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# Dependence of some mechanical properties of elastic bands on the length and load time

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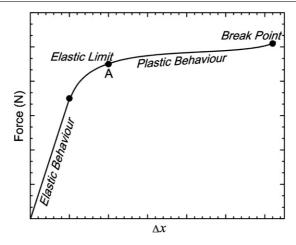
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### **Abstract**

We present a study of the maximum stress supported by elastics bands of nitrile as a function of the natural length and the load time. The maximum tension of rupture and the corresponding variation in length were found by measuring the elongation of an elastic band when a mass is suspended from its free end until it reaches the breaking point. The work done by non-conservative forces was calculated by means of mechanical hysteresis cycles. Measurement of the hysteresis cycle was performed by the controlled addition and subtraction of masses to produce the band's stretching and contraction processes. It was found that the tension of rupture of an elastic band is independent of its natural length and that it decreases linearly with increasing load time. The mechanical work lost in the hysteresis cycle increases linearly with the natural length of the elastic band. An analysis using a state equation of the thermodynamics of rubber bands was performed obtaining good agreement with the experimental results. Also shown is the dependence on the work done by non-conservative forces in repetitive processes of cyclic hysteresis. This experiment is suitable to be implemented in experimental physics courses as an example of showing the characteristics of elastic materials; furthermore it can be done with inexpensive equipment.

# 1. Introduction

The deformation processes experienced by elastic materials when they are subjected to external forces give rise to mechanical phenomena of great importance in the study of their physical properties. Elastomers are widely used in industrial and technological applications due to their deformation, adhesion and damping properties, as well as their resistance to natural phenomena such as humidity, heat and chemicals. The efficiency of elastic bands can be increased by mixing with others materials, which allows them to be used, for example, in systems operating at high temperatures or experiencing a concentrated force. It is also interesting to study



**Figure 1.** General behaviour of an elastic material represented in diagrams of force F as a function of the deformation  $\Delta x$ , where A represents the point of permanent deformation.

the phenomena of stress and fatigue that produce changes in the elastic's internal structure. Nitrile elastomers are characterized, among other properties, by having an optimal resistance to oil and other solvents and low water swelling, making them useful in a large variety of applications [1]. In particular, for this study we used nitrile rubber cord, which is normally used for making many kinds of seals or for the manufacture of o-rings, as an elastic band [2]. In the present paper we analyse the maximum load supported by nitrile elastic bands subjected to mechanical forces. Also, the elastic bands' behaviour under conditions of prolonged loading time  $t_l$  and how their natural length  $l_0$  influences their mechanical properties are analysed. Likewise, we show how repetitive mechanical hysteresis cycles diminish the work W done by non-conservative forces and how this effect produces variations in the properties of elastic bands as result of the progressive change of their internal structure. This experiment is optimal for being implemented in studies of the mechanical properties of elastics, since the materials and the measuring equipment required to perform the experiment can easily be found in any undergraduate physics laboratory. Finally, it is suggested that this experiment can be included in a laboratory course to study the phenomena of elasticity and deformation, which involve concepts related to Hooke's law, work done by non-conservative forces, breakdown tension, mechanical hysteresis and cyclic hysteresis. The present paper is organized as follows. In section 2 we review some theoretical aspects. Section 3 describes the main features of the experimental setup. Then in sections 4-7 we present the experimental results. Finally, we present the conclusions of this paper.

### 2. Theoretical aspects

The behaviour exhibited by an elastic material is represented through diagrams of force F as a function of the elongation  $\Delta x$ , as shown in figure 1. If the force is directly proportional to the deformation, it is said that the material obeys Hooke's law (elastic behaviour). Such materials undergo deformations when an external force is exerted on them, but recover their shape when the force is eliminated. However, there is an elastic limit, which when exceeded causes these materials not to be able to recover their initial shape when the external force is removed. If the elastic limit is exceeded the deformation of the material increases rapidly to a point of

permanent change in structure (point A in figure 1). When the applied force is very large, the material experiences a plastic deformation (plastic behaviour) and subsequently reaches its breaking point [3].

