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# Determination of surface tension coefficient of liquids by diffraction of light on capillary waves

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## Abstract

This paper describes a simple technique for determining the coefficient of the surface tension of liquids, based on laser light diffraction on capillary waves. Capillary waves of given frequency are created by an exciter needle acting on the surface of liquid and represent a reflective diffraction grating, the constant of which (the wavelength of capillary waves) can be determined based on a known incidence angle of light (grazing angle). We obtain the coefficient of the surface tension of liquids by applying the dispersion relation for capillary waves and analyze the difficulties that arise when setting up and conducting the experiment in detail.

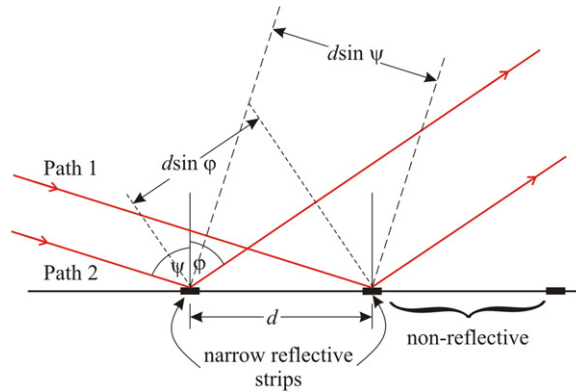
(Some figures may appear in colour only in the online journal)

## 1. Introduction

Methods for determining the surface tension of liquids based on light diffraction on capillary waves are very significant because they are nondestructive procedures [1, 2].<sup>3</sup> This method is particularly important in situations where we have very small samples or where the perturbation of the system by the devices used in conventional methods for determining the coefficient of surface tension is unacceptable. This is the case, for instance, when it is necessary to measure the parameters of thin films on the surface of liquid. This method is particularly applicable in biology, where thin films are used for cell membrane modeling.

By setting up, conducting and analyzing experiments of this type, in addition to mastering yet another method for determining the coefficient of surface tension, students can see the unity of natural phenomena and the richness of the results that can be obtained in a physical experiment. Mechanical and electromagnetic waves are in fact very often taught in universities

<sup>3</sup> The first measurement of the coefficient of surface tension based on the analysis of waves in the fluid, mentioned in literature was in 1890 and was made by Lord Rayleigh [3].



**Figure 1.** Path difference of rays observed on a reflection diffraction grating.

as part of specific courses, and consequently they are often treated as completely independent and separate natural phenomena; in this experiment, however, the focus is on their interaction and its associated quantities.

Since capillary waves serve as a reflective diffraction grating for light, the physical basics of diffraction on a reflection grating are presented in section 2. In section 3 we describe the basics of the experiment, with details of measuring the appropriate quantities relevant for later determination of the coefficient of surface tension. In section 4, we analyze the measurement results, and in section 5 there are concluding remarks, mainly related to the difficulties that may arise during the set-up and performance of the experiment. We also indicate the possibility of other, more complex measurements based on laser light diffraction on capillary waves.

## 2. Capillary waves as a reflection diffraction grating for laser light

Unlike the transmission diffraction grating in which the light passes through a series of narrow slits located between the opaque parts of a glass or plastic plate, with reflection gratings the light is reflected from a series of parallel, close, smooth surfaces and is absorbed or diffused by the surface areas located between them (figure 1). Consequently, the destructive interference does not occur along every path where the reflected angle is not equal to the incidence angle, but on some of them the constructive interference will occur. As a result, a series of bright spots appears on the screen, rather than just one.

Path 1 is longer by  $d \sin \psi$  from path 2 before the arrival of rays to the smooth part of the grating, but it is shorter from it by  $d \sin \varphi$  after reflection ( $d$  is the distance between adjacent reflective parts). When this difference is equal to the whole number of wavelengths of light  $\Lambda$

$$d(\sin \psi - \sin \varphi) = m\Lambda, \quad m = 0, \pm 1, \pm 2, \dots, \quad (1)$$

the waves reflected from the adjacent reflective surfaces are in phase and this leads to the constructive interference [4–6]. Zeroth-order interference maximum ( $m = 0$ ) is in the direction of the so-called geometric reflection, for which  $\psi = \varphi$ .

