ECE-205
Exam 3
Winter 2012

Calculators and computers are not allowed. You must show your work to receive credit.

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Problem 6 ________/15

Total ___________
1) (15 points) For the following transfer functions, determine the unit step response of the system. Do not forget any necessary unit step functions.

a) \( H(s) = \frac{e^{-3s}}{(s+1)^2} \)

b) \( H(s) = \frac{1}{s^2 + 4s + 8} \)

\[ y(t) = \frac{1}{s} H(s) \]

\[ G(s) = \frac{1}{(s+1)^2} = A + \frac{B}{s+1} + \frac{C}{(s+1)^2} \]

\[ A = 1 \quad C = -1 \]

\[ X(s), u(t) \rightarrow \infty \quad 0 = A + B \quad B = -A = -1 \]

\[ y(t) = (1 - e^{-t} - te^{-t}) u(t) \]

\[ y(t) = \frac{1}{s} \left[ 1 - e^{-t} - te^{-t} \right] u(t) = y(t) \]

b) \( y(t) = \frac{1}{s} H(s) \)

\[ G(s) = \frac{1}{(s+2)^2 + 4} \]

\[ A = \frac{1}{8} \quad \text{as} \quad u(t) \rightarrow \infty \quad 0 = A + C \quad C = -\frac{1}{8} \]

\[ b = -2 \quad \frac{1}{-8} = -\frac{1}{16} + \frac{B}{2} \quad -2 = -1 + 8B \quad B = -\frac{1}{8} \]

\[ y(t) = \frac{1}{8} \left[ 1 - e^{-2t} \sin(2t) - e^{-2t} \cos(2t) \right] u(t) \]
2) **(15 points)** Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function \( G_p(s) = \frac{3}{s+5} \)

![Block diagram](image)

a) Determine the settling time of the plant alone (assuming there is no feedback)

\[ T_S = \frac{4}{5} \]

b) Determine the steady state error for plant alone assuming the input is a unit step (simplify your answer)

\[ e_{ss} = 1 - \frac{3}{5} = \frac{2}{5} = e_{ss} \]

c) For a proportional controller, \( G_c(s) = k_p \), determine the closed loop transfer function \( G_0(s) \)

\[ G_0(s) = \frac{3k_p}{s+5+3k_p} \]

d) Determine the settling time of the closed loop system, in terms of \( k_p \)

\[ T_S = \frac{4}{s+3k_p} \]

e) Determine the steady state error of the closed loop system for a unit step, in terms of \( k_p \) (simplify your answer)

\[ e_{ss} = 1 - \frac{3k_p}{s+3k_p} = \frac{5}{s+3k_p} = e_{ss} \]

f) For an integral controller, \( G_c(s) = \frac{k_i}{s} \), determine the closed loop transfer function \( G_0(s) \) and the steady state error for a unit step in terms of \( k_i \)

\[ G_0(s) = \frac{3k_i}{s(s+5)+3k_i} \]

\[ e_{ss} = 0 \]
3) (20 points) For the following circuit,

\[ \text{Determines expressions for the following in terms of the parameters given}
\]

a) the zero input response (ZIR)

\[ I_{\text{in}}(t) = \frac{V_{\text{out}}(1d)}{R_b} + \frac{V_{\text{out}}(1d) - V(t)}{C} = -CV(t) + V_{\text{out}}(1d) \left[ \frac{1}{R_b} + C \right] \]

b) the zero state response (ZSR)

\[ I_{\text{in}}(t) + CV(t) = V_{\text{out}}(1(\theta)) \left[ \frac{R_bCt + 1}{R_b} \right] \]

\[ V_{\text{out}}(1(\theta)) = \left[ \frac{R_b}{R_bCt + 1} I_{\text{in}}(1(\theta)) + \left( \frac{R_bC}{R_bCt + 1} \right) \right] \]

\[ h(\theta) = \frac{V_{\text{out}}(1d)}{I_{\text{in}}(1d)} = \frac{R_b}{R_bCt + 1} = H(\theta) = \frac{R_b}{R_bCt + \frac{1}{R_bC}} = \frac{1}{C + \frac{1}{R_bC}} \]

\[ I_{\text{in}}(t) = \frac{1}{C} e^{-\frac{t}{R_bC}} u(t) \]
4) **(20 points)** An LTI system has impulse response, input, and output as shown below. Determine numerical values for the parameters $a$, $b$, $c$, $d$ and $e$. Note that the diagrams are not to scale!

![Diagram of h(t), x(t), y(t)]

- $t-1 = a$, $t = a+1 = 3$, $a = 2$
- $t-1 = b$, $t = b+1 = 6$, $b = 5$
- $-4 = (1)(2)(e)$, $e = -2$
- $t-2 = c$, $t = c+2 = 8$, $c = 6$

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5) **(15 points)** Determine the transfer function for the following circuit in terms of the parameters given. For full credit you must simplify your result as much as possible.

\[
\frac{I(s)}{R_a + \frac{1}{C_m s}} + \frac{V_{1(s)}}{R_b} + \frac{V_{2(s)}}{C_m s} = 0
\]

\[
\frac{I(\epsilon)}{R_0 C_m + 1} = -V(\epsilon) \left[ C_m s + \frac{1}{R_b} \right] = -V(\epsilon) \left[ \frac{R_b C_m s + 1}{R_0} \right]
\]

\[
\frac{V(\epsilon)}{I(s)} = \frac{R_b C_m s}{(R_0 C_m + 1)(R_b C_b s + 1)}
\]
6) (15 points) Simplify the following expressions as much as possible. Do these problems in the time-domain.

a) \[ y(t) = e^{-3(t-1)} \delta(t-2) \]
\[ y(t) = e^{-2(t-1)} \delta(t-2) \]
\[ e^{-2} \delta(t-2) = y(t) \]

b) \[ y(t) = e^{-2(t-1)} * \delta(t-2) \]
\[ y(t) = \int_{-\infty}^{\infty} e^{-2(\lambda-1)} \delta(t-\lambda-2) d\lambda \]
\[ = e^{-2(t-1)} \int_{-\infty}^{\infty} \delta(t-\lambda-2) d\lambda \]
\[ = e^{-2(t-1)} \delta(t-3) = y(t) \]

c) \[ y(t) = \delta(t-1) * \delta(t-2) \]
\[ y(t) = \int_{-\infty}^{\infty} \delta(t-\lambda-1) \delta(t-\lambda-2) d\lambda \]
\[ = \delta(t-3) = y(t) \]

d) \[ y(t) = \int_{-1}^{t-1} e^{-3(t-3)} d\lambda \]
\[ = e^{-2t} \int_{t-1}^{t-3} e^{-3\lambda} d\lambda \]
\[ = e^{-2t} \left[ -e^{-3\lambda} \right]_{t-1}^{t-3} \]
\[ = e^{-2t} \left[ e^{-2} - e^{-3(t-1)} \right] \]
\[ = e^{-2t} - e^{-3(t-1)} \]
\[ = y(t) \]