In materials such as rubber that can be stretched to a length several times greater than its original length, it is found that the force F is not proportional to deformation. Although this material does not follow Hooke's law, it has the property of recovering its original length when the applied external force is small. The deformation and relaxation process of elastic materials, such as experienced by a rubber band, implies a loss of energy reflected in the increase of temperature. This energy loss is determined by diagrams of force F as a function of the elongation and relaxation, known as mechanical hysteresis cycles. This phenomenon shows that the restoring force exerted by an elastic material increases with the deformation experienced by the material due to the increased load. By contrast, the restoring force decreases as the elastic begins to recover its natural length when the load is removed [3]. Previous studies of the mechanical behaviour of rubber bands show that the area of the hysteresis cycle is related to the work W done by non-conservative forces, and that such work increases linearly with the band's natural length  $l_0$  [4, 5]. In addition, the energy stored in a rubber band and the behaviour of the hysteresis cycle under stretching and torsion processes have been studied [6]. Fatigue and stress analysis of elastic bands show that the material's breaking point is due to the existence of fissures in its internal structure [7]. The aim of the present study is to determine the mechanical behaviour of nitrile elastic bands as a function of their natural length  $l_0$ , the time during which the load is applied (which we call load time  $t_l$  in this paper) and their mechanical hysteresis cycles. Furthermore, an analysis is performed using a state equation, which, based on previous studies of the thermodynamic behaviour of rubber bands [8, 9], is given by the equation

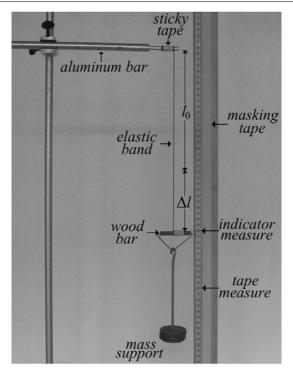
$$F = AT\left(\frac{l}{l_0} - \frac{l_0^2}{l^2}\right) = AT\left(\sigma - \frac{1}{\sigma^2}\right),\tag{1}$$

where A is a positive constant that depends on the composition and geometrical parameters of the elastic band [10],  $l_0$  is the band's natural length, F is the applied force at its free end, l is the length of the band when the force F is applied and T is the absolute temperature. The term  $\sigma = l/l_0$  corresponds to the stretching of the band relative to its natural length and the term  $1/\sigma^2$  is related to compressive effects that are also present when the elastic band is stretched [11].

### 3. Experimental details

To carry out the experiment, nitrile rubber cords purchased at a store specializing in elastic materials were used as an elastic band [12]. An elastic strip of 30 m length was bought in order to ensure that all bands used in this experiment have been manufactured under similar conditions. This is an important factor, since the properties of the elastic band depend strongly on its preparation method and variations in the manufacturing process produce changes in its internal structure. Nitrile rubber is obtained from a copolymer of acrylonitrile and butadiene, so its properties depend on the concentration of each compound at the moment of manufacture [13].

Figure 2 shows a photograph of the experimental setup used to study the mechanical behaviour of the elastic bands. Note that the elastic bands are attached on the one end to an aluminium bar that is fixed in a holder and on the other to a small wooden bar that is left free. The aluminium bar has a diameter of  $D_a = 6.4 \times 10^{-3}$  m, and the wooden bar has a diameter of  $D_w = 6.0 \times 10^{-3}$  m. Two transverse holes are drilled into each of the bars, through which



**Figure 2.** Photograph of the experimental setup. Note that the upper and lower ends of the elastic band are wrapped and secured around the aluminum and wooden bars, respectively. Masking tape, which served to register the position of the indicator in some experiments, was placed beside the tape measure.

