Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

You must show your work to receive credit.

Problem 1  ________/15
Problem 2  ________/15
Problem 3  ________/10
Problem 4  ________/15
Problem 5  ________/15
Problem 6-15 ________/30

Total  _______________
1) **(15 points)** Consider the circuit shown in the figure below which is initially at rest:

![Circuit Diagram](image)

a) Assume during charging \((0 \leq t < 0.002 \text{ seconds})\) that the time constant \(\tau = 0.001 \text{ seconds}\), the static gain is \(K = 2\) and the input is a step with amplitude \(A = 2 \text{ volts}\). Determine the voltage on the capacitor at time \(t = 0.002 \text{ seconds}\)

b) Assume during discharging \((t \geq 0.002 \text{ seconds})\) that the time constant \(\tau = 0.002 \text{ seconds}\). Determine the voltage on the capacitor at time \(t = 0.006 \text{ seconds}\)

c) Sketch the voltage on the capacitor from \(t = 0 \text{ seconds}\) to \(t = 0.010 \text{ seconds}\). The most important part of this graph is the shape of the curves (i.e., what does the voltage on the capacitor look like when it is charging compared to when it is discharging)

\[
u_c(t) = (\nu_c(t_0) - \nu_c(\infty)) e^{-\frac{(t-t_0)}{\tau}} + \nu_c(\infty)
\]

\[\text{a) } K = 2 \quad A = 2 \quad t_0 = 0 \quad \nu_c(\infty) = 0 \quad \nu_c(\infty) = KA = 4 \quad \tau = 0.001 \quad t = 0.002 \]

\[
u_c(0.002) = (0 - 4)e^{-2} + 4 = 4(1 - e^{-2}) = 3.458 \text{ V}
\]

\[\text{b) } K = 0 \quad t_0 = 0.002 \quad \nu_c(t_0) = 3.458 \text{ V} \quad \nu_c(\infty) = 0 \quad \tau = 0.002 \quad t = 0.006 \]

\[
u_c(0.006) = (3.458 - 0)e^{-2} + 0 = 3.458e^{-2} = 0.46 \text{ V}
\]
2) (15 points) For the following differential equation, assume the input is \( x(t) = 4u(t) \) (the input is equal to four for time greater than zero), and the initial conditions are \( y(0) = y'(0) = 0 \)

\[
y''(t) + 2y'(t) + 3y(t) = 3x(t)
\]

Determine the solution and put your final answer in a box. Be sure to use the initial conditions to solve for all unknowns. You must show all your work to receive credit. You cannot just use the answer to problem 5, though you can check your answer if you want.

\[
\lambda = -2 \pm \frac{\sqrt{4-12}}{2} = -2 \pm \sqrt{8} = -2 \pm 2\sqrt{2} = -4 \pm 2\sqrt{2}
\]

\[3y_p = 3\lambda, \quad y_p(t) = 4\]

\[y(t) = 4 + ce^{-t}\sin(\sqrt{2}t + \theta)\]

\[y(0) = 0 = 4 + c\sin(\theta) \quad c = \frac{-4}{\sin(\theta)}\]

\[y'(t) = -ce^{-t}\sin(\sqrt{2}t + \theta) + c\sqrt{2}e^{-t}\cos(\sqrt{2}t + \theta)\]

\[y'(0) = 0 = -\sin(\theta) + \sqrt{2}\cos(\theta) = 0\]

\[
\frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta) = \sqrt{2} \quad \theta = 45^\circ
\]

\[c = \frac{-4}{\sin(45^\circ)} = -4.90\]

\[y(t) = 4 - 4.90e^{-t}\sin(\sqrt{2}t + 45^\circ)\]
3) (10 points) For the following first order differential equation,

\[ \frac{1}{t} \dot{y}(t) + y(t) = \cos(t)x(t) \]

determine an expression for the output assuming \( t_0 = 0 \) and \( y(t_0) = y(0) = 0 \).

\[ \dot{y}(t) + ty(t) = t \cos(t)x(t) \]

\[ \frac{d}{dt} \left( y(t) e^{t^2/2} \right) = e^{t^2/2} t \cos(t)x(t) \]

\[ y(t) e^{t^2/2} = \int_0^t e^{\lambda^2/2} \lambda \cos(\lambda)x(\lambda) d\lambda \]

\[ y(t) = \int_0^t e^{\lambda^2/2} \lambda \cos(\lambda)x(\lambda) d\lambda \]
4) (15 points) For the following two op-amps circuits, we can write \( v_{\text{out}}(t) = G \cdot v_{\text{in}}(t) \). Determine the value of \( G \) for each circuit.

