DETECTION OF NONSTATIONARY RANDOM SIGNALS IN COLORED NOISE

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ABSTRACT

This paper decribes a novel method for detecting nonstationary signals in colored noise. A first order complex autoregressive, or AR(1), signal model is used which restricts the application of the detector to low order signals, i.e., those which are well modeled by a low order AR process and have only a single spectral peak. The detector assumes the noise covariance is stationary and known. The likelihood function is estimated in the frequency domain because the model simplifies, and the nonstationary frequency estimate can be obtained by an algorithm which approximates the Viterbi algorithm. The AR model parameters are then used to form the appropriate covariance matrix and the approximate likelihood is calculated. Therefore, the detector uses efficient approximations to approximate the generalized likelihood ratio test (GLRT). Simulation results are shown to compare the detector with the known signal likelihood ratio test.

1. INTRODUCTION

A number of interesting algorithms for detecting a nonstationary random signal have been suggested in the literature. Typically, these methods either require that the signal be narrowband, or assume a priori information about the signal's time-frequency distribution, or "scattering function." The detector described below takes advantage of a time-varying first order complex Gaussian autoregressive model assumption to detect a low order Gaussian signal. A low order Gaussian signal is defined here to mean one which is well modeled by a low order autoregressive (AR) model and which has only one spectral peak. The signal model is commonly referred to as an AR(1) model. Although the detector requires that the signal frequency vary slowly,

no other information is assumed about the signal's frequency path. The frequency path is estimated using a near maximum likelihood technique, and the detector is based on an approximation to the generalized likelihood ratio test (GLRT). Note that with an AR(1) signal model, the detector easily generalizes to wideband signals by simply reducing the model's pole radius.

2. PROBLEM

The following mathematical model gives a framework for describing the detection algorithm. Let the received signal be a zero mean complex Gaussian random vector $\underline{x}$. Then let $\underline{x}(p)$ be defined as a subset or block of $\underline{x}$ with

$$\underline{x}(p) = [x(pM), x(pM+1), \ldots, x(pM+(M-1))]^T \quad (1)$$

where M is the number of samples per block, and P is the number of blocks in the observation. Define $\mathbf{K}_x(p,q) = E[\underline{x}(p)\underline{x}'(q)]$ to be the cross-covariance matrix of any two blocks of $\underline{x}$. There are two possible hypotheses about $\underline{x}$, H_0 and H_1:

$$H_0: \quad \mathbf{K}_x(p,p) = \mathbf{K}_n(p,p) \qquad \text{for all } p$$
$$H_1: \quad \mathbf{K}_x(p,p) = \mathbf{K}_s(p,p) + \mathbf{K}_n(p,p) \quad \text{for all } p$$
$$(2)$$

Since the nonstationary signal frequency is required to be slowly varying, the blocklength, M, can be chosen so that the signal frequency is approximately stationary for the duration of each block. This assumption allows the use of stationary signal detection techniques on the blocks. For instance, the GLRT can be approximated by estimating the signal frequency at each block and then combining the results from each block to find the likelihood for the entire observation. This procedure is described further below.

3. BLOCK DETECTION

It is possible to compute an unrestricted GLRT such as

$$\max_{\mathbf{K}_1} l_1(\underline{x}|\mathbf{K}_1) - l_0(\underline{x}) \overset{H_1}{\underset{H_0}{\gtrless}} 0 \qquad (3)$$

where l_i is the log likelihood function under H_i. However, an unconstrained $\mathbf{K}_1$ has too many free parameters to maximize reasonably. By applying the block-stationary assumption and the AR signal model, the problem can be simplified. Since $\underline{s}$ is assumed to be a nonstationary AR(1) process, $\mathbf{K}_1(p,p)$ can be written entirely in terms of the signal parameters, $G, r, \theta(p)$, (signal gain, pole radius, and pole angle, respectively) and the noise covariance $\mathbf{K}_n$. Then the alternative detector can be computed as

$$\max_{G,r,\underline{\theta}} \sum_{p=0}^{P-1} l_1(\underline{x}(p)|G, r, \theta(p)) - l_0(\underline{x}(p)) \overset{H_1}{\underset{H_0}{\gtrless}} 0 \qquad (4)$$

This alternative detector makes the implied assumption that $\mathbf{K}_x(p,q) = 0$ when $p \neq q$; therefore it only approximates the GLRT. This assumption is simply that the blocks are independent of one another. Although this is only true for the white signal and noise case, in many cases when the blocksize is large, only nearby samples in adjacent blocks have any significant correlation. Clearly, there are performance tradeoffs between the accuracy of the block stationary assumption which requires a small blocksize, and the accuracy of the block independence assumption which improves with larger blocksizes.

