ECE-205
Exam 1
Winter 2009

Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

You must show your work to receive credit.

Problem 1 __________
Problem 2 __________
Problem 3 __________
Problem 4 __________
Problem 5 __________

Total _______________
1) **(15 points)** Derive the governing differential equation for the following first order circuit. You can use any method you want (except for copying). You do not need to put your answer in a standard form.
2) (30 points) Assume we have a first order system with the governing differential equation

\[ 2y(t) + 4y(t) = 8x(t) \]

The system is initially at rest, so \( y(0) = 0 \). The input to this system is

\[
x(t) = \begin{cases} 
0 & t \leq 0 \\
3 & 0 < t \leq 1 \\
-2 & 1 < t \leq 1.5 \\
1 & 1.5 < t 
\end{cases}
\]

Determine the output of the system in each of the above time intervals. *Simplify your final answer as much as possible and box it.*
3) **(15 points)** Assume we have a first order system with the governing differential equation

\[ 2\dot{y}(t) + 3y(t) = e^{-t}x(t - 1) \]

The initial time is \( t_0 = 1 \) and initial value \( y(1) = 2 \). Use the method of integrating factors to determine the output \( y(t) \) as a function of the (unknown) input \( x(t) \). *Simplify your answer as much as possible and box it.*
4) (20 points) For the second order circuit below, derive the governing second order differential equation for the output $y(t)$ and input $x(t)$. You do not need to put it into a standard form.
5) **(20 points)** The form of the under damped \((0 < \zeta < 1)\) solution to the second order differential equation

\[
\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = K\omega_n^2 x(t)
\]

for a step input \(x(t) = Au(t)\) is

\[
y(t) = KA + ce^{-\zeta\omega_n t}\sin(\omega_d t + \phi)
\]

where \(c\) and \(\phi\) are constants to be determined and the damped frequency \(\omega_d = \omega_n\sqrt{1-\zeta^2}\)

a) Using the initial condition \(\dot{y}(0) = 0\) show that \(\tan(\phi) = \frac{\sqrt{1-\zeta^2}}{\zeta}\)

b) We can express the relationship in part a using the following triangle. Fill in the blanks and then use this triangle determine an expression for \(\sin(\phi)\).

\[
\begin{align*}
\text{r} &= \underline{\phantom{0}} \\
x &= \underline{\phantom{0}} \\
y &= \underline{\phantom{0}}
\end{align*}
\]

\[
\phi
\]

c) Use your answer to part b, and the initial condition \(y(0) = 0\) to determine the remaining unknown constant, and write out the complete solution for \(y(t)\).