the elastic bands will pass in order to hold them firmly. It is important that the holes made in the two bars do not have sharp edges, because they could cut the bands. The elastic bands are placed as follows: one end of the band is inserted through one of the holes in the aluminium bar and is wrapped around the metal bar until six laps are completed. Next the elastic band is passed through the second hole of the aluminium bar. Then the same procedure is repeated for the wooden bar. After adjusting to the desired length, sections of the elastic band that are wrapped around the bars are covered with tape to prevent them from slipping when the load is put on the mass support. We attach a hook to the edges of the wooden bar to suspend the masses that exert the force at the free end of the elastic band. The natural length  $l_0$  and the diameter of the elastic bands were measured by using a Vernier caliper. The mass of the wooden bar and the mass support together have a value of  $m_{ws} = 75 \times 10^{-3}$  kg, and are included in the values of the suspended masses. During the experiment, standard masses of 10, 50, 100 and 248 g were used. The length of the bands was measured using a measuring tape graduated in millimetres, and the time was recorded with a stopwatch that measures to hundredths of a second. Therefore, in all results shown below, the experimental error in recording the data is less than the size of the dots shown in the graphs. The maximum cost per metre of the bands is 50 cents, which shows that this is a low-cost experiment and can easily be implemented in undergraduate physics laboratories. Finally, we should mention that the details of how each one of the experimental variables measured will be presented in their corresponding results section.

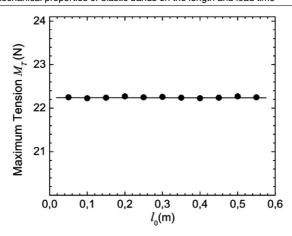
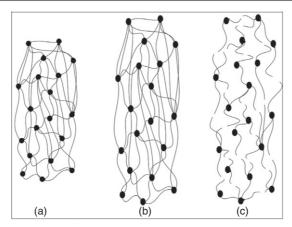


Figure 3. Maximum tension of rupture  $M_T$  as a function of the elastic band's natural length  $l_0$ .

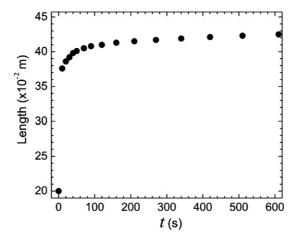
### 4. Relation between the maximum load and the band length

The first part of the study of the mechanical behaviour of nitrile elastic bands was carried out to determine if there exists any dependence of the maximum load that the band can resist just before it reaches the breaking point on its natural length  $l_0$ . In order to analyse this behaviour, we used the experimental setup described in the previous section. We used 11 nitrile elastic bands of  $d = 2 \times 10^{-3}$  m diameter, with lengths between 0.05 m  $\leq l_0 \leq$  0.55 m (increases of 0.05 m). The maximum load supported by the elastic bands was found by adding masses  $m_s$  in a controlled way at time intervals of 2 s. Initially, the mass was increased by 100 g up to 2.2 kg. After this value, the increase in mass was in steps of 10 g, in order to more accurately determine the maximum tension for breaking the elastic band. Figure 3 shows that the maximum tension of rupture  $M_T$  supported by the elastic band is independent of its natural length  $l_0$  and corresponds to a value close to 22.25 N. That is, the bands support a mass of  $m_s = 2.27$  kg before reaching the breaking point.

