Project 1: Analyzing Craps

MA 381

Due Monday September 29, 2003

1. Suppose you have 2 mutually exclusive events A and B, where $P(A) + P(B) < 1$ (so other events can occur). Experiments are conducted iteratively until either event A or event B occurs, at which point the experiment stops. Show that the probability that event A occurs before event B occurs is
$$\frac{P(A)}{P(A) + P(B)}.$$

2. The game of craps is a dice game where the player throws a (fair) pair of dice. If the first roll is a 7 or an 11, the player wins. If the roll is a 2,3, or 12, the player loses. If it is anything else, the player continues to roll. On subsequent rolls, if the player matches their first roll before rolling a 7, the player wins. If a 7 appears first, the player loses. The player continues rolling until one of these events occurs. Your job is to find the probability of the player winning. The best approach is to use conditional probability arguments. For this approach, what event would be best to "condition" on?

3. Now, let's make it interesting. Suppose you could alter the dice so that the probabilities change. For example, suppose each die was "shaved" so that the probabilities of rolling a value k are
$$\begin{aligned} P(\text{ roll } 1) &= P(\text{roll } 2) = P(\text{roll } 5) = P(\text{roll } 6) = \frac{1}{6} - \varepsilon \\ P(\text{roll } 3) &= P(\text{roll } 4) = \frac{1}{6} + 2\varepsilon, \end{aligned}$$
for some $\varepsilon > 0$.
   (a) Find the probability of winning as a function of ε.
   (b) Plot the probability of winning as a function of ε.
   (c) For what value of ε is the probability of winning $= \frac{1}{2}$?