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A laboratory activity on the eddy current brake

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Abstract

The aim of this paper is to introduce a simple and low-cost experimental setup that can be used to study the eddy current brake, which considers the motion of a sliding magnet on an inclined conducting plane in terms of basic physical principles. We present a set of quantitative experiments performed to study the influence of the geometrical and electromagnetic properties of the magnet on the magnetic drag force. This video-based experiment is ideal for the study of kinematic graphs and the application of Newton's laws. Video motion analysis software enables students to make precise measurements of the magnet's position at incremental times during its motion, thus allowing them to quantify electromagnetic induction phenomena. The equipment needed for this experiment and data collection software are present in most physics teaching laboratories or are inexpensive and available.

(Some figures may appear in colour only in the online journal)

1. Introduction

The physics laboratory has long been a distinctive feature of physics education. It has been given a central role in the teaching and learning of physics at school and undergraduate levels in universities. The literature indicates that science educators have suggested that there are academically rich benefits in the learning and understanding of physics based on laboratory work [1]. Experiments play a wide spectrum of functions in physics instruction since they have a high motivational value, promote the development of practical skills, stimulate scientific inquiry methods and help make abstract concepts visible [2]. In this context, a large variety of simple setups for qualitatively demonstrating Faraday's and Lenz's laws are used in introductory physics courses. The slowing down of a magnet falling through a vertical conducting tube is a classic classroom demonstration of magnetic induction phenomena [3–5]. The changing magnetic flux caused by the falling magnet induces eddy currents in the tube that oppose the increase in the flux. The interaction between the induced eddy currents and the magnet leads to a damping force, which causes the magnet to fall with a much reduced speed. As the force on the magnet increases with its speed, the magnetic force eventually balances the weight of the magnet to give a constant velocity. One shortcoming of this demonstration is that the magnet remains out of view whilst inside the tube. Another demonstration of Lenz's law is the slowing down of a metal sheet passing through a magnetic field [6]. Although this magnetic braking is the basis of some technological applications [7], very few quantitative laboratory experiments in the area of electromagnetic induction can be performed with readily available equipment [8].

While a mass subject to a retarding force that is proportional to its velocity is often discussed in physics classes, students are rarely able to perform experiments involving such systems [9]. Our experiment provides students with such an opportunity. Other examples of motion under the action of retarding forces include skydiving, falling raindrops and the remarkable 'flying' seeds of some trees [5]. In such phenomena, the bodies begin falling in transient accelerated motion, a regime that soon changes to uniform motion under the combined action of the gravitational force and a retarding force. The constant speed during this final motion is known as the 'terminal speed'.

Multimedia elements are of increasing importance for physics instruction. In particular, an affordable digital video would offer students the opportunity to actively engage in kinematics and dynamics in an introductory physics course [2, 10] as video analysis allows motions that cannot be easily measured in a traditional lab setting to be studied. Furthermore, appropriate use of a digital video's frame-advance features, and 'marking' the position of a moving object in each frame, would enable students to more accurately determine the position of an object at much smaller time increments than is currently possible with traditional timing devices. Once the student has collected the position and time data, these values can be used to determine velocity and acceleration. The use of video analysis in physics teaching has had positive effects, mainly on student attitude and motivation [11]. A variety of commercial and free applications can be used for video analysis. Indeed, with the relevant technology becoming increasingly less expensive, video analysis has become a key instructional tool in introductory physics courses.

Herein, we would like to present a simple and inexpensive apparatus that makes use of video analysis to study magnetic braking. This experiment has already been used in our undergraduate physics laboratory as a quantitative demonstration of the effects of electromagnetic induction associated with Faraday's law. It consists of a magnet sliding on an inclined track with a non-magnetic conducting base. The students have found this experiment to be a very interesting laboratory exercise as the video analysis enables them to see a demonstration of the setup and make quantitative measurements. We present a set of quantitative investigations on the effect of the geometrical and electromagnetic properties of the magnet on the eddy current brake.

2. Theoretical analysis

When a magnet slides down the slope, a magnetic flux change occurs in the metal track. In accordance with Faraday's law, this flux change induces an electromotive force (EMF), and circulating currents, known as eddy currents, are generated inside the metal. As a result, a magnetic force is exerted on that portion of eddy current that lies within the external magnetic field created by the magnet. This force is transmitted to the metal track and is the retarding force associated with the magnetic braking. Although this is the force acting on the induced current from the magnet, Newton's third law equates this with the force on the magnet arising from the current in the metal.