When an elastic band supports a load, it undergoes an elongation from its natural length  $l_0$ to a final length l, but the elongation of the elastic band is not only experienced at its extremes, but along its entire length. If the elastic band is uniform, each volume element should stretch by the same proportion as the whole band. For this reason the breaking point should not depend on the length of the elastic band. One possible explanation for the elastic band breakdown is represented in the structural model proposed in figure 4. Since the internal structure of the band is formed by polymers chains (see figure 4(a)), the tensions exercised on the band produces the gradual appearance of fissures and deformations (see figure 4(b)), leading to the breakdown of the polymeric links that constitute the band (see figure 4(c)). So the breaking point is reached only when enough tension is generated to break the polymer chains that make up the band. Since the bands have similar characteristics, it is to be expected that the breakdown tension is the same, and independent of the natural length  $l_0$ . In the structural model, it is important to keep in mind that polymers are large molecules constructed through the repetition of one or a few structural units, which are equal or very similar to the monomer or starting material that forms the polymer. The joining of the structural units in some cases can form a linear chain, but it can also form branched or interconnected chains. The length of the polymer chain is determined by the number of repeating units in the chain, which is known as the degree



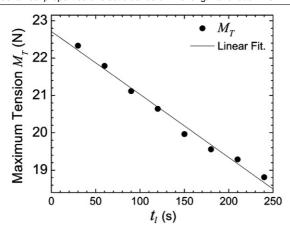
**Figure 4.** Structural model of the gradual deformation of the polymer chains that make up the band: (a) band undeformed, (b) band loaded ( $f \ll f_{\text{Max}}$ ) and (c) elastic band near the breaking point, in which the breakdown of most of the polymer chains is represented.



**Figure 5.** Variation of the length of the elastic bands a function of the load time  $t_l$ , for a unique load of  $m_s = 2.27$  kg. The natural length of the band is  $l_0 = 0.2$  m.

of polymerization, which can vary over a wide range of values depending on the preparation conditions [14].

Figure 5 shows the variation of the elastic band length l as a function of the load time  $t_l$ . The load was exerted by suspending a unique mass  $m_s = 2.27 \,\mathrm{kg}$ , all at once, and the natural length of the band was  $l_0 = 0.2 \,\mathrm{m}$ . Note that we chose a mass value that causes the break up of the band. As soon as the mass  $m_s$  is suspended, the length of the band begins to change rapidly. Therefore, in order to register its variation, the indicator position was marked on the masking tape that was placed next to the tape measure (see figure 2), for certain time intervals. Upon termination, the variation in the length of the band was determined with the tape measure according to the markings on the masking tape. It can be seen that the length of the band changes appreciably during the first 10 s that the load is acting. So we see that the length variation is slower and continues to increase until reaching the breaking point (at about



**Figure 6.** Maximum breakdown tension  $M_T$  of the elastic band as a function of the load time  $t_l$ .

600 s). We can say that the polymer chains that constitute the band are breaking under the action of the load, up to a moment when the internal structure fails. This behaviour is similar for all bands regardless of their natural length  $l_0$ .

### 5. Influence of load time $t_i$ in the breaking of the band

Another important aspect in the analysis is that the elastic band's maximum tension of rupture  $M_T$  is strongly related to the load time  $t_l$ , which is defined as the time elapsed before increasing or decreasing the mass  $m_s$  that is placed as a load at the free end of the elastic band. In order to analyse the dependence of the  $M_T$  on the load time  $t_l$  the experimental setup shown in figure 2 was again used. In this part of the experiment, eight bands with a natural length of  $l_0 = 0.2$  m and a diameter of  $d = 2 \times 10^{-3}$  m were used. The load mass  $m_s$  was increased in increments of 0.1 kg for load time intervals between  $30 \le t_l \le 240$  s, (which was varied in increments of 30 s). The increase in mass continued up to reaching the breaking point of each one of the elastic bands. Figure 6 shows the variation of the band's maximum tension of rupture  $M_T$  as a function of the load time  $t_l$ . The results show that  $M_T$  decreases linearly with the load time  $t_l$  following a functional relationship given by

$$M_T = -0.017t_l + 22.7, (2)$$

where the linear correlation coefficient of adjustment is 0.99. The value 22.7 N corresponds to the breaking tension of the band for a load time of  $t_l = 0$  s, that is, the case in which the force produced by the suspended mass  $m_s$  at the free end of the band is equal to 22.7 N. The -0.017 kg ms<sup>-3</sup> value is a parameter that among other things could depend on the diameter and the material with which the elastic band is manufactured. This behaviour is because the band undergoes a progressive deformation with the load time  $t_l$ . As the load time  $t_l$  increases the deformation experienced by the band due to the mass  $m_s$  suspended from its free end becomes greater. For this reason, we need a smaller external force to reach the breaking point. At the microscopic level one can imagine that for a greater  $t_l$  there is an increase in the number of polymer chains that break progressively due to the external force acting on the band.