For further application it is convenient to introduce the angle  $\theta$  between the incident ray and the plane of the grating, for which this applies  $\theta + \psi = \pi/2$  and the angle  $\delta$  between the direction of the geometrically reflected ray and the direction of the first constructive interference  $\psi - \varphi_1 = \delta$  (for  $m = 1$ ). It is shown that in this case  $\Lambda = 2d \sin \frac{\delta}{2} \sin \left( \theta + \frac{\delta}{2} \right)$ .

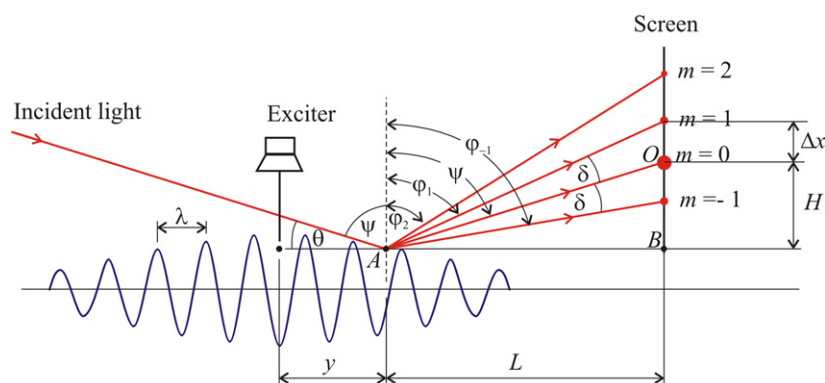


Figure 2. Sketch of the experiment.

If we apply the same procedure to the first ray of constructive interference, but on the other side of the geometrically reflected ray for which  $\varphi_{-1} - \psi = \delta$ , by an analogous procedure relation  $\Lambda = 2d \sin \frac{\delta}{2} \sin \left( \theta - \frac{\delta}{2} \right)$  is obtained. Adding these two equations leads to the important formula

$$\Lambda = d \sin \frac{\delta}{2} \left( \sin \left( \theta + \frac{\delta}{2} \right) + \sin \left( \theta - \frac{\delta}{2} \right) \right). \quad (2)$$

The previous relation can be written using the wave number of capillary waves [7–9]

$$k = \frac{2\pi}{\Lambda} \sin \frac{\delta}{2} \left( \sin \left( \theta + \frac{\delta}{2} \right) + \sin \left( \theta - \frac{\delta}{2} \right) \right). \quad (3)$$

### 3. The experiment

The dispersion relation for capillary waves [7, 10–12]  $\omega^2 = \sigma k^3 / \rho$  (the Kelvin relation) can, if written as

$$\sigma = \frac{\rho \cdot \omega^2}{k^3} = \frac{\rho v \lambda^3}{2\pi} \quad (4)$$

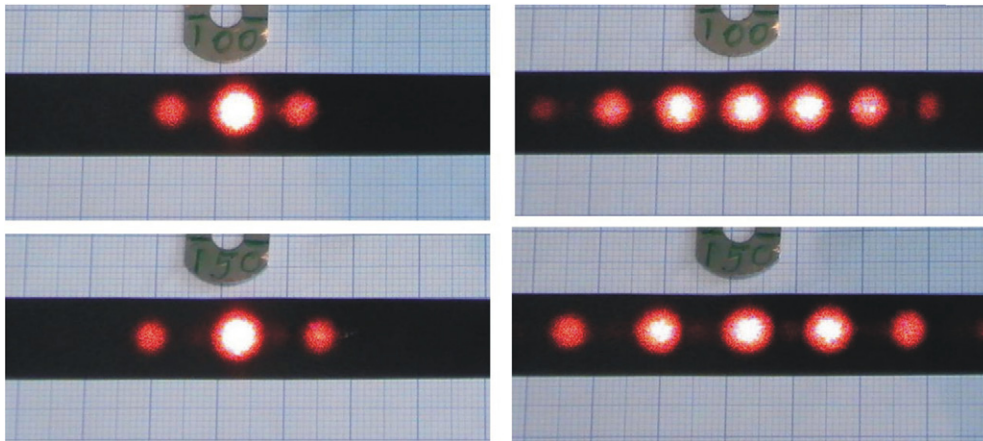
be used to experimentally determine the coefficient of surface tension. For this purpose it is necessary to measure/know the density of the fluid on which the capillary waves are generated, their wavelength and frequency. A sketch of the experimental set-up for measuring these quantities is shown in figure 2.

