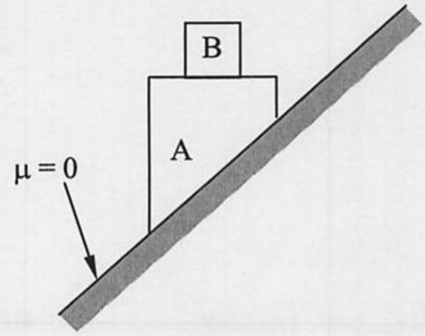
$\vec{V}_B = \vec{V}_A - \vec{V}_{A/B}$

$\begin{Bmatrix} 2 \\ 2 \end{Bmatrix} - \begin{Bmatrix} 3 \\ 4 \end{Bmatrix}$

$\vec{V}_B = \begin{Bmatrix} -1 \\ -2 \end{Bmatrix}$

1.3) The system shown is released from rest in the position shown. Qualitatively the normal force between block A and B is (circle one) (3 pts)

a) less than the weight of B
b) equal to the weight of B
c) greater than the weight of B
d) not enough information is given



1.4) A plane is traveling as shown. At the instant shown r is increasing at a <u>constant</u> rate of 300 ft/s and the angle, θ, is decreasing at a <u>constant</u> rate of 0.18 rad/s. Determine the speed of the plane. (3 pts)

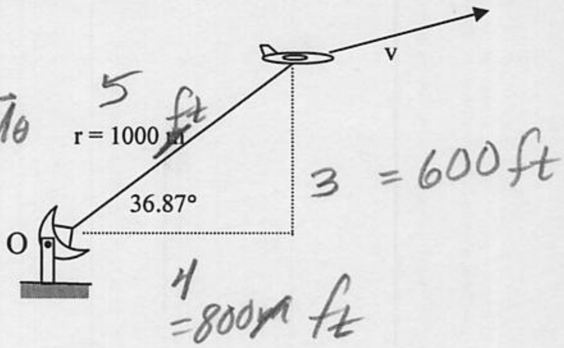
$v = \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta$

$= 300\vec{u}_r - (1000)(0.18)\vec{u}_\theta$

$v = \sqrt{300^2 + 180^2}$

$v \cong 350 \text{ ft/s}$

a) 120 ft/s
b) 180 ft/s
c) 300 ft/s
d) 350 ft/s
e) 480 ft/s



1.5) Determine the acceleration of the plane shown in problem 1.4. (3 pts)

$\ddot{r} = \ddot{\theta} = 0$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta$

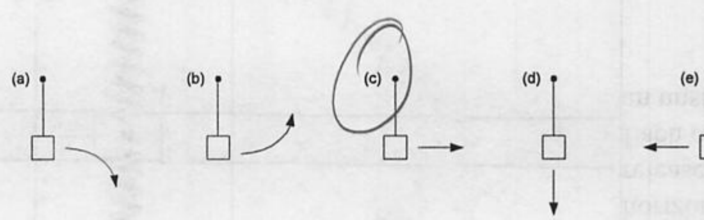
$= (-1000)(0.18)^2\vec{u}_r + (2)(300)(-0.18)\vec{u}_\theta$

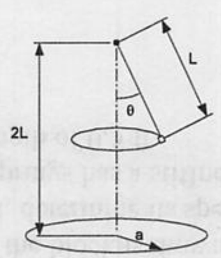
$= -32.4\vec{u}_r - 108\vec{u}_\theta$

$\boxed{|a| \cong 112.8 \text{ ft/s}}$

a) 0 ft/s
b) 32.4 ft/s
c) 108 ft/s
d) 112.8 ft/s
e) 140.4 ft/s

1.6) A bucket is attached to a rope of length L and is made to revolve in a horizontal circle. The bucket is rotating <u>counterclockwise</u> when viewed from the ceiling. Drops of water fall from the bucket and strike the floor along the perimeter of a circle of radius a. Which sketch accurately reflects the path of a drop of water **<u>as viewed from the ceiling</u>**? (3 pts)



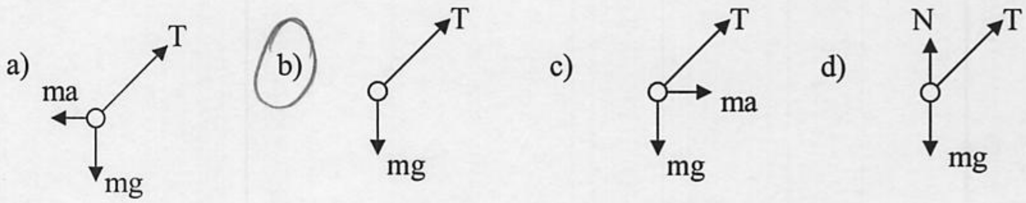


1.7) A weight at the end of a string moves at a constant speed around a circle path in a horizontal plane as shown. Circle ALL the following statements that are correct. (3 pts)

There will be a non-zero component of acceleration

a) in the same direction as the velocity
b) directed radially inward towards the center of the circular path
c) directed radially outward from the center of the circular path
d) in the vertical direction due to gravity

1.8) For the weight shown in Problem 1.7, the correct free body diagram is

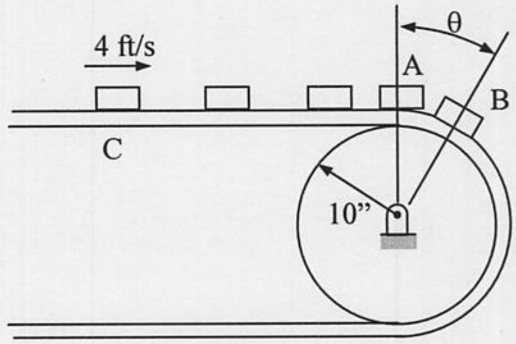


Name ______________________________ 36 pts

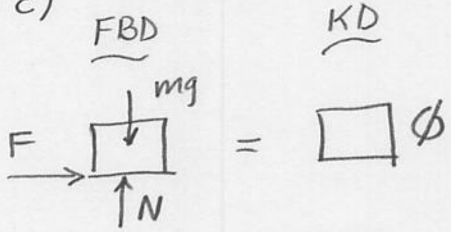
ES204 Examination I **Problem 2** Dec. 14, 2001

