Ray tracing - intersection

COMP575

Overview

- Homework
- Sphere intersection
- Triangle intersection

Homework review

- Homework 5

Sphere intersection

- Two ways to derive
  - Algebraic - from formula
  - Geometric - from shapes

Sphere intersection - algebraic

- Ray equation:
  - $\mathbf{p} = \mathbf{e} + t\mathbf{d}$
  - $\mathbf{e}$ is origin, $\mathbf{d}$ is direction
- Implicit sphere equation:
  - $(x-x_c)^2 (y-y_c)^2 (z-z_c)^2 - R^2 = 0$
  - $(\mathbf{p}-\mathbf{c})\cdot(\mathbf{p}-\mathbf{c}) - R^2 = 0$
  - $\mathbf{c}$ is sphere center, R is radius

Sphere intersection - algebraic

Ray $p = \mathbf{e} + t\mathbf{d}$ Sphere $(\mathbf{p} - \mathbf{c}) \cdot (\mathbf{p} - \mathbf{c}) - R^2 = 0$

Substitute ray in: $(\mathbf{e} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{e} + t\mathbf{d} - \mathbf{c}) - R^2 = 0$

Expand: $(\mathbf{d} \cdot \mathbf{d})t^2 + 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{c})t + (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - R^2 = 0$

Quadratic:

$$A = (\mathbf{d} \cdot \mathbf{d})$$
$$B = 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{c})$$
$$C = (\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - R^2$$

Sphere intersection - algebraic

Use quadratic eqn:

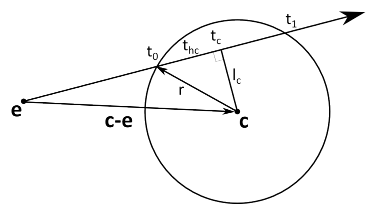
$$t = \frac{-\mathbf{d}\cdot(\mathbf{e}-\mathbf{c}) \pm \sqrt{(\mathbf{d}\cdot(\mathbf{e}-\mathbf{c}))^2 - (\mathbf{d}\cdot\mathbf{d})((\mathbf{e}-\mathbf{c})\cdot(\mathbf{e}-\mathbf{c}) - R^2)}}{(\mathbf{d}\cdot\mathbf{d})}$$

- Discriminant (square root part)
  - - : no intersect
  - 0 : tangent, one intersect
  - + : two intersects

Sphere intersection - geometric

- From geometry of ray and sphere

Sphere intersection - geometric



Assuming d is unit vector

Ray to sphere: $\mathbf{ec} = \mathbf{c} - \mathbf{e}$

Project on direction: $t_c = \mathbf{ec}\cdot\mathbf{d}$

Dis to closest: ${l_c}^2 = \mathbf{ec}\cdot\mathbf{ec} - {t_c}^2$

Hit offset dis: ${t_{hc}}^2 = R^2 - {l_c}^2$

Sphere intersection - geometric

Final geometric solution:

$$\mathbf{ec}\cdot\mathbf{d} \pm \sqrt{R^2 - \mathbf{ec}\cdot\mathbf{ec} + (\mathbf{ec}\cdot\mathbf{d})^2}$$

Sphere intersection - geometric

Geometric:

$$\mathbf{ec}\cdot\mathbf{d} \pm \sqrt{R^2 - \mathbf{ec}\cdot\mathbf{ec} + (\mathbf{ec}\cdot\mathbf{d})^2}$$

Algebraic:

$$t = \frac{-\mathbf{d}\cdot(\mathbf{e}-\mathbf{c}) \pm \sqrt{(\mathbf{d}\cdot(\mathbf{e}-\mathbf{c}))^2 - (\mathbf{d}\cdot\mathbf{d})((\mathbf{e}-\mathbf{c})\cdot(\mathbf{e}-\mathbf{c}) - R^2)}}{(\mathbf{d}\cdot\mathbf{d})}$$

Sphere intersection - geometric

Geometric:

$$\mathbf{ec}\cdot\mathbf{d} \pm \sqrt{R^2 - \mathbf{ec}\cdot\mathbf{ec} + (\mathbf{ec}\cdot\mathbf{d})^2}$$

Algebraic with unit direction:

$$t = -\mathbf{d} \cdot (\mathbf{e} - \mathbf{c}) \pm \sqrt{(\mathbf{d} \cdot (\mathbf{e} - \mathbf{c}))^2 - ((\mathbf{e} - \mathbf{c}) \cdot (\mathbf{e} - \mathbf{c}) - R^2)}$$

Triangle intersection

What is a triangle?
Plane with bounded region. A triangle's region is defined by its 3 vertices. These vertices can be used to form bounding halfplanes.

Triangle intersection

- Plane intersection
  - Does the ray even hit the plane the triangle is on?

We need an equation for a plane...
Planes can be defined as a direction and a distance from the origin. This can be expanded to a point on the plane, and a direction. This is a more useful definition for us, so we will use it.

Triangle intersection

- Plane intersection

Ray $p = \mathbf{e} + t\mathbf{d}$ Plane $(\mathbf{p} - \mathbf{a}) \cdot \mathbf{n} = 0$

Substitute:

$$(\mathbf{e} + t\mathbf{d} - \mathbf{a}) \cdot \mathbf{n} = 0$$

Solve for t:

$$t = \frac{(\mathbf{a} - \mathbf{e}) \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}$$