The equipment for measuring comprises a digital signal generator, an amplifier, an oscilloscope/frequency meter handyscope<sup>4</sup> HS2, a camera tripod, speakers, an exciter, a micrometer, a styrofoam plate of about 2 cm in depth, a screen, a helium–neon (He–Ne) laser, a charged-coupled device (CCD) camera, a meter tape, a thermometer, a metal plate and a notebook computer.

The speaker was taken from the computer (COMPAQ 19190 THAI AD8DB 356080-001 A) and we taped the exciter-needle to its paper diaphragm with the tip touching the water surface. It was excited by the digital signal generator (Iskra MA 3605) by low power sine signals in the frequency range from 50 to 400 Hz, in steps of 25 Hz<sup>5</sup>. The speaker was fixed to

<sup>4</sup> A type of multifunctional computer controlled measuring instrument.

<sup>5</sup> Capillary waves can also be excited by the optical action of laser radiation; the technique is called 'laser-induced surface deformation'.



**Figure 3.** Diffraction patterns for water for  $\nu_1 = 100$  Hz and  $\nu_2 = 150$  Hz. (a) Smaller signal generator output power and (b) larger output power. There are more spots in fringe pattern (b), but there is no difference in distance ( $\Delta x$ ) between them.

a metal plate located on a stand, with three degrees of movement—back and forth, side to side, up and down—and with it we could adjust the position of the speaker. The selected frequency values were read on the function generator display and controlled by the frequency meter on the computer [7, 8, 11].

The frequency generator and laser were separated from the table on which the plate with the liquid was positioned. We used the He–Ne laser ‘Iskra’ as a light source, with power  $P = 2$  mW and wavelength  $\Lambda = 632.8$  nm (red). It was set to a distance  $D = 1$  m from the surface of the liquid and fixed with two stands and two handles at a height of 10–15 cm. The laser was then finely adjusted so that the beam of light fell sideways with a small angle  $\theta$  to the surface of liquid. The laser beam was narrow and had a small angular width but it ‘spread’ on the surface of the liquid due to falling sideways and hitting a few crests of capillary waves. Special care should be taken that the laser beam falls outside the scope of the radius  $y$  (figure 2) and away from the walls of the container and also not hitting the exciter. The position of the capillary wave exciter needle was chosen so that the needle was in the plane with the incidence and the referent ray. In our experiment, the modulation point  $M$  (the point where the exciter needle touches the surface of the liquid) was 1–2 cm away from the incidence point of the laser beam.

The frequencies used were also recorded in photographs (figure 3) and matt black paper was placed where the diffraction fringe patterns were generated to make them more obvious. The number of diffraction maximums depends on the selected frequency and the output power of the signal generator. With increased frequency, we observed an increase in the distance  $\Delta x$  between the first adjacent spots for  $m = \pm 1$  and vice versa. The horizontal distance  $AB = L = (4.39 \pm 0.01)$  m, from the incidence point of the laser beam into the plate with the liquid to the screen, was measured by a meter. The point of incidence of the laser beam on the surface of liquid was determined by moving the small plate (or the wire near the surface of water) from the exciter to the laser beam, until the time the beam was completely obscured.