778 C A Triana and F Fajardo

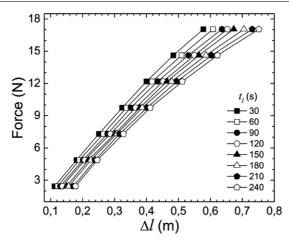


Figure 7. Force F as a function of the elongation  $\Delta l$  undergone by the elastic band; the load time  $t_l$  is taken as a parameter.

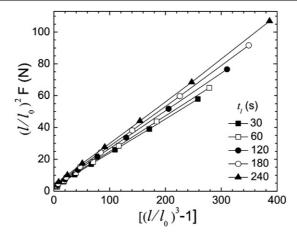
### 6. Influence of the load time $t_l$ on the elongation of the elastic band

Figure 7 shows the force F as a function of the elongation  $\Delta l$  suffered by the elastic bands, when the load time  $t_l$  is taken as a parameter. In all the cases we used a band with a natural length of  $l_0=0.2$  m and increments of mass of  $\Delta m_s=0.248\,\mathrm{kg}$ . As shown in figure 7, the elongation  $\Delta l$  suffered by the elastic bands is greater for longer load times  $t_l$ , due to the progressive stretching of the polymer chains that form its internal structure. Note that for small values of the applied force the elastic bands exhibit a behaviour that closely follows Hooke's law. Nevertheless, Hooke's law is not satisfied for large values of the applied force, since gradual stretching should begin to cause breaks in the bonding polymer chains that exist inside the band. This leads to a permanent deformation of the band structure which causes the applied force not to be proportional to the displacement.

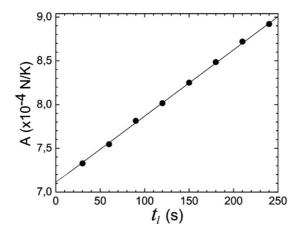
Next we show the results of the fit to the state equation for an elastic band proposed by G Savarino and M R Fisch [8], which is given by equation (1). In this part of the experiment we wanted to see if there is any dependence between the constant A and the load time  $t_l$ , during which the elastic band is subjected to a tension. According to equation (1) and the results shown in figure 7, it is possible to make a graph of  $(l/l_0)^2F$  as a function of  $((l/l_0)^3 - 1)$ , because these variables are determined experimentally [8]. For clarity, in figure 8 only the results for five of the load times  $t_l$  shown in figure 7 are displayed. In figure 8 we can see that the quantity  $(l/l_0)^2F$  shows a direct relationship with  $((l/l_0)^3 - 1)$ , which is similar to the result obtained by Savarino and Fisch [8]. The linear adjustments in all cases have a linear correlation coefficient greater than 0.99. By calculating the slopes of these lines, we can obtain the constant A for each one of the bands subjected to different loading times  $t_l$ , since in agreement with equation (1) we have [8]

$$A = \frac{(l/l_0)^2 F}{T((l/l_0)^3 - 1)},\tag{3}$$

where  $(l/l_0)^2 F/T((l/l_0)^3 - 1) = m'$  corresponds to the calculated slopes of the lines shown in figure 8 and T = 298 K is the absolute temperature T of the elastic band.



