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A laboratory experiment on inferring Poiseuille's law for undergraduate students

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Abstract

In this paper a laboratory experiment is proposed to infer Poiseuille's law. A simple set-up based on two flasks joined by a detachable tube allows one to measure using tubes of different radii and different lengths. One of the flasks is connected to a vacuum pump to control the pressure differential between the tube extremes. The influence on the flow of different radii, lengths, pressures and viscosities can be studied in a didactic way by measuring the flow rate for each of these variables. The experiment can be performed getting together the students in groups, so that each group concentrates on the effect on the flow of a specific variable, leaving the rest fixed. After putting together all these results the law of Poiseuille can be obtained as a very good approach.

1. Introduction

The study of the flow of real fluids through tubes is of considerable interest in basic science (physics, chemistry, etc), in biomedical science (hemodynamics, biorheology, etc) (Karnak *et al* 2001) and in engineering (chemical engineering, industrial engineering, oil drilling, etc) (Mott 1996). Students thereof are familiarized with the theory of the laws regulating flow, though such knowledge is typically limited to laminar flow through a cylindrical tube for small Reynolds numbers (Jou *et al* 1994).

Such behaviour is attributable to Poiseuille's law, expressed as follows (Landau and Lifshitz 1984):

$$Q = \frac{\pi R^4}{8\eta L}\Delta P, \tag{1}$$

where Q is the flow rate or flow volume per unit time, R and L are the radius and length of the tube, respectively, η is the viscosity of the fluid and ΔP is the pressure difference between the ends of the tube.

In undergraduate laboratories, experiments based on Poiseuille's law are carried out using a fixed tube diameter and fixed fluid experimental set-up in the exercise consisting of measuring the output obtained after varying the pressure difference. Another version of the experiment involves determination of the viscosity of a fluid using glass capillary viscometers (Ostwald, Canon-Fenshke, etc) that are improved versions of the former.

The experiment proposed in this study goes further and takes into account the teaching potential of laboratory exercises (Salinas 1994, Hofstein and Lunetta 2004), which allow students to understand concepts that are difficult to explain in theory classes in both fluid dynamics and fluid static (Loverude *et al* 2003, Heron *et al* 2003, Maroto *et al* 2002). The experiment, based on a constructivist view of science learning (Salinas 1994), consists of the experimental inference of Poiseuille's law and can be carried out once students are familiar with the concept of viscosity, but do not necessarily know the Poiseuille's law.

2. Experimental set-up and procedure

2.1. First part: postulating the hypothesis

In the first part the teacher poses the problem to be solved: the physical law that allows calculation of the flow rate in a tube is needed due to its multiple scientific and technological applications (it would be of interest to enumerate them).

Effecting order to obtain this law, we should first define the variables involved in the problem. The students are faced with the following question: 'What does the flow rate of a tube depend upon?' Develop a hypothesis. The students' previous experience with related cases, such as the flow from garden hoses, the flow out of a syringe, etc, can help in this process.

After a period of individual thought about the question, groups of students are formed to allow them to discuss the question among themselves, since peer discussion is very useful for learning (Cox and Junkin 2002). The conclusion is then eventually reached that the variables that intervene in the problem are the pressure difference between the ends of the tube (ΔP), tube radius R and length L, and the circulating fluid, represented by the viscosity η. It is also advisable to analyse the limiting situations in which these variables are either zero or very large. We face a function $Q = f(\Delta P, R, L, \eta)$.

2.2. Second part: experimental design

The experimental set-up needed to infer Poiseuille's law must allow variation of all the magnitudes that intervene in fluid flow through tubes (ΔP, R, L and η). Moreover, the set-up should be as simple and easy to handle as possible. One option is to allow the students to try to design an experimental set-up satisfying these requirements.

The set-up proposed in the present study (figure 1) consists of a container flask CF with the study fluid, and a receptor flask RF for collecting the liquid that passes through the tube T in a period of time t. In this latter flask it is possible to modify the pressure, measured with a manometer M, by means of a vacuum regulating system. The tubes are easily replaced by means of the wing nuts R on both flasks. Flask RF must be easily separable from the rest of the assembly. Key B in turn is used to connect RF to the vacuum system or atmosphere.