For the first circuit:
\[
V^+ = \frac{V_{\text{in}} R_b}{R_a + R_b} \quad \frac{V^+}{R_a} + \frac{V_{\text{out}}}{R_c} = 0 \quad V_{\text{out}} = -\frac{R_e}{R_a} \cdot \frac{R_b}{R_c} \cdot V_{\text{in}}
\]

For the second circuit:
\[
\frac{V_{\text{in}}}{R_a} + \frac{V^*}{R_b} = 0 \quad V^* = -\frac{R_b}{R_a} V_{\text{in}} \quad V^* = \frac{V_{\text{out}} R_d}{R_e} \quad V_{\text{out}} = \frac{R_c + R_d}{R_c} \cdot \left(-\frac{R_b}{R_a}\right) \cdot V_{\text{in}}
\]
\( G \)
5) (15 points) The form of the under damped ($0 < \zeta < 1$) solution to the second order differential equation

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = K\omega_n^2 x(t)$$

for a step input $x(t) = Au(t)$ is

$$y(t) = KA \left[ 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi) \right]$$

where the damped frequency is $\omega_d = \omega_n \sqrt{1-\zeta^2}$, $\sin(\phi) = \sqrt{1-\zeta^2}$, and $\tan(\phi) = \frac{\sqrt{1-\zeta^2}}{\zeta}$

a) Determine an expression for the maximum value of the output, which occurs at the time to peak, $T_p = \frac{\pi}{\omega_d}$. Hence you are to determine $y(T_p)$. Note that $\sin(\pi + \phi) = -\sin(\phi)$. To receive credit you must simplify your answer as much as possible.

b) Determine an expression for $y(\infty)$, the steady state value of the output.

c) Determine an expression for the Percent Overshoot (PO), defined as

$$PO = \left[ \frac{y(T_p) - y(\infty)}{y(\infty)} \right] \times 100\%$$

To receive full credit you must simplify your answer as much as possible.

\[ a) \quad y(T_p) = KA \left[ 1 - \frac{e^{-\frac{\pi}{\omega_d} \sqrt{1-\zeta^2}}}{\sqrt{1-\zeta^2}} \sin \left( \frac{\pi}{\omega_d \sqrt{1-\zeta^2}} + \phi \right) \right] = KA \left[ 1 + e^{-\frac{\pi}{\sqrt{1-\zeta^2}} \sin(\phi)} \right] = KA \left[ 1 + e^{-\frac{\pi}{\sqrt{1-\zeta^2}} \sin(\phi)} \right] = y(T_p) \]

\[ b) \quad y(\infty) = KA \]

\[ c) \quad PO = \left[ \frac{y(T_p) - y(\infty)}{y(\infty)} \right] \times 100\% = e^{-\frac{\pi}{\sqrt{1-\zeta^2}}}) \times 100\% = PO \]
Problems 6-15, 3 points each, no partial credit (30 points)

Problems 6 and 7 refer to the following circuit:

6) The Thevenin resistance seen from the ports of the capacitor is

a) \( R_a = R_a + R_b \)  b) \( R_{th} = R_c \)  c) \( R_{th} = R_c \ || (R_a + R_b) \)  d) \( R_{th} = R_a + R_b + R_c \)  e) none of these

7) The static gain for the system is

a) \( K = 1 \)  b) \( K = \frac{R_c}{R_a + R_b + R_c} \)  c) \( K = \frac{R_a + R_b}{R_a + R_b + R_c} \)  d) \( K = \frac{R_c}{R_a + R_b} \)  e) none of these

Problems 8 and 9 refer to the following circuit

8) The Thevenin resistance seen from the ports of the capacitor is

a) \( R_{th} = R_a + R_b \)  b) \( R_{th} = R_c \)  c) \( R_{th} = R_c \ || (R_a + R_b) \)  d) \( R_{th} = R_a + R_b + R_c \)  e) none of these

9) The static gain for the system is

a) \( K = 1 \)  b) \( K = R_c \)  c) \( K = R_a + R_b \)  d) \( K = R_c \ || (R_a + R_b) \)  e) none of these
Problems 10 and 11 refer to the following circuit

10) The Thevenin resistance seen from the ports of the inductor is
   a) $R_{th} = R_a + R_b \parallel R_c$  b) $R_{th} = R_c + R_a \parallel R_b$  c) $R_{th} = R_a + R_b$  d) $R_{th} = R_a + R_c$  e) none of these

11) The static gain for the system is
   a) $K = 1$  b) $K = \frac{R_b}{R_a + R_b}$  c) $K = \frac{R_a}{R_a + R_b}$  d) $K = \frac{R_b}{R_c + R_b}$  e) none of these
Problems 12 and 13 refer the following graph showing the response of a second order system to a step input.

12) The percent overshoot for this system is best estimated as
   a) 180%  b) 150%  c) 100%  d) 80%  e) 60%  f) 50%
   \[
   \frac{1.8 - 1}{1} = 0.8
   \]

13) The static gain for this system is best estimated as
   a) 1.8  b) 1.2  c) 1.00  d) 0.83  e) 0.5
   \[
   k(1.2) = 1
   \]
   \[
   k = 0.83
   \]
Problems 14 and 15 refer the following graph showing the response of a second order system to a step input.

14) The percent overshoot for this system is best estimated as
   a) 300%   b) -300%   c) 200%   d) -200%   e) 33%   f) -33%
   \[ \frac{-2 - (-1.5)}{-1.5} = \frac{-0.5}{-1.5} = \frac{1}{3} \]

15) The static gain for this system is best estimated as
   a) -3   b) 3   c) 2.5   d) -2.5   e) 1.5   f) -1.5