### 3.1. Measuring the angle of incidence $\psi$

To determine the values of incident angle  $\psi = \pi/2 - \theta$  we measured the grazing angle of the laser beam  $\theta$ . The value of this angle is calculated approximately from  $\theta = \arctan H/L$

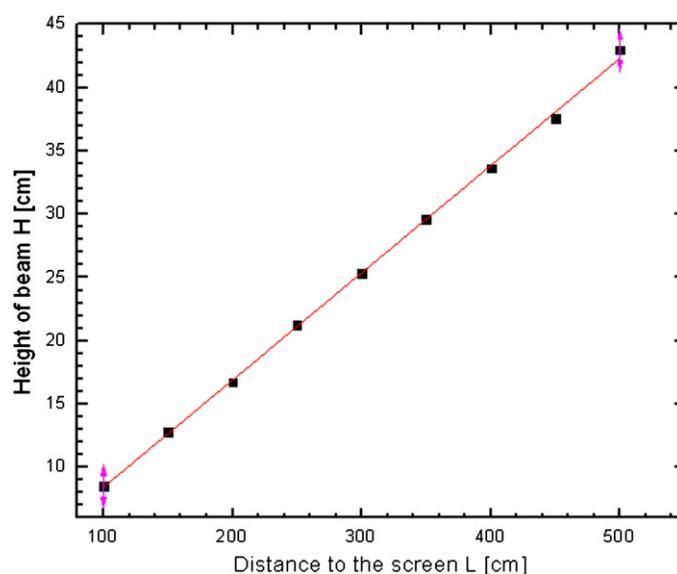


Figure 4. Determining the grazing angle  $\theta$ .

[10, 11, 13, 14]. To measure this angle, a graph was recorded, showing the dependence of height  $H$  as a function of the distance  $L$ , shown in figure 4.

The height  $H$  was measured in the vertical plane of the screen, from the reference level of the horizontal surface of liquid in the plate to the point  $O$ , which is the laser beam geometrically reflected from the calm surface of liquid. For more accurate measurement of height we used 'Wasserwaage' (a transparent hose filled with water). In this way, the position of reference point ( $B$ ) in the liquid surface level was determined more accurately. From the graph (figure 4), we read the angle  $\theta$  from the inclination of the straight line.

### 3.2. Measuring the angle of diffraction $\delta$

The angle of diffraction  $\delta$  was determined from the approximate formula for small angles  $\delta = \arctan \Delta x/L$  [4, 10, 13, 14] by measuring the distances  $\Delta x$  and  $L$ . In order to measure the distance  $\Delta x$ , a series of photographs of diffraction patterns was recorded by a CCD camera, and for analysis of these images and measurement of  $\Delta x$ , a computer program written in Delphi was used. The motive for using the program for measuring the distance  $\Delta x$  was to obtain the value of this distance with absolute error  $\Delta(\Delta x) = 0.11$  mm, less than the measurement error of a meter or ruler [1, 11]. Photographs made by a CCD camera were imported onto the computer and analyzed. The distance between the spots on both sides of the central maximum peak was given in pixels in the snapshot. To convert pixels to meters at the selected zoom, we scanned the distance of 5 cm in the photograph. Thus, for example, a 100-times zoom of the distance of 5 cm corresponds to  $N = 458$  pixels, so the value of a pixel is 1 pixel =  $50 \text{ mm}/458 = 0.11 \text{ mm/pixel}$ . Then the required distance  $\Delta x$ , in meters, is easily obtained as the product of the number of pixels from the central maximum to the maximum of  $m$ th order and the value of a pixel measured in this particular way.<sup>6</sup>

<sup>6</sup> To determine the actual position of diffraction peaks, the centers of bright spots are the important factor. The position of the diffraction peaks' spot centers is estimated by the intensity of their red color and their dimensions. Errors can occur due to subjective readings; therefore, these are to be taken into account and reduced to a minimum.