In our experiment we have used tubes T of radii 0.61, 0.88, 0.91 and 1.11 mm. For each radius there are tubes measuring 10, 15, 20, 30 and 40 cm in length. The experiment requires the use of fluids with viscosity values that are independent of the flow velocity (Newtonian fluids). These fluids can be obtained, e.g., with different concentrations of glycerol and water,

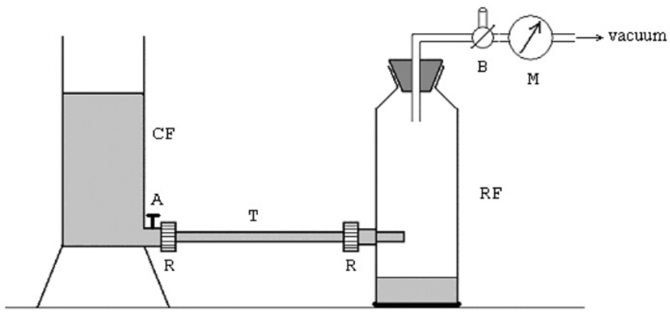

Figure 1. Experimental set-up.

presenting viscosities between about 0.1 and 1 Pa s, determined by any available viscometer (like the Ostwald or Canon-Fenshke type). We used a Brookfield DV II rotary viscometer and selected six glycerol and water solutions with viscosities between 0.110 Pa s and 0.934 Pa s.

The measurement method is as follows. Flask RF is weighed with an electronic balance of accuracy in the mg range (Mettler Toledo PB 3003), and is placed in the experimental set-up (figure 1). The study fluid has been placed in flask CF, with key A closed.

The vacuum system is turned on to obtain a pressure difference ΔP between the ends of the tube T (we assume the fluid height in CF to be in the order of 10 cm, as a result of which the hydrostatic pressure can be considered negligible), measured with the manometer M. Key A is then opened, and the liquid circulating through T accumulates in RF. After a period of time t, measured with a chronometer, key A is closed, and flask RF is removed and weighed. The mass of liquid divided by its density (as determined previously) yields the liquid volume that has passed through the tube. Dividing this volume by the time t then yields the flow rate through tubes of different radii and lengths, for the liquids of different viscosities.

2.3. Third part: measurements

Since we are studying the influence of the four variables ΔP, R, L and η, the class can be divided into groups in which each group studies the influence of one or two of the variables simultaneously, depending on the time assigned to the exercise and the material available in the laboratory. Below are detailed the two ways of proceeding. As an example, the first group studies the relationship between the flow rate and ΔP and L simultaneously, while the other two groups only study the influence of η and R, respectively.

2.3.1. Group I. Relationship between output and pressure difference and length. This group of students always uses the same liquid (as a result of which viscosity remains constant) and tubes with the same radius. As an example, for $R = 0.88$ mm, the values obtained for output as a function of ΔP for each of the lengths are shown in figure 2.

The experimental values corresponding to each length have been fitted using lineal functions of the type $Q = m\Delta P$. The values of m obtained, and their corresponding correlation coefficients r^2, are reported in table 1.

The values of r^2 obtained confirm the goodness of the lineal fits, as a result of which it can be concluded that the flow rate and pressure differences established between the ends of the tube are directly proportional.

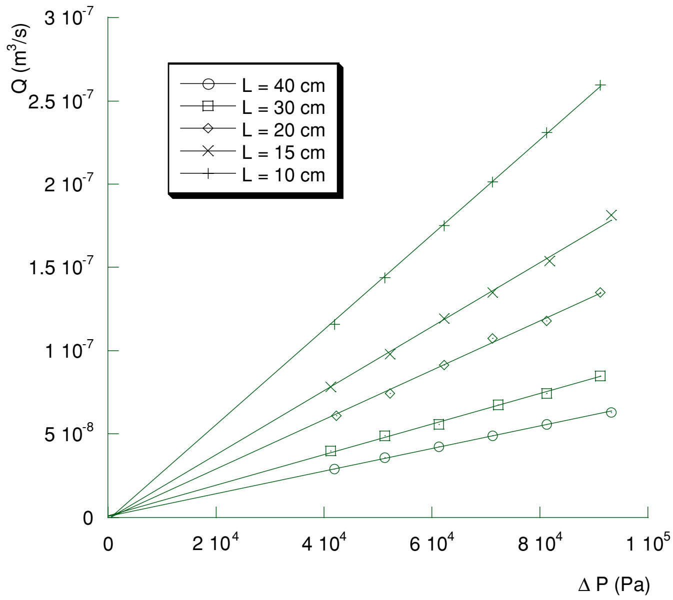

Figure 2. Flow rate Q as a function of the difference of pressure ΔP for tubes of $R = 0.88$ mm and different lengths, L.