As a first approximation, we can consider the magnetic braking force to be $-bv$, where b is a damping coefficient that depends on the parameters of the system and v is the velocity of the magnet [3, 12, 13]. With respect to the magnet, the metal plane moves with velocity v and the motional EMF in any length element will be proportional to v. As the velocities involved are small, such transformation to a magnet frame is reasonable.

If m is the mass of the sliding magnet, then its motion, due to its weight, the magnetic braking force and the mechanical friction, is given by Newton's second law as

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = mg\sin\theta - bv - \mu mg\cos\theta, \tag{1}$$

where g is the acceleration due to gravity, θ is the inclination angle of the metal sheet and μ is the coefficient of kinetic friction between the magnet and the plane. The term corresponding to mechanical friction becomes less significant as the inclination angle θ increases. The solution to this differential equation with the initial condition $v(0) = 0$ yields the expression for the speed of the magnet as a function of time:

$$v = v_T[1 - \exp(-t/\tau)], \tag{2}$$

where the time constant τ and terminal speed v_T are given by

$$\tau = \frac{m}{b} \quad \text{and} \quad v_T = \frac{mg\sin\theta - \mu mg\cos\theta}{b}. \tag{3}$$

On the other hand, the equation corresponding to v_T can be obtained by considering that, as the acceleration of the magnet when it slides down the slope at constant speed is zero, the net force on the magnet must also be zero. As the motion of the magnet is due to the combined action of a retarding force and a constant force, the magnet motion consists of an initial transient accelerated regime followed by uniform terminal motion, where the constant speed is precisely the terminal speed v_T given by equation (3). The expression for the displacement of the magnet as a function of time starting from rest can be obtained from equation (2) as

$$s = v_T\tau\left[\frac{t}{\tau} - 1 + \exp\left(-t/\tau\right)\right]. \tag{4}$$

This law describes the magnet's motion and can be employed immediately to fit the experimental data collected in the experimental section by adjusting the fit parameter v_T and τ. Note that this fitting task may be better accomplished for large values of t (i.e. $t \gg \tau$), where $s = v_T\ t$. The time at which the exponential term is negligible, and therefore, the velocity that is approximately v_T, depends on the time constant τ that is inversely proportional to the damping coefficient b.

3. Apparatus and procedure

The apparatus used in these experiments is a clear plastic table (62 cm × 28 cm) with two rectangular copper bases (62 cm × 2 cm) attached. The metal tracks were fabricated from the same stock to ensure the same conductivity for all of them. A thin plastic sheet (2 mm of thickness) is fixed to the top of the copper bases to provide a smooth sliding surface for the magnet (see figure 1), thereby ensuring that the kinetic friction coefficient is the same for all experiments. The plastic table is mounted on a non-magnetic support with different positions that define the slope angle of the inclined plane. The angles between the track and the floor are determined using a protractor. We have used two copper-based tracks with different thicknesses 1 (Cu1) and 3 mm (Cu3). The commercial magnets used in the experiments, made up of NdFeB, have a cylindrical pellet shape. The dimensions, magnetic field strength and mass of the magnets are given in table 1. A Hall effect gaussmeter was used to measure

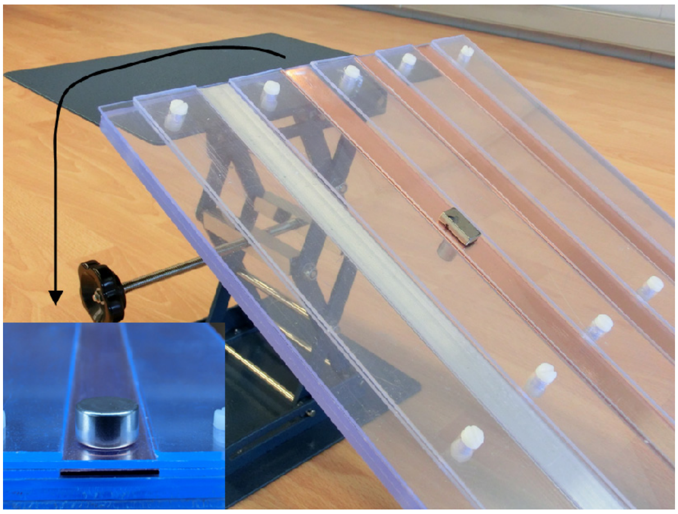

Figure 1. Photograph of the experimental setup.

Table 1. Main characteristics of the magnets.