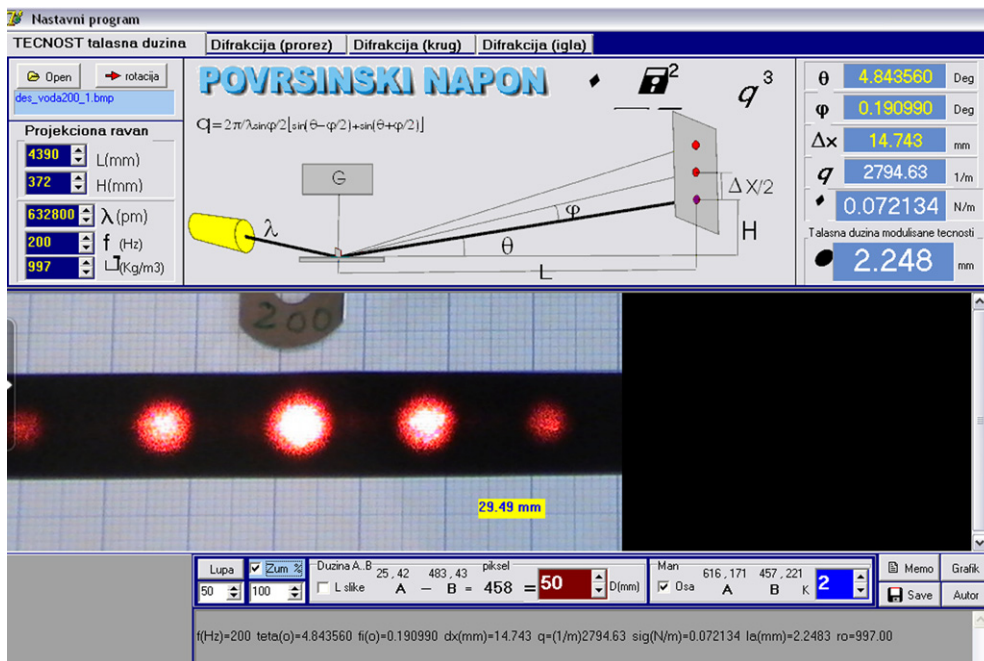


Figure 5. A screenshot of the software used.

Based on the measured values of the angle  $\theta$  and the diffraction angles  $\delta$ , using the relation (3) the software calculates the wavelength of capillary waves, i.e. their wave number that is needed to determine the coefficient of surface tension using the relation (4).

#### 4. Analysis of the results

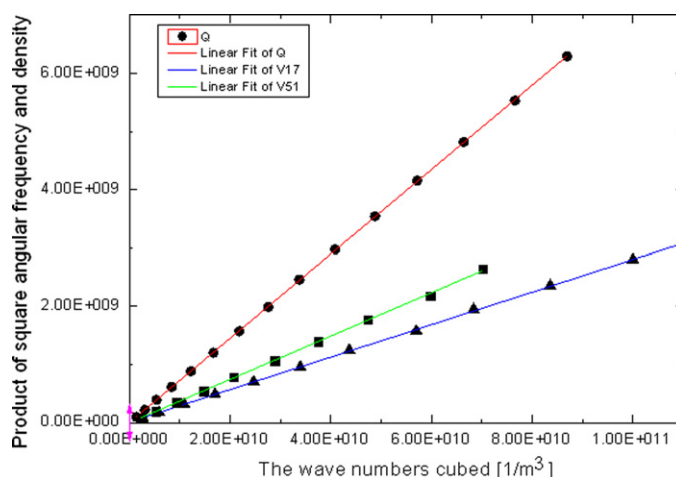
Three fluids were used in the experiment: distilled water, and 70% and 50% ethyl alcohol. Using the values of the frequency of capillary waves  $\nu$ , distance  $\Delta x$ , angle  $\theta$ , the known wavelength of laser light  $\Lambda$ , the distance  $L$  and the values of diffraction angles  $\delta$ , the program (figure 5) gives the numerical values of capillary waves wavelength  $\lambda$  and the coefficient of surface tension  $\sigma$ . Table 1 presents the results of measurements for distilled water while the data tables for other liquids are not shown because of their enormous size; only the appropriate graphs are presented.

Using the procedure described, and based on measurement results, we obtained the coefficient of surface tension  $\sigma = (0.0727 \pm 0.0005) \text{ N m}^{-1}$ .

In figure 6, for the three liquids mentioned, we showed the linear dependences  $\omega^2 \rho$  on  $k^3$ . In table 2 there are values of the coefficient of surface tension obtained (using the Origin package) based on the inclination of the corresponding straight lines shown in figure 6. Table 2 also shows the generally accepted value of the coefficient of surface tension (according to [15]) for water and ethyl alcohol.

