ECE-320 Linear Control Systems

Winter 2012, Exam 1

No calculators or computers allowed, except for Problem 6 when you should use Matlab’s sisotool.

You must simplify your answers as much as possible, or points will be deducted.

Problem 1 _____/24
Problem 2 _____/12
Problem 3 _____/8
Problem 4 _____/8
Problem 5 _____/24
Problem 6 _____/24
Total _____/100
1) (24 points) Consider the following simple feedback control block diagram. The plant is $G_p(s) = \frac{3}{s+5}$. The input is a unit step.

a) Determine the settling time, steady state error for a unit step input, and the bandwidth of the plant alone (assuming there is no feedback).

b) Assuming a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function, $G_c(s)$.

c) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of $k_p$ so the steady state error for a unit step is 1/4, and the corresponding settling time for the system.

d) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of $k_p$ so the settling time is 4/11 seconds, and the corresponding steady state error.

e) Assuming a proportional controller, $G_c(s) = k_p$, determine the value of $k_p$ so the bandwidth is 17 rad/sec.

\[ T_s = \frac{4}{\frac{1}{k_p}} \quad \text{sec} \quad e_{ss} = 1 - \frac{3}{5} = \frac{2}{5} \quad \text{BW} = \frac{5}{k_p} \text{ rad/sec} \]

\[ G_c(s) = \frac{3k_p}{s+3k_p} \]

\[ e_{ss} = \frac{5}{s+3k_p} = \frac{1}{4} \quad k_p = 5 \quad T_s = \frac{4}{s+5} = \frac{4}{20} = \frac{1}{5} = T_s \]

\[ T_s = \frac{y}{s+3k_p} = \frac{4}{11} \quad k_p > 2 \quad e_{ss} = \frac{5}{11} \]

\[ \text{BW} = \frac{s+3k_p}{1} \quad k_p = 4 \]
2) **(12 points)** For the following questions, refer to the following graph showing the input and output of a second order system. For this system the input is a step of amplitude 2. (You can leave your answers as fractions.)

![Graph of input and output of a second order system](image)

**a)** What is the static gain of the system? \( K \cdot 2 = 8 \quad (K = 4) \)

**b)** What is the percent overshoot? \( PO = \frac{14 - 8}{8} \times 100\% = \frac{6}{8} \times 100\% = 75\% \)

**c)** What is the steady state error? \( ess = 2 - 8 = -6 \)
3) (8 points) For the following systems, assume \( G_c(s) = \frac{1}{s+2} \) and \( G_p(s) = \frac{1}{s+5} \)

\[
\begin{array}{c}
\textstyle \sum \\
\rightarrow \quad G_c(s) \quad \rightarrow \\
\downarrow \quad \text{G}_p(s) \quad \rightarrow \\
\end{array}
\]

a) Determine the position error constant \( K_p \) 
\[
K_p = \left. \frac{1}{10} \right|_{s \to 0} = \lim_{s \to 0} G_c(10) G_p(10)
\]

b) Determine the steady state error for a unit step input.

\[
e_{ss} = \frac{1}{1 + K_p} = \frac{10}{11} = e_{ss}
\]
4) (8 points) For the following systems, assume $G_c(s) = \frac{3}{s}$ and $G_p(s) = \frac{1}{s+4}$

\[ \begin{array}{c}
\sum \\
\downarrow \ \\
G_c(s) \rightarrow G_p(s)
\end{array} \]

a) Determine the velocity error constant $K_v$.

\[ \left( \frac{\dot{V}}{V} \right) = \lim_{s \to 0} s \left( G_c(s) + G_p(s) \right) \]

b) Determine the steady state error for a unit ramp input.

\[ \varepsilon_{ss} = \frac{1}{K_v} = 2 \]
5) (24 points) For a system with the transfer function \( H(s) = \frac{1}{(s+1)(s+2)^2} \)

a) Determine the impulse response \( h(t) \)

\[
H(s) = \frac{1}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}
\]

\[
h(t) = (e^{-t} - e^{-2t} - te^{-2t})u(t)
\]

\[
A = 1 \quad C = -1
\]
\[
x_{\$}(t) \rightarrow \infty \quad 0 = A + B \quad B = -A = -1
\]

b) Determine the unit step response.

\[
Y(s) = \frac{1}{s(s+1)(s+2)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} + \frac{D}{(s+2)^2}
\]

\[
y_{\$}(t) = \left( \frac{1}{4} - e^{-t} + \frac{3}{4} e^{-2t} + \frac{1}{2} te^{-2t} \right)u(t)
\]

\[
A = \frac{1}{4} \quad B = -1 \quad D = \frac{1}{2}
\]
\[
x_{\$}(t) \rightarrow \infty \quad 0 = A + B + C
\]
\[
C = -A - B = -\frac{1}{4} + 1 = \frac{3}{4}
\]
6) (24 points) (sisotool problem)

Consider the plant

\[ G_p(s) = \frac{100}{s^2 + 2s + 20} \]

Design a PID controller using sisotool with complex zeros so that

\[ T_s \leq 1.0 \text{ sec} \]
\[ P.O. \leq 10\% \]

In addition, your controller must be designed so that

\[ k_p \leq 0.5 \]
\[ k_i \leq 5 \]
\[ k_d \leq 0.1 \]

Write your final values for \( k_p \), \( k_i \), \( k_d \), and the transfer function of the controller in the space below.

\[ k_p = \]
\[ k_i = \]
\[ k_d = \]
\[ G_c(s) = \]