**Figure 8.**  $(l/l_0)^2 F$  as a function of  $(l/l_0)^3 - 1$ .

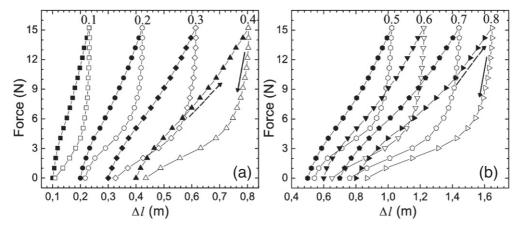


**Figure 9.** Constant A from the state equation as a function of the load time  $t_l$ .

The results presented in figure 9 shows that the constant A increases linearly with the load time  $t_l$ , following the functional relationship given by

$$A = 7.65 \times 10^{-7} t_l + 7.1 \times 10^{-4}. \tag{4}$$

We believe that the constant values in equation (4) could depend on some characteristics of the band, such as the type of material from which it is made, the diameter and the internal structure. When we compare the values of A obtained for a load time of  $t_l = 0$  s and one of  $t_l = 240$  s, we observe that there is an increase of close to 25%. This may be because when the load time  $t_l$  is increased, the friction between the polymer chains that make up the band's internal structure also increase, and consequently their breakdown increases. This fact is verified experimentally because of the increased temperature of the band's surface when it is subjected to a large amount of tension for a long time. From equation (4) we can infer that for the elastic bands used in this experiment, if the load time is less than  $t_l = 10$  s, the expected change in the value of constant A of the state equation (1) must be less than 1%.

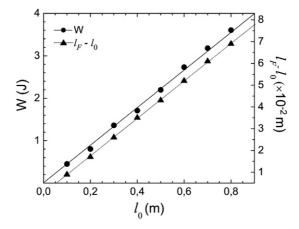


**Figure 10.** Mechanical hysteresis cycle of elastic bands with different natural lengths  $l_0$  (a) for  $0.1 \text{ m} \leqslant l_0 \leqslant 0.4 \text{ m}$  and (b) for  $0.5 \text{ m} \leqslant l_0 \leqslant 0.8 \text{ m}$ . The dashed arrow indicates the loading process and the continuous arrow corresponds to the unloading process of the elastic band.

# 7. Influence of the natural length of the band on the mechanical hysteresis loop

Another important aspect in the study of the behaviour of elastic bands is to analyse the dependence of the mechanical hysteresis cycle on the natural length  $l_0$ , and calculate the work W done by non-conservative forces in the hysteresis cycle. For this purpose, the experimental setup described in section 3 was again used. We worked with eight bands with a diameter of  $d = 2 \times 10^{-3}$  m, and with natural lengths  $l_0$  that varied in increments of 0.1 m in the range 0.1 m  $\leq l_0 \leq$  0.8 m. In order to perform the deformation of the elastic band, masses  $m_s$  were added in the range  $0.1 \,\mathrm{kg} \leqslant m_s \leqslant 1.5 \,\mathrm{kg}$  with increments of  $0.1 \,\mathrm{kg}$ , at time intervals of 30 s. When the value  $m_s = 1.5$  kg was reached, and in order to effect the relaxation of the elastic band, the masses were withdrawn every 30 s with decreases of 0.1 kg. During the process of loading and unloading the masses, measurements of the elongation  $\Delta l$  experienced by the elastic bands were carried out. Appropriate control of the load time  $t_i$  is very important, since, as shown in the previous section, the band is deformed gradually when subjected to a load, and the elongation depends on the loading time  $t_l$ , which determines the correct measurement of the hysteresis cycle. In order to carry out this measurement, a procedure similar to that explained for collecting the data for figure 5 was followed. That is, the position of the indicator of the length of the band was marked on the masking tape that was adjacent to the tape measure (see figure 2). The above procedure allowed increasing and decreasing the masses and registering the change in the length of the elastic band very quickly (t < 2 s), thereby obtaining good control of  $t_l$ .