Table 1. Values of the proportionality factor m, $Q = m\ \Delta P$, for tubes of different lengths (L).

L (m)	m ($\mathrm{m^3\ s^{-1}\ Pa\ s^{-1}}$)	r^2
0.10	$(2.83 \pm 0.01) \times 10^{-12}$	0.999
0.15	$(1.91 \pm 0.01) \times 10^{-12}$	0.999
0.20	$(1.47 \pm 0.01) \times 10^{-12}$	0.998
0.30	$(0.928 \pm 0.007) \times 10^{-12}$	0.998
0.40	$(0.685 \pm 0.003) \times 10^{-12}$	0.999

Output dependence upon tube length On the other hand, it is seen that the values of the proportionality factor m reported in table 1 vary with the length of the tube, L. The graphic representation of m versus L is shown in figure 3.

Posterior fitting of $m = f(L)$ has been carried out with a potential function of the type

$$m = K \cdot L^{K_1}. \tag{2}$$

The values of K and K_1 obtained are (all expressed in SI)

$$K = (2.86 \pm 0.01) \times 10^{-13} \qquad K_1 = -(1.00 \pm 0.02) \qquad r^2 = 0.999. \tag{3}$$

The value of the exponent of L, $K_1 = -1$, shows that m, and therefore Q, is inversely proportional to the length of the tube.

2.3.2. Group II. Output dependence upon fluid viscosity. To study the influence of the viscosity, the second group of students measures liquids with different viscosities, the rest of the variables remaining constant. For example, six solutions of glycerol in water can be prepared, with different viscosities, repeating the experiment with each of them using a tube

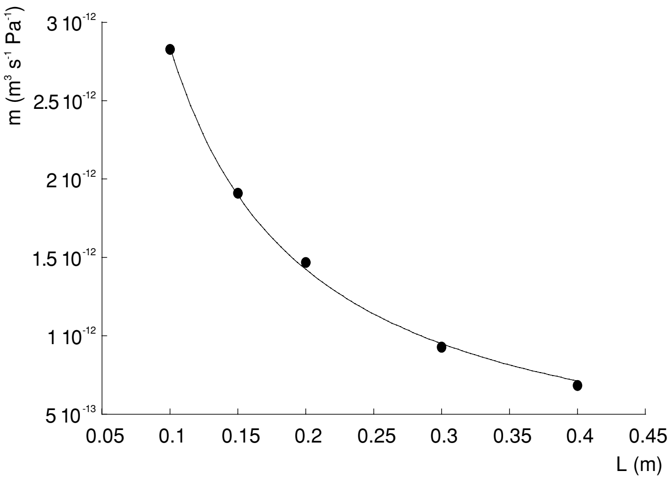

Figure 3. Value of the proportionality factor m (table 1) as a function of the tube length, L.

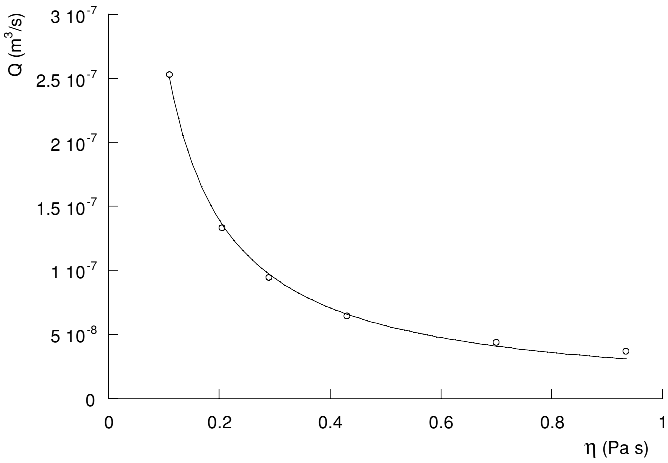

Figure 4. Plot of the flow rate, Q, as a function of viscosity, η (tube length $L = 30$ cm and radius $R = 0.91$ mm).

(30.0 ± 0.1) cm in length and with a radius of (0.91 ± 0.01) mm, the different liquids being passed through the tube for 300 s at a fixed ΔP of $(31 \pm 1)\ 10^3$ Pa.