4. MULTIPARAMETER MAXIMIZATION

The detector given in (4) is simple to compute except for the multiparameter maximization. A few reasonable simplifications, however, can make this maximization tractable. First of all, values of r and G may be known a priori. Even if they must be estimated, the accuracy of these parameters is not as important to detection performance as the frequency parameter because the detector is relatively insensitive to perturbations in these parameters [1]. Second, the frequency parameters need not be continuous but if the necessary required accuracy is known, can be limited to a finite number of discrete values defined in advance. Discretizing the parameters substantially reduces the size of the maximization problem. The use of the FFT tends to suggest the discrete frequencies associated with it, but the method is in no way restricted to this choice of discrete frequency values.

The search for the maximum likelihood path through the frequency values is equivalent to a tree-searching problem since both the time blocks and the frequencies are treated discretely. This treatment is yet another way to take advantage of the slowly varying frequency assumption. Because a tree-searching approach applies, the Viterbi algorithm can be used to find the optimum solution [2]. If P is the number of blocks in the observation $\underline{x}$, and V is the number of discrete values of θ allowed, the Viterbi algorithm requires $O(V^2 P)$ computations, which may be overly expensive when a large frequency resolution is needed.

Now consider restricting the signal to have a slowly-varying frequency parameter. The detector in (4) chooses $\theta(p)$ to maximize l_1, but in low SNR problems this can give a discontinuous signal frequency path even when the actual path is smooth. To minimize erroneous path discontinuities it is useful to place a restriction on the rate of frequency change. The slowly-varying assumption is taken to mean that the frequency parameter can vary only up or down by one discrete value, or bin, from one block to the next. This reduces the computational requirements of the Viterbi algorithm to $O(VP)$. Based on simulations, the one bin restriction significantly improves the estimator's performance, even when there is a fairly rapid rate of frequency change.

In addition to the assumptions and restrictions given above, further computational savings can be obtained by utilizing efficient approximations to the likelihood function and the Viterbi algorithm. The "stack" algorithm, an efficient approximation to the Viterbi algorithm, is thoroughly described in [3], and will not be covered here. An efficient approximation to the likelihood function is suggested in [4], which has been used by the authors to reduce the computation necessary to implement the new detector. A thorough description of this approximation is given below.

5. APPROXIMATE LIKELIHOOD

In (4) above, the log likelihood function can be written, without the constant terms which cancel, as

$$l_i(\underline{x}(p)) = -\frac{1}{2}\ln|\mathbf{K}_i(p,p)| - \frac{\underline{x}'(p)\mathbf{K}_i^{-1}(p,p)\underline{x}(p)}{2} \qquad (5)$$

which requires $O(M^2)$ multiplies to compute. However, the problem can be converted to the frequency domain by letting

$$\tilde{\underline{x}}(p) = \mathbf{W}\underline{x}(p) \qquad (6)$$

and

$$\tilde{\mathbf{K}}_i(p,p) = \mathbf{W}\mathbf{K}_i(p,p)\mathbf{W}' \qquad (7)$$

where $\mathbf{W}$ is the FFT matrix and $\tilde{\ }$ represents conversion to the spectral domain. Then (5) becomes

$$l_i(\tilde{\underline{x}}(p)) = -\frac{1}{2}\ln|\tilde{\mathbf{K}}_i(p,p)| - \frac{\tilde{\underline{x}}'(p)\tilde{\mathbf{K}}_i^{-1}(p,p)\tilde{\underline{x}}(p)}{2} \quad (8)$$

which has the same order of complexity as before, and gives identical results. However, because the FFT transformation tends to diagonalize $\tilde{\mathbf{K}}_i$, it is possible to approximate (8) by

$$l_i(\tilde{\underline{x}}(p)) = -\frac{1}{2}\ln|D(\tilde{\mathbf{K}}_i(p,p))| - \frac{\tilde{\underline{x}}'(p)D(\tilde{\mathbf{K}}_i^{-1}(p,p))\tilde{\underline{x}}(p)}{2}$$

$$(9)$$

where $D()$ is defined as the operator which sets the off-diagonal elements to zero. Now (9) can be evaluated with a vector multiply instead of a matrix multiply, for a significant computational savings. On the other hand, (9) is no longer the exact likelihood function. The approximate likelihood function has been found to be good for a variety of signals, but best for wideband signals which have a greater tendency towards diagonalizing in the frequency domain [1].

6. RESULTS

Figure 1 shows the short-time Fourier transform (STFT) of a synthetic nonstationary AR(1) signal superimposed on harmonic noise. This signal is an example of the type of problem that motivates the detection algorithm described above. Note that with an observation length of many thousands of samples, quadratic

detection is computationally prohibitive. Clearly the signal would be difficult to detect, even if its non-frequency parameters (pole radius and signal power) were known. Because the signal is the only nonstationary process in the data, it is possible to estimate the noise covariance by a stationary estimator that tends to smear and de-emphasize the nonstationary signal. With an estimate of the signal parameters, detection of the signal is straightforward. Since the estimation technique is based on maximizing the likelihood function, detection performance should approach the generalized likelihood ratio test's performance.