Magnet	Mass (g)	Magnetic field (mT)	Diameter (mm)
A	2.02 ± 0.01	246 ± 1	9 ± 1
M1	9.38 ± 0.01	248 ± 1	18 ± 1
M2	16.08 ± 0.01	248 ± 1	18 ± 1
M3	18.82 ± 0.01	248 ± 1	18 ± 1
C	18.82 ± 0.01	371 ± 1	18 ± 1

the strength of the magnetic field at 2 mm from the centre of one of the flat surfaces of the magnet.

For a given angle, if the magnet is placed in a position with no metal base and then released, it accelerates significantly under the force of gravity as it slides, as expected. However, students are surprised and curious about the relatively low speed of the magnet as it slides down on the copper-based track.

In order to produce a velocity profile of the magnet's progress down the inclined plane, students record the magnet's motion using a digital camera. The motion of the magnet is studied by analysing a series of frames extracted from the resulting video. Individual frames are obtained using Tracker (free Java software available at http://www.cabrillo.edu/~dbrown/tracker/), which has the advantage of being readily available, open source and easy to use. The magnet's position is determined from each frame as its distance from a fixed point along the plane and then scaled into real meters using an object of known length found in the same frame. Although we have successfully analysed experiments using video clips recorded with a camera at 30 frames per second, our experience shows that it is better to have more data points available in order to determine the position–time curves under experimental conditions when the magnetic braking force is not so strong. We solved this problem by using a relatively inexpensive Casio Exilim high-speed camera that can record at a speed of 240 frames per second.

To obtain accurate data, it was necessary to take care in several aspects of the video-recording process. For example, care must be exercised in positioning the camera to ensure

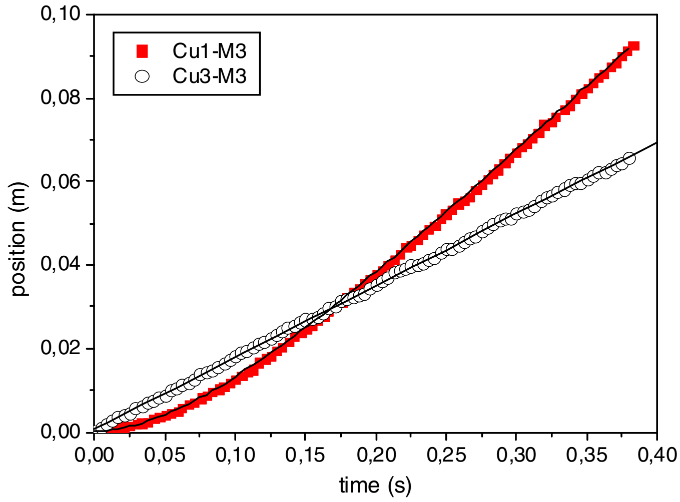

Figure 2. M3 magnet position versus time at $\theta = 31.1°$. The solid line for Cu1 track data corresponds to predictions from equation (4).

that the image plane is parallel to the plane of the magnet's motion. We placed lights on the floor, in addition to those in the ceiling, so that the magnet was uniformly illuminated. To calibrate the camera's timing, we videotaped a digital timer and confirmed that the frames were 1/240 s. As part of their laboratory activity, students are expected to set up the experiment, measure the position of the sliding magnet as a function of time from the video analysis, display the values obtained graphically and explain their experimental observations and results theoretically.

4. Results and discussion

4.1. Influence of the magnetic field intensity

An effective way for the students to learn the meaning of a physical quantity is to perform the experiments in which they determine the magnitude of that quantity in terms of the other quantities involved in its definition. To show the importance of the magnetic field of the magnet on the eddy current brake, we carried out experiments with two magnets identical in all respects except their magnetic field strength (magnets M3 and C). Composite plots of position versus time are shown in figures 2 and 3 for M3 and C magnets, respectively. These results correspond to an inclined plane with a slope angle of $31.1°$. In all the experiments, the magnet is initially stationary on the track, thus meaning that the initial velocity is zero. From figure 2, one can note that the shape of the Cu3 track plot corresponds to a constant-velocity motion. The magnet reaches terminal speed almost immediately and slides down at a constant velocity as the net force approaches zero. The line in this figure is the least-squares fit to the data ($r > 0.9998$), and the slope represents the terminal speed of the magnet under magnetic braking, mechanical friction and gravitational forces. No initial transient acceleration regime was detected in this experiment, as the magnetic braking force is so large that the sliding magnet reaches its terminal speed in such a short period of time (less than 0.004 s) that the plot appears to be a simple straight line. In contrast, as can be seen in this figure, the data from the experiment with Cu1 slab clearly show a transient acceleration regime for the sliding magnet as the magnetic braking force is somewhat weaker. The Cu1 track data