Based on the measurements and data presented in table 1, Kelvin dispersion relation (4) can be checked, and it is more convenient to write it in the form

$$\omega = (\sigma/\rho)^{1/2} k^{3/2}. \quad (5)$$



**Figure 6.** Linearized dependence  $\omega^2\rho$  on the wave number  $k^3$  for water (dots), 70% alcohol (triangles) and 50% alcohol (rectangles).

**Table 1.** Measurement results for water:  $\rho = 997 \text{ kg m}^{-3}$ ,  $H = 37.2 \text{ cm}$ ,  $\theta = 0.084 \text{ rad}$ ,  $\Lambda = 0.6328 \text{ }\mu\text{m}$  at  $t = 25 \text{ }^\circ\text{C}$ .

No	n (Hz)	$\Delta x$ (m)	$\delta$ (rad)	$k$ (1/m)	$\omega^2 \rho$ (Hz <sup>2</sup> kg m <sup>-3</sup> )	$\lambda$ (m)	$\sigma$ (N m <sup>-1</sup> )
1	50	0.005 74	0.001 31	1101.66	9.8E + 07	0.005 70	0.073 57
2	75	0.007 70	0.001 75	1453.70	2.2E + 08	0.004 32	0.072 04
3	100	0.009 23	0.002 10	1750.86	3.9E + 08	0.003 59	0.073 31
4	125	0.010 87	0.002 48	2032.34	6.1E + 08	0.003 09	0.073 24
5	150	0.012 16	0.002 77	2300.25	8.9E + 08	0.002 73	0.072 74
6	175	0.013 44	0.003 06	2554.19	1.2E + 09	0.002 46	0.072 31
7	200	0.014 72	0.003 35	2794.29	1.6E + 09	0.002 25	0.072 13
8	225	0.015 89	0.003 62	3019.97	2E + 09	0.002 08	0.072 32
9	250	0.017 06	0.003 89	3231.54	2.5E + 09	0.001 94	0.072 87
10	275	0.018 11	0.004 13	3443.31	3E + 09	0.001 82	0.072 88
11	300	0.019 16	0.004 36	3654.44	3.5E + 09	0.001 72	0.072 56
12	325	0.020 33	0.004 63	3851.87	4.2E + 09	0.001 63	0.072 72
13	350	0.021 26	0.004 84	4049.41	4.8E + 09	0.001 55	0.072 59
14	375	0.022 31	0.005 08	4246.84	5.5E + 09	0.001 48	0.072 24
15	400	0.023 14	0.005 27	4430.17	6.3E + 09	0.001 42	0.072 40

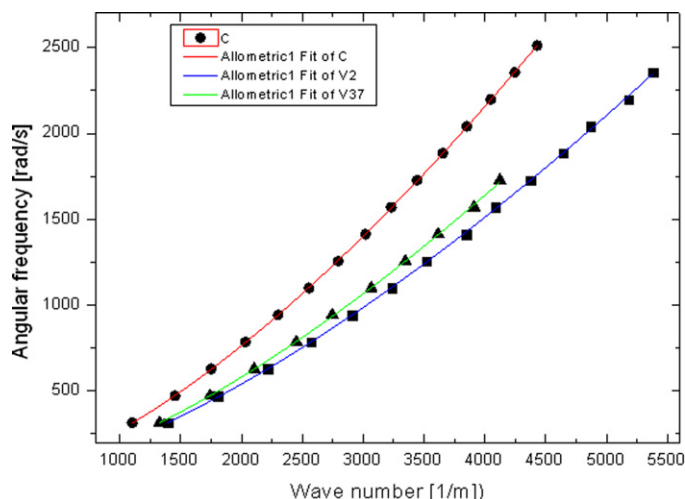
**Table 2.** The values of the coefficient of surface tension obtained from the inclination of the lines in figure 6.

