Midterm Exam 3

ECE205 Dynamical Systems

Midterm Exam 3
5/12/11

NAME: ___________________________ CM: __________

- You must **show work** to receive partial and full credit.
- Put a box around your final answer and it must include units, if necessary.
- Time allowed: 50 minutes.

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<thead>
<tr>
<th>Question #</th>
<th>Possible Points</th>
<th>Awarded Points</th>
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1) (10 points) For the following multiple choice questions circle the letter next to the correct answer.

The following transfer function is for questions i, ii, and iii.

\[ H(s) = \frac{1}{(s + 4)(s^2 + 4s + 4)(s^2 + 2s + 5)} \]

i) Which of the following is not a characteristic mode of the system?
   a) \( e^{-4t} \)
   b) \( te^{-2t} \)
   c) \( e^{-2t} \)
   d) \( e^{t} \cos(2t) \)
   e) \( e^{-t} \sin(2t) \)

ii) The best estimate of the settling time for this system is
   a) 4 seconds
   b) 2 seconds
   c) 1 second
   d) 0.2 seconds
   e) 8 seconds

iii) The dominant pole(s) of this system are
   a) -2 and -2
   b) -1 + 2j and -1 - 2j
   c) -4
   d) -20
   e) 0

iv) How many of the following impulse responses represent unstable systems?
   \[ h_1(t) = [t + e^{-t}]u(t) \]
   \[ h_2(t) = e^{-2t}u(t) \]
   \[ h_3(t) = [2 + \sin(t)]u(t) \]
   \[ h_4(t) = [1 - t^3 e^{-0.1t}]u(t) \]
   \[ h_5(t) = [1 + t + e^{-t}]u(t) \]
   \[ h_6(t) = [te^{-t} \cos(5t) + e^{-2t} \sin(3t)]u(t) \]
   a) 0
   b) 1
   c) 2
   d) 3
   e) 5

v) Which of the following transfer functions represents a stable system?
   \[ G_a(s) = \frac{s-1}{s+1} \]
   \[ G_c(s) = \frac{s}{s^2 - 1} \]
   \[ G_b(s) = \frac{1}{s(s+1)} \]
   \[ G_d(s) = \frac{s+1}{(s+1+j)(s+1-j)} \]
   \[ G_e(s) = \frac{(s-1-j)(s-1+j)}{s} \]
   \[ G_f(s) = \frac{(s-1-j)(s-1+j)}{(s+1-j)(s+1+j)} \]
   a) all but \( G_c \)
   b) only \( G_a, G_b, \) and \( G_d \)
   c) only \( G_a, G_d, \) and \( G_f \)
   d) only \( G_a \) and \( G_f \)
   e) only \( G_a \) and \( G_d \)
2) **(20 points)** For the following circuit,

Write the output, $V_{out}(s)$ in terms of $V_{in}(s)$, $R$, $C$ and $v(0^-)$. Identify the ZSR (zero state response) and the ZIR (zero input response). (You can leave your answer in the s-domain)
3) **(20 points)** Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \frac{3}{s+5}$.

![Block Diagram Image]

a) Determine the settling time of the plant alone (assuming there is no feedback).

b) Determine the steady-state error due to a unit step input of the plant alone (assuming there is no feedback).

c) For a proportional controller, $G_c(s) = k_p$.
   
   i) Determine the closed loop transfer function $G_c(s)$.

   ii) What is the settling time in terms of $k_p$?

   iii) What is the steady state error due to a unit step input, in terms of $k_p$?
4) **(20 points)** The following figure shows three different circuits, which are subsystems for a larger system. We can write the transfer functions for these systems as

\[
G_a(s) = \frac{-K_{\text{low}} \omega_{\text{low}}}{s + \omega_{\text{low}}} \quad G_b(s) = \frac{-K_{\text{high}} s}{s + \omega_{\text{high}}} \quad G_c(s) = -K_{\text{ap}}
\]

Determine the parameters \(K_{\text{low}}, \omega_{\text{low}}, K_{\text{high}}, \omega_{\text{high}}, \) and \(K_{\text{ap}}\) in terms of the parameters given (the resistors and capacitors).
5) (20 points) For the following transfer functions, determine the **impulse response** of the system. Do not forget any necessary unit step functions.

a) \( H(s) = \frac{e^{-t}}{s+2} \)

For the following transfer functions, determine the **unit step response** of the system. Do not forget any necessary unit step functions.

b) \( H(s) = \frac{1}{(s+1)^2} \)

c) \( H(s) = \frac{1}{s^2+4s+20} \)
6) **(10 points)** For the following signal flow graph, determine the transfer function between the input and output using Mason’s gain formula. You do not need to simplify your final answer.

![Signal Flow Graph](image-url)

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EQUATION SHEET

\[ \mathcal{L}\{\delta(t)\} = 1 \]
\[ \mathcal{L}\{u(t)\} = \frac{1}{s} \]
\[ \mathcal{L}\{tu(t)\} = \frac{1}{s^2} \]
\[ \mathcal{L}\left\{ \frac{t^{m-1}}{(m-1)!} u(t) \right\} = \frac{1}{s^m} \]
\[ \mathcal{L}\{e^{-at}u(t)\} = \frac{1}{s+a} \]
\[ \mathcal{L}\{te^{-at}u(t)\} = \frac{1}{(s+a)^2} \]
\[ \mathcal{L}\left\{ \frac{t^{(m-1)}}{(m-1)!} e^{-at}u(t) \right\} = \frac{1}{(s+a)^m} \]
\[ \mathcal{L}\{\cos(\omega_0 t)u(t)\} = \frac{s}{s^2 + \omega_0^2} \]
\[ \mathcal{L}\{\sin(\omega_0 t)u(t)\} = \frac{-\omega_0}{s^2 + \omega_0^2} \]
\[ \mathcal{L}\{e^{-at}\cos(\omega_d t)u(t)\} = \frac{s + \alpha}{(s + \alpha)^2 + \omega_d^2} \]
\[ \mathcal{L}\{e^{-at}\sin(\omega_d t)u(t)\} = \frac{\omega_0}{(s + \alpha)^2 + \omega_d^2} \]
\[ \mathcal{L}\{dx(t)/dt\} = sX(s) - x(0^-) \]
\[ \mathcal{L}\{d^2x(t)/dt^2\} = s^2X(s) - sx(0^-) - \dot{x}(0^-) \]
\[ \mathcal{L}\{x(t-a)\} = e^{-at}X(s) \]
\[ \mathcal{L}\{e^{-at}x(t)\} = X(s+a) \]
\[ \mathcal{L}\left\{ \frac{t}{a} \right\}, a > 0 = aX(as) \]

Initial Value Theorem: if \( x(t) \leftrightarrow X(s) \) \( \lim_{t \to 0^+} x(t) = \lim_{s \to \infty} sX(s) \)

Final Value Theorem: if \( x(t) \leftrightarrow X(s) \) \( \lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s) \)

Second Order System Properties

Percent Overshoot: \( P.O. = e^{-\zeta \pi} \times 100\% \)

If \( \beta = \frac{P O_{\text{max}}}{100} \) then \( \zeta = \frac{-\ln(\beta)}{\pi} \sqrt{1 + \left( \frac{-\ln(\beta)}{\pi} \right)^2} \)

\( \theta = \cos^{-1}(\zeta) \) Time to Peak:

\( T_p = \frac{\pi}{\omega_n}, \quad \omega_n = \omega_0 \sqrt{1 - \zeta^2} \)

2% Settling Time: \( T_s = \frac{4}{\zeta \omega_n} = 4\tau \)