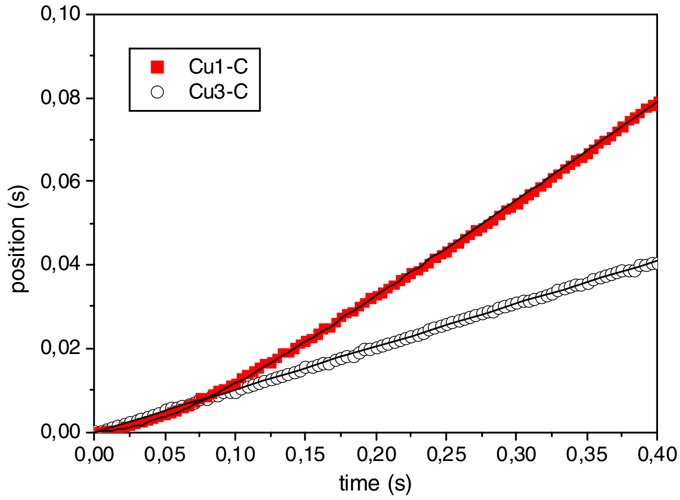

Figure 3. C magnet position versus time at $\theta = 31.1°$. The solid line for Cu1 track data corresponds to predictions from equation (4).

were fitted to equation (4) and show an excellent agreement between the theoretical curve and the experimental data. The initial transient accelerated motion of the magnet before it reaches its terminal speed is clearly visible. The slope of the graph subsequently becomes constant as the velocity approaches v_T. As expected, the greater the metal thickness, the lower the magnet's terminal speed. This is a natural consequence of Faraday's and Lenz's laws: a greater metal thickness means larger induced eddy currents in the metal and therefore a larger opposing force on the magnet. On the other hand, the same behaviour is observed in figure 3 for the experiments with C magnet. However, inspection of the spacetime diagrams in figures 2 and 3 shows that they differ in terms of the duration of the first motional regime (accelerated motion) and the terminal speed. In the case of the M3 magnet, the constant speed region is achieved later than in the C magnet experiment. The calculated value of τ are 0.087 and 0.065 for M3 and C magnets, respectively. The τ values obtained from our experiments are consistent with those published in the literature for a falling magnet inside a vertical conducting pipe (values between 0.029 and 0.08 s, depending on the experimental conditions) [3, 14]. The terminal speed of the M3 magnet is higher than the terminal speed of the C magnet for a slab with the same thickness. That is, the greater the magnetic field intensity, the lower the values for v_T and τ.

The above process was repeated for four values of θ (angle of the inclined plane). Dividing the terms of the terminal speed equation (see equation (3)) by cos θ gives

$$\frac{v_T}{\cos\theta} = \frac{mg}{b}(\tan\theta - \mu). \qquad (5)$$

It can be seen from equation (5) that the dragging coefficient b and the kinetic friction coefficient μ can be estimated from the slope and the y-intercept values, respectively, by plotting $v_T/\cos\theta$ against $\tan\theta$ and performing a least-squares fit. Indeed, a straight line fits the experimental data plotted in figure 4 well (all fits present $r > 0.9997$). The calculated values are summarized in table 2, examination of which reveals a decrease of the damping coefficient as the magnetic field strength of the magnet decreases. In other words, the magnetic braking force increases with increasing the magnetic field strength of the magnet, thus meaning that the terminal speed is lower (for the same value of θ). It is also interesting to note that the terminal

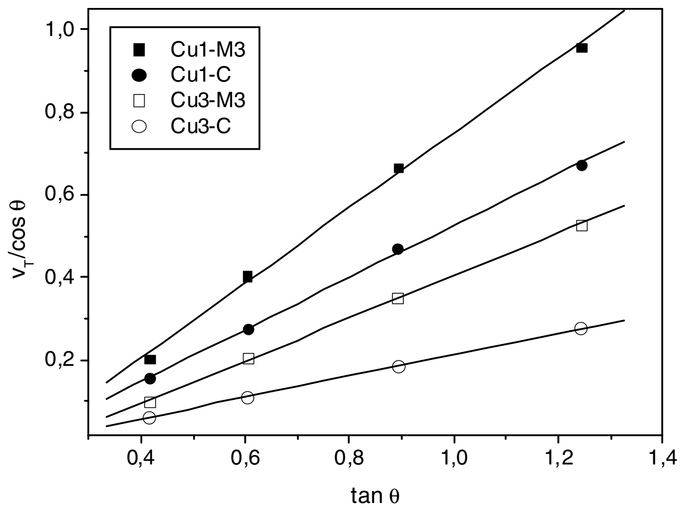

Figure 4. Plot of $v_T/\cos\theta$ as a function of tan θ for M3 and C magnets.

