ECE-320: Linear Control Systems
Homework 7

Due: **Thursday** January 30 at the beginning of class

1) **(Easy)** Show that \( \sum_{l=-\infty}^{n} \delta(l) = u(n) \) and \( \sum_{l=-\infty}^{n} \delta(l-k) = u(n-k) \)

2) **(Easy)** For impulse response \( h(n) = \delta(n) + 2\delta(n-2) + 3\delta(n-3) \) and input \( x(n) = \frac{1}{2} u(n-2) \), determine the output \( y(n) \) (this should be easy).

3) For impulse response \( h(n) = \left( \frac{1}{2} \right)^n u(n) \) and input \( x(n) = u(n) \), show that the system output is \( y(n) = 2 \left[ 1 - \left( \frac{1}{2} \right)^{n+1} \right] u(n) \)
   a) by evaluating the convolution sum \( y(n) = \sum_{k=-\infty}^{k=\infty} x(n-k)h(k) \)
   b) by evaluating the convolution sum \( y(n) = \sum_{k=-\infty}^{k=\infty} x(k)h(n-k) \)

   *Note that this is the unit step response of the system.*

4) For impulse response \( h(n) = \left( \frac{1}{3} \right)^{n-2} u(n-1) \) and input \( x(n) = \left( \frac{1}{2} \right)^n u(n-1) \), show that the system output is \( y(n) = 9 \left[ \left( \frac{1}{2} \right)^{n-1} - \left( \frac{1}{3} \right)^{n-1} \right] u(n-2) \) by evaluating the convolution sum \( y(n) = \sum_{k=-\infty}^{k=\infty} x(n-k)h(k) \)

5) For impulse response \( h(n) = \left( \frac{1}{2} \right)^{n-3} u(n-1) \) and input \( x(n) = \left( \frac{1}{4} \right)^{n+1} u(n-2) \), show that the system output is \( y(n) = \left[ \left( \frac{1}{2} \right)^n - \left( \frac{1}{4} \right)^{n-1} \right] u(n-3) \) by evaluating the convolution sum \( y(n) = \sum_{k=-\infty}^{k=\infty} h(n-k)x(k) \)
6) For impulse response \( h(n) = \left( \frac{1}{3} \right)^{n+1} u(n-2) \) and input \( x(n) = \left( \frac{1}{2} \right)^{n-2} u(n+1) \), show that the system output is \( y(n) = \frac{16}{9} \left[ \left( \frac{1}{2} \right)^n - \left( \frac{1}{3} \right)^n \right] u(n-1) \) by evaluating the convolution sum \( y(n) = \sum_{k=-\infty}^{\infty} h(n-k)x(k) \).

7) For \( h(n) = \left( \frac{1}{a} \right)^n u(-n) + a^n u(n) \) with \( |a| < 1 \), using the two-sided z transform to show that

\[
H(z) = \frac{1}{1 - az} + \frac{1}{1 - az^{-1}}
\]

and the region of convergence is \(|a| < |z| < \frac{1}{|a|}\).