Figures 10(a) and (b) shows the mechanical hysteresis cycle experienced by the elastic bands as a function of the variation of the length  $\Delta l$ . In these two figures the net work W done by non-conservative forces corresponds to the area between the curves limited by the filled points and the empty points. The direction of the dashed arrow indicates the loading process of the elastic band (filled points) and the curve travelling in the direction of the continuous arrow corresponds to the unloading process of the band (empty points). Note in figures 10(a) and (b) that as the natural length  $l_0$  becomes greater, the hysteresis cycle area also increases. At a microscopic level, subjecting the band to a hysteresis cycle produces a stress on the internal structure, which is manifested in an increase of the temperature at its outer surface [6].



**Figure 11.** Mechanical work W done by non-conservative forces as a function of the natural length  $l_0$  of the elastic band (circles). Also shown is the variation in length  $\Delta l = l_F - l_0$  as a function of the natural length  $l_0$  of the elastic bands, after subjecting them to a hysteresis cycle (triangles).

The temperature increase indicates the presence of dissipative forces due to friction between the polymer chains within the band. As a result of the temperature variations and the processes of deformation and rupture of polymer chains that form the band, deterioration in the structure is generated, which produces the loss of its elastic properties. The mechanical work W lost in the hysteresis cycle can be calculated. For this we take into account that the difference between the energy stored during the band stretching and the energy recovered during its relaxation process is equal to the net work W done by non-conservative forces, which, as was mentioned above corresponds to the area enclosed by the hysteresis cycle. Another important aspect to note is that after completing a hysteresis cycle, the final length  $l_F$  of the band is greater than its natural length  $l_0$ . This variation in the band length is a consequence of the breaking of the polymer chains that make up its internal structure. These breaks produce a plastic deformation, which causes the band not to recover its natural length  $l_0$ .

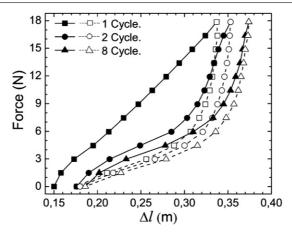
Figure 11 shows the mechanical work W done by non-conservative forces as a function of the elastic band's natural length  $l_0$ . The results show that the work W lost during the hysteresis cycle increases linearly with the natural length  $l_0$  of the elastic bands, following a functional relationship given by

$$W = 4.59l_0. (5)$$

The slope of the linear fit in figure 11 evidently could depend on some characteristics of the elastic band, including the type of material with which it is manufactured and its diameter, as well as the load time  $t_l$ . Also in figure 11 is shown the variation in length  $\Delta l = l_F - l_0$ , as a function of the natural length  $l_0$  of the elastic bands, after subjecting them to a hysteresis cycle. According to the results it can be see that there is a directly proportional relationship between the elongation  $\Delta l$  and the natural length  $l_0$  of the elastic bands, which is determined by the expression

$$\Delta l = l_F - l_0 = 8.6 \times 10^{-2} l_0, \tag{6}$$

where the constant value  $8.6 \times 10^{-2}$  corresponds to the unitary deformation by tension after the hysteresis cycle is performed, which refers to the relative variation of the band's shape and dimensions. The unitary deformation by tension is defined as  $l_F - l_0/l_0 = \Delta l/l_0$ , that is

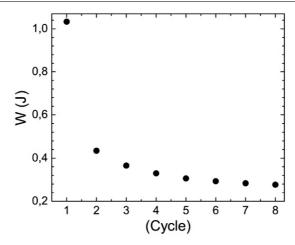


**Figure 12.** Successive measurements of the mechanical hysteresis cycle for the same elastic band; for clarity only three cycles out of eight are shown. The filled points correspond to the loading process and the empty points correspond to the unloading process.

the ratio between the increase in the band length and its natural length. This result shows that after the hysteresis cycle is performed, the unitary deformation by tension of the elastic bands studied in this paper corresponds to 8% of its natural length  $l_0$ .