A series of small packages, each weighing 0.75 lb, are discharged from a conveyor as shown. The belt moves at a constant speed. Knowing that the coefficient of static friction between each package and the belt is 0.4, and the coefficient of kinetic friction is 0.35, determine,

a) the normal force the belt exerts on the package at point C
b) the friction force at point C
c) the force exerted by the belt on the package just after if passes point A
d) the angle θ defining the point B where the package first slips relative to the belt. **NOTE: for part d) just derive the necessary equations so that the angle could be determined. You do not need to solve the equations.**

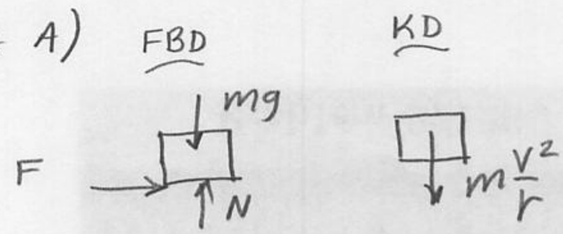


at c)



$$\boxed{F = 0; \qquad N = mg}$$

at A)

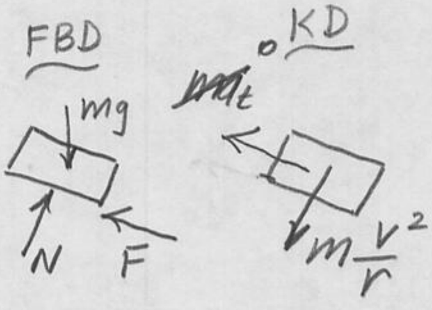


$$\Sigma F_{nFBD} = \Sigma F_{nKD}$$

$$N - mg = -m\frac{v^2}{r}$$

$$\boxed{N = mg - m\frac{v^2}{r}}$$

at B)



$$\Sigma F_{nFBD} = \Sigma F_{nKD}$$

$$N - mg\cos\theta = -m\frac{V_2^2}{r}$$

$$N = mg\cos\theta - m\frac{V_2^2}{r} \quad (1)$$

$$\Sigma F_{tFBD} = \Sigma F_{tKD}$$

$$F - mg\sin\theta = \emptyset \quad (2)$$

Package slips when $F > \mu_s N$ required for motion $a_t = \emptyset$

Name ______________________________________ 40 pts

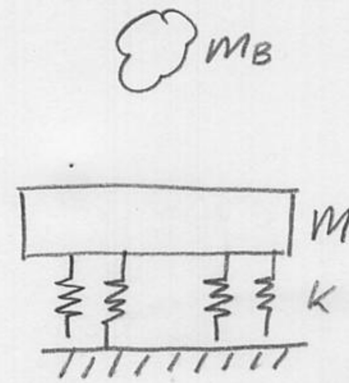
ES204 Examination I **Problem 3** Dec. 14, 2001

The Caterpillar 769D dumptruck, W = 10 tons, is used at the Ajax Mine just east of Terre Haute. A 15 ton boulder is dropped 20 feet from a crane into its bed. Assuming the coefficient of restitution is 0.8, the *four* springs in the suspension are initially compressed 1" and that the springs can deflect no more than 3 additional inches, determine the spring constant k.

DO NOT SOLVE THIS PROBLEM - **SET IT UP ONLY**
Your solution should consist of a table of unknowns/equations and a numbered collection of equations. Clearly define all your terms.

unk	eqs
V_1'	1
V_T'	2

Model



Segment Ø - free fall

system = boulder

strategy = energy

$E_{sys_1} - E_{sys_0} = \cancel{W}^{10}$

$\frac{1}{2}mv_1^2 - mgh_0 = \emptyset$

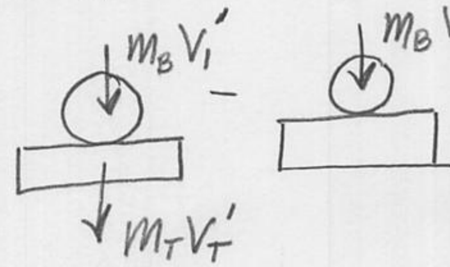
$V_1 = \sqrt{2gh_0}$

$= \sqrt{(2)(32.2)(20)}$

$V_1 = 35.89 \text{ ft/s}$

Segment 1: collision

After - Before = During



$= \emptyset$

$(\downarrow)\ m_B V_1' + m_T V_T' - m_B V_1 = \emptyset \quad (1)$

COR:

$$e = \frac{-(V_T' - V_1')}{V_T - V_1} \quad (2)$$

By guessing V_T', V_1' $\downarrow$, sign errors are minimized.

Segment 2: energy system truck & suspension:

$$E_{sys_2} - E_{sys_1} = \emptyset = W$$

3" { ______ initial
- - - - - - datum

$$\cancel{E_{K_2}}^{0} + \cancel{E_{G_2}}^{0} + E_{S2} - E_{K_1} - E_{G_1} - E_{S_1} = \emptyset$$

(3) $$4\left(\frac{1}{2}k\left(\frac{4}{12}\right)^2\right) - \frac{1}{2}mv_+'^2 - mg\left(\frac{3}{12}\right) - 4\left(\frac{1}{2}k\left(\frac{1}{12}\right)^2\right) = \emptyset$$

solve for k.