An example frequency estimate track is shown in Figure 2. The noise covariance was estimated from the signal and noise data using a stationary covariance estimator. For this estimator, no a priori knowledge of the noise was needed and only the nonstationarity of the signal and true values for G and r were assumed known. The signal frequency estimator uses the assumption that the frequency is slowly varying, as described above, to reduce discontinuities. This technique gives much better results than individual-block peak picking methods. Given such an estimate as input, it is clear that the detector can perform nearly as well as one with a known signal covariance.
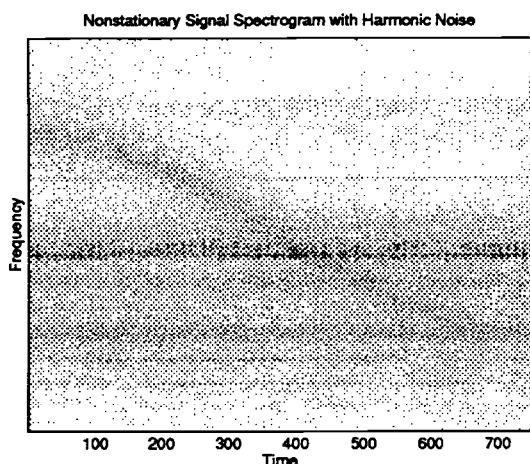

Figure 1: The spectrogram is the magnitude of 256 point FFTs with a rectangular window. The data blocks are non-overlapping. The "harmonic noise" is actually measured sonar data.
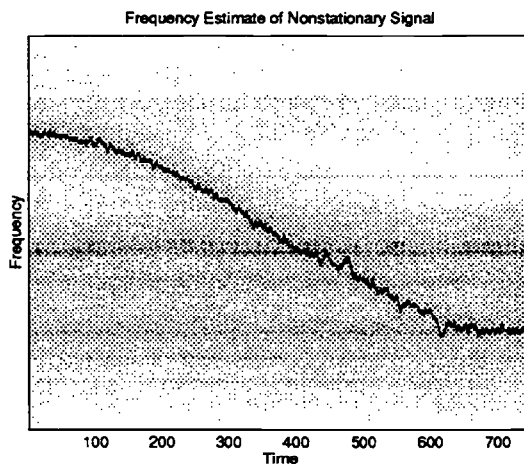

Figure 2: The spectrogram is the same as in Figure 1. The true instantaneous frequency of the nonstationary signal is a half-period of a sinusoid.

In order to verify the detection algorithm's behavior and compare it to a reasonable standard, simulations have been run for the case where the signal is an AR(1) process with a pole located at $\sqrt{0.9}\exp(i\pi/4)$. The observed sequence was 256 samples long and the noise was white. Although this case does not take advantage

of the detector's ability to deal with nonstationary signals and colored noise, it allows a simple comparison to be made with the likelihood ratio test (LRT) for the known signal case, which is the optimum detector. For the simulations, trials were performed at 1dB intervals from -25dB SNR to 5dB with the maximum probability of false alarm set to approximately 0.1. For the LRT case there were 100 trials for each SNR value, and for the new detector there were 300 trials per SNR value. Note that neither the LRT or the proposed detector are constant false alarm rate detectors. The simulation results are shown in Figure 6. More iterations were required for the new detector because the variance of the probability estimates was larger than for the LRT detector. The larger variance is reduced in the figure by a 3 sample median smoothing of the probability curve for both detectors.
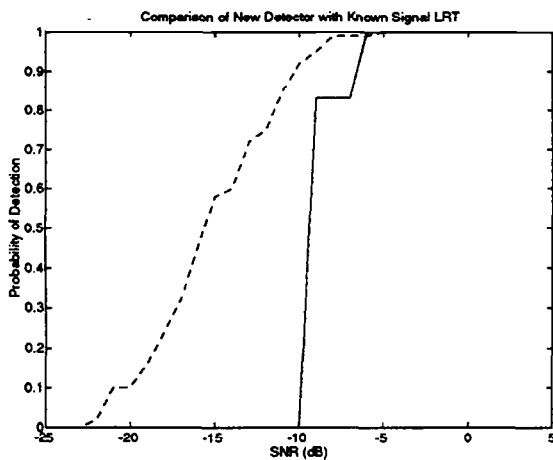

Figure 3: Comparison of the likelihood ratio test detector with known signal (dashed line) with the new detector (solid line). The threshold is set so that the maximum probability of false alarm is approximately 0.1. Both probability curves are median smoothed.

7. CONCLUSIONS

Based on these results, the proposed new detector is clearly detecting signals in the same region of SNR as the optimum known signal detector. Although the known signal detector is somewhat more sensitive in the lower SNR region, this is expected, since more information is available to it. The reduced sensitivity as well as the larger variance of the probability estimates for the new detector are at least partly due to a nonstationary signal assumption, which the LRT detector does not make. In the future the authors intend

to present a more detailed performance analysis of the detector and include estimation of other AR model parameters in the algorithm.

8. REFERENCES

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