Liquid	$\sigma$ (E-3 N m <sup>-1</sup> )	$\sigma$ (E-3 N m <sup>-1</sup> ) from [15]
Distilled water	72.42 ± 0.09	71.9 ± 0.04
70% ethyl alcohol	27.90 ± 0.02	22.00
50% ethyl alcohol	37.24 ± 0.03	

This is why graphs for all the measured liquids were drawn (figure 7). Fitting was done in the Origin package using the allometric function  $y = ax^b$ , where  $y = \omega$ ,  $a = (\sigma/\rho)^{1/2}$ ,  $x = k$  and  $b = 1.5$ .

Table 3 shows the constants  $a$  and  $b$  for these liquids. One can note the good agreement between the theoretical value of the constant  $b$  and its experimental value.





**Figure 7.** Kelvin dispersion relation of capillary waves for the three liquids used.

**Table 3.** Allometric checking of the dispersion relation for capillary waves for the three liquids.

liquid	Constant $a$	Experiment $b$	Theory $b$
Distilled water	$0.0087 \pm 0.0002$	$1.497 \pm 0.002$	1.5
70% ethyl alcohol	$0.0067 \pm 0.0005$	$1.496 \pm 0.011$	1.5
50% ethyl alcohol	$0.0068 \pm 0.0005$	$1.484 \pm 0.008$	1.5

## 5. Concluding remarks

The apparatus described in this paper and the procedure for using it are very simple and can be implemented for both general physics courses at university and high-school physics courses. Although they have already been described in the literature, many problems can be encountered during the set-up and realization of the experiment. The apparatus for measurement is very sensitive when the measurements are carried out in real school conditions. Thus, for example, the flickering of interference pattern images is caused by any external vibrations; the sound of sirens, aircraft engines, the screeching of car brakes, etc. These effects could not be completely eliminated and consequently the diffraction images obtained are imperfect, and there are vertical vibrations. Some of these influences can be eliminated by using CCD cameras with the 'picture freeze' feature.

During our work we noticed that the following steps enhance the accuracy of the results.

- (1) Determination of height  $H$  (which directly impacts the accuracy of determining the angle  $\theta$ ) using Wasserwaage is much simpler and more precise for school laboratories, which usually do not have the professional equipment that would allow more sophisticated measurements.
- (2) For measuring the diffraction maximum distance  $\Delta x$  it is better to use a CCD camera and perform a computer analysis of the images to achieve more accurate measurements.
- (3) The position of point  $A$  (the place of incidence of the laser beam into the plate) was determined using a thin wire, by moving it horizontally near the surface of the liquid. The

position of point *A* is defined at the same place where the wire fully covers the incident laser beam, which is manifested by a lack of interference spots on the screen.

The area with the dimension *y* around the exciter is the area of the nonlinear regime, and this can be a source of many mistakes, due to which the position of point *A* is very difficult to determine. The laser beam, therefore, must not be directed towards the area of fluid excitation with very pronounced nonlinear effects or too far away. It is necessary to choose the place (usually 1 to 2 cm in front of or behind the exciter) from which to obtain clear diffraction images (sharper fringe patterns) on the screen.

The correct use of the camera<sup>7</sup> and a good choice of position for it are very important factors. The camera should not be situated far away from the screen because the images are then small and useless, but nor should it be too close to the screen as the bright central maximum will then cover the other higher order diffraction peaks, making the photographs poor for analysis. If the camera is positioned at a slant, it introduces mistakes that are evident in the reading and measurement of the distance between the diffraction peaks  $\Delta x$  in the photographs.

Finally, it is worth mentioning that during analysis and measurement the viscosity of the fluid has been neglected. Viscosity causes the attenuation of waves and by monitoring the rate of attenuation we can, by a non-contact method, determine the coefficient of fluid viscosity. It was Stokes who first proposed this more than a century ago [1,16] and a practical difficulty is that we need to measure the amplitude of capillary waves accurately and based on that determine the attenuation coefficient, the value of which is affected directly by the viscosity of liquids. This also represents a challenge for further work on the setting up of this type of experiment that could be performed in every school or faculty laboratory for general physics courses.

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<sup>7</sup> CCD camera JVC GZ-MG 330HE (35x optical zoom, 30GB), Victor Ltd, Japan.