Chapter 6 — Conservation of Angular Momentum

1. Define, explain, compare and contrast the following terms and concepts:

Motion of a rigid body
  Rectilinear translation vs. Curvilinear translation
  Rotation about a fixed axis
  General motion
Rotational motion
Angular position: θ [radians]
  Angular velocity: ω [radians/second]
  Angular acceleration: α [radians/(seconds)^2]

Angular momentum about origin O
  angular momentum about the origin O for a particle: $\mathbf{L}_o = \mathbf{r} \times m\mathbf{V}$

  specific angular momentum about the origin O : $\mathbf{l}_O = \mathbf{r} \times \mathbf{V}$

    where $\mathbf{r}$ is the position vector with respect to the origin O

  right-hand rule sign convention
  vector nature of angular momentum
  units of angular momentum (N·m·s, lbf·ft·s)

Application of Accounting Principle for Angular Momentum

  rate of accumulation of angular momentum with the system

    amount of angular momentum about origin O: $\mathbf{L}_{o,sys} = \int_{V_{sys}} \left(\mathbf{r} \times \mathbf{V}\right)\rho dV$

    mass moment of inertia about a single axis: $\boxed{I_G = \int_{V_{sys}} r^2 \ \rho \ dV}$

    relation between mass moment of inertia, angular momentum, and angular velocity

  transport rate of angular momentum across system boundaries

    transport with forces

      torques or moments of an external force about origin O: $\sum \mathbf{r} \times \mathbf{F}_{ext}$

      torque or moment of a couple: $\mathbf{M}_o$

  mass transport of angular momentum about the O: $\sum \dot{m}(\mathbf{r} \times \mathbf{V})_{in} - \sum \dot{m}(\mathbf{r} \times \mathbf{V})_{out}$

generation/consumption of angular momentum within the system
Empirical Result =====> Angular momentum is conserved !

Conservation of Angular Momentum (about the origin *O*)

rate form:

$$\frac{d\mathbf{L}_{o,sys}}{dt} = \underbrace{\sum \mathbf{M}_{o,external}}_{\text{Due to couples}} + \underbrace{\sum\left(\mathbf{r}\times\mathbf{F}_{external}\right)}_{\text{Due to forces}} + \underbrace{\sum_{in}\dot{m}_i(\mathbf{r}\times\mathbf{V})_i - \sum_{out}\dot{m}_e(\mathbf{r}\times\mathbf{V})_e}_{\text{Due to mass transport}}$$

Angular Impulse

SPECIAL CASE: Plane, Translational Motion of a Closed, Rigid System

angular momentum about origin *O*: $L_{o,sys} = \mathbf{r}_G \times m\mathbf{V}_G$

where $\mathbf{r}_G$ = the position vector of the center of mass with respect to the origin.
$\mathbf{V}_G$ = the velocity of the center of mass.

Conservation of Angular Momentum:

$$\frac{d\mathbf{L}_{o,sys}}{dt} = \sum \mathbf{r}\times\mathbf{F}_{ext} + \sum \mathbf{M}_o$$

$$\frac{d}{dt}\left(\mathbf{r}_G \times m\mathbf{V}_G\right) = \sum \mathbf{r}\times\mathbf{F}_{ext} + \sum \mathbf{M}_o$$

$$\underbrace{\cancel{\left[\frac{d\mathbf{r}_G}{dt}\times m\mathbf{V}_G\right]}}_{\text{since } \mathbf{V}_G\times\mathbf{V}_G=0} + \left[\mathbf{r}_G \times m\frac{d\mathbf{V}_G}{dt}\right] = \sum \mathbf{r}\times\mathbf{F}_{ext} + \sum \mathbf{M}_o$$

$$\mathbf{r}_G \times m\frac{d\mathbf{V}_G}{dt} = \sum \mathbf{r}\times\mathbf{F}_{ext} + \sum \mathbf{M}_o$$

where
$\mathbf{r}$ = the position vector with respect to the origin.
$\mathbf{r}_G$ = the position vector of the center of mass with respect to the origin.

2. Apply conservation of angular momentum to solve problems involving
   (1) steady-state open or closed systems,
   (2) static (stationary) closed system,
   (3) closed, stationary, rigid-body systems,
   (4) translating, closed, rigid body systems, i.e. systems with $\omega=0$ and $\alpha=0$.

(See item number 2 on the linear momentum objective page to see necessary steps.)