The experimental values of Q obtained have been graphically represented (figure 4) and fitted by means of a potential function of the type

$$Q = K_2 \eta^{K_3}. \tag{4}$$

The values of K_2 and K_3 obtained (expressed in SI) are

$$K_2 = (2.88 \pm 0.14) \times 10^{-8} \qquad K_3 = -(0.98 \pm 0.03) \qquad r^2 = 0.998. \tag{5}$$

The students' group should reach the conclusion that the value of K_3 (exponent of η) is -1; consequently, Q is inversely proportional to the viscosity.

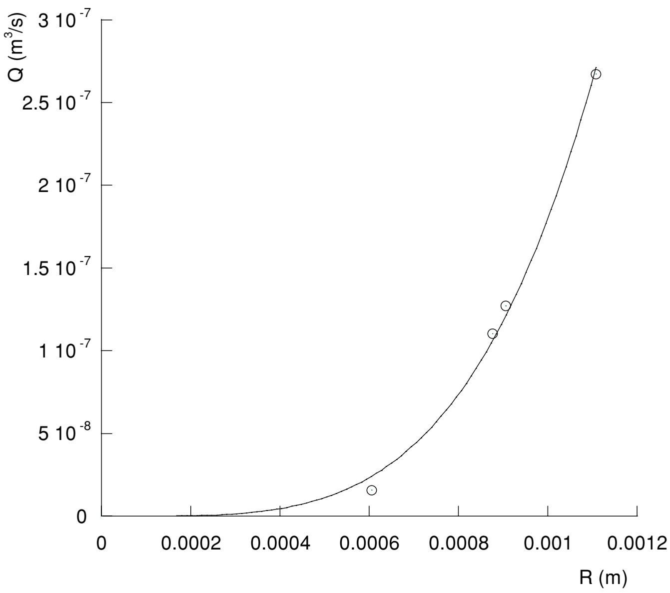

Figure 5. Plot of the flow rate, Q, as a function of the radius, for tubes of equal length (0.30 m) and the same glycerol solution.

2.3.3. Group III. Output dependence upon the radius of the tube. In this case, using one of the glycerol solutions ($\eta = 0.166$ Pa s and $\rho = 1.233 \times 10^3$ kg m^{-3}), tubes of equal length ($L = 0.30$ m) and a fixed ΔP of $(60 \pm 1)\ 10^3$ Pa, the volume output is measured for tubes of different radii.

The values of Q obtained have been plotted as a function of the radius (figure 5), and fitting by means of a potential function of the type

$$Q = K_4 R^{K_5} \tag{6}$$

yields the following values of K_4 and K_5:

$$K_4 = (14.76 \pm 0.18) \times 10^{-4} \qquad K_5 = (3.9 \pm 0.2) \qquad r^2 = 0.997. \tag{7}$$

From the value of K_5 (the exponent to which the radius R is raised), the output is inferred to be directly proportional to the fourth power of the radius.

2.4. Fourth part: inference of Poiseuille's formula

Eventually, dimensional analysis can help in determining the exponents more precisely. Dimensions of Q are $L^3\ T^{-1}$, and $ML^{-1}\ T^{-2}$ and $ML^{-1}\ T^{-1}$ for ΔP and η respectively, which suggest that they can be related as $\Delta P/\eta$. Therefore R and L are involved in a form of dimensions L^3. Therefore $K_1 = -1$, $K_3 = -1$ and $K_5 = 4$.

However, the students should realize that the adimensional coefficients involved can be obtained not by dimensional analysis but experimentally. From the global results obtained, it can be concluded that the formula sought is of the following type:

$$Q = K_6 \frac{R^4}{\eta L} \Delta P. \tag{8}$$

Based on the existing experimental data, the different working groups can now calculate the value of K_6, followed by determination of its mean value. With the above data, $K_6 = 0.41 \pm 0.04$ (adimensional)—a result that is practically coincident with the value of $\frac{\pi}{8} = 0.393$ from Poiseuille's equation (1). The law regulating fluid flow through tubes is thus demonstrated.

3. Conclusions

With the proposed exercise, the student is able to infer Poiseuille's law, which determines the flow rate of a fluid through a tube by means of an experimental design that clearly illustrates the different steps of the scientific method.

The proposed set-up is relatively simple and inexpensive, and offers very satisfactory results. An added advantage is that the layout can also be used as a capillary viscometer for both Newtonian and non-Newtonian fluids, since it allows us to modify the flow velocity by varying the pressure difference between the ends of the tube.

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