### 8. Cyclic repetition of mechanical hysteresis curve

In this section we show the evolution of the mechanical hysteresis cycle when the loading and unloading process of the elastic band is performed eight times in repetitive cycles. To determine these measurements we used a band with a natural length of  $l_0 = 0.15$  m and a diameter of  $d = 2 \times 10^{-3}$  m. To measure the hysteresis cycle, we used the same procedure explained in the previous section, except that after completion of the first cycle, seven cycles were measured successively using the same elastic band. To stretch the band, we added masses  $m_s$  in the range  $0.15 \,\mathrm{kg} \leq m_s \leq 1.8 \,\mathrm{kg}$  with increments of  $0.15 \,\mathrm{kg}$ . Then, to effect the relaxation of the band, the masses were removed with decrements of 0.15 kg. The load time was always fixed at  $t_l = 30$  s. Figure 12 shows the evolution of the hysteresis cycle for a nitrile elastic band as successive cycles are performed. For clarity, only the hysteresis curves for the first, second and eighth cycle are shown. The loading process of the elastic band is represented by filled points and the unloading process is represented by empty points. As mentioned above, the area enclosed within the hysteresis cycle corresponds to the mechanical work W done by non-conservative forces. Figure 12 shows that the area of the first cycle is much greater than that of the second, and then the area decreases slowly as each new cycle is carried out. This behaviour once again shows that as we carry out each cycle of loading and unloading, inside the elastic band breaking and rearrangement of the polymer chains that make up its internal structure are evidently produced. Also, the work W done by non-conservative forces changes in each one of the hysteresis cycles as a function of the band length after each cycle is finished. Note that between the first and second cycle, the band length changes dramatically (see in figure 12 the first and the final data point of the hysteresis cycles), so this is the interval where there are major changes in its internal structure and the greatest loss of its elastic properties is produced. However, after the second hysteresis cycle, the band length no longer undergoes large deformations.



**Figure 13.** Work W done by non-conservative forces in each one of the hysteresis cycles as a function of the cycle for the same elastic band.

Figure 13 shows the work W done by non-conservative forces in each one of the hysteresis cycles as a function of the cycle for the same elastic band. Note that the work W done by non-conservative forces decreases rapidly during the second hysteresis cycle and then decreases more slowly, tending to a limit value. This would indicate that the band experienced the greatest loss of its elastic properties during the first hysteresis cycle. After eight hysteresis cycles are carried out, it is found that the band can be deformed applying a very small force. At the end of the eight cycles, the band cannot recover its initial shape; that is, it has undergone a permanent deformation as a result of its loss of elastic properties. At the structural level, we might think that every time a hysteresis cycle is performed more links of the polymer chains, which are what determine the band's elastic properties, are broken.

# 9. Conclusions

Several properties of the mechanical behaviour of elastic bands of nitrile were studied. The elongation of the band was measured as a function of the load time for which it was subjected to tension and as well as a function of the variation of its natural length. Also, the dependence of the mechanical hysteresis on the length of the elastic band and on the number of times that the cycle is repeated was shown. Some of the results found are (i) the maximum tension for breaking the band is independent of its natural length, (ii) the maximum tension for breaking the band decreases linearly with the load time, (iii) the amount of mechanical work lost during the hysteresis cycle of the elastic band increases linearly with the natural length and (iv) the work lost in processes of repetitive hysteresis cycles decreases with the number of cycles to which the elastic band is subjected. The above phenomena are related to internal structural changes within the elastic band. To explain some of the results, a state equation for elastic bands was used, which shows good agreement with the experimental results. This experiment is suitable for introductory physics courses because it allows an analysis of the elastic properties of a common material with a wide variety of applications. Finally, it is a very inexpensive experiment through which it is possible to illustrate concepts such as tension, Hooke's law, non-conservative forces, work, mechanical hysteresis and cyclical hysteresis.

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