Table 2. Damping coefficient (b) and kinetic friction coefficient (μ) values as a function of the magnet used in the experiment.

System (track–magnet)	b (N s m^{-1})	μ
Cu1–M3	0.204 ± 0.007	0.18 ± 0.03
Cu3–M3	0.367 ± 0.006	0.22 ± 0.02
Cu1–C	0.294 ± 0.008	0.18 ± 0.03
Cu3–C	0.71 ± 0.01	0.18 ± 0.01

speed increases with θ and decreases with the metal thickness. The magnetic braking force depends on the intensity of the currents induced in the metal. The metal offers less resistance to the eddy currents as its thickness increases, thus implying that the current increases, thereby resulting in greater damping and a longer sliding time.

It is interesting to remark the discrepancy of the b value obtained for the C magnet on the Cu3 slab. This experiment corresponds with a situation where the magnetic field strength is strong and the metal thickness is large. The calculated damping coefficient is too high. The ratio of the damping coefficient for M3 and C magnets on the Cu1 slab is 0.69, whereas this ratio for the Cu3 slab is 0.53. Then, the expected value of the damping coefficient for the C magnet on the Cu3 slab is 0.53. A probable explanation for the existence of this large magnetic drag force is the existence of a significant magnetic field created by the eddy currents, which we have not taken into account.

In order to determine the coefficient of kinetic friction between the magnet and the plastic surface that covers all metal slabs, we have plotted the distance travelled by the magnet on a plastic track without a metal base as a function of time in figure 5. This plot shows a parabolic trend, where the parabolic axis of symmetry lies parallel to the distance axis with the vertex at (0, 0), as expected from the equation of motion $x(t) = 0.5\ at^2$, corresponding to a uniformly accelerated motion with an initial velocity of zero. The acceleration can be obtained from the gradient (equal to $0.5a$, in this example the experimental acceleration value is $a = 2.01$ m s^{-2}) of the straight line fitted to the data of the distance versus squared-time graph (inset in figure 5). This acceleration is used to calculate the coefficient of kinetic friction. The experiment was

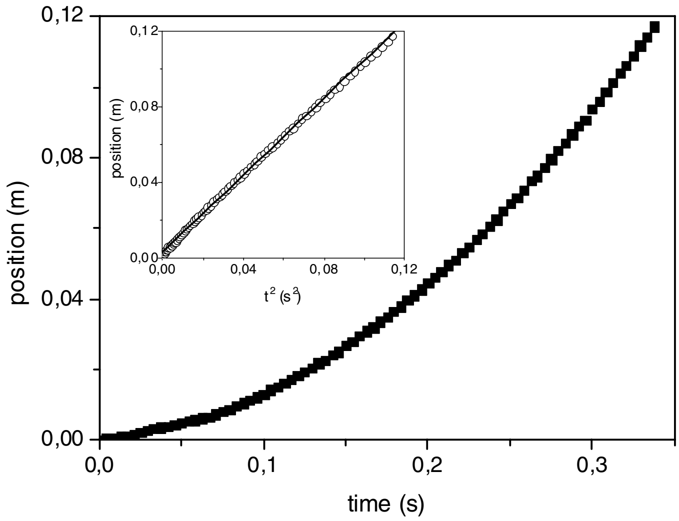

Figure 5. Magnet position versus time for a plastic track.

repeated for different magnets several times at different values of θ and results averaged to obtain better accuracy. The final calculated value for the kinetic friction coefficient was $\mu = 0.195 \pm 0.002$. This value agrees well with the mean value obtained for the kinetic friction coefficients collected in table 2 ($\mu = 0.19 \pm 0.03$).

Introductory physics students consistently encounter difficulties in interpreting kinematic graphs. The experiments reported in this section allow us to achieve, amongst others, the educational goal of clarifying the difference between (uniformly and non-uniformly) accelerated motion and motion at a constant speed.

4.2. Influence of magnet size

The purpose of this section is to experimentally determine the influence of the geometry of the magnet footprint on the damping coefficient. The magnetic force exerted on the magnet depends on the portion of the eddy current that is within the magnetic field created by the magnet. This effective eddy current length is related to the pole projection area of the magnet [12, 15]. We present experiments carried out with two magnets with the same magnetic field strength but different geometry (A and M1 magnets) at different angles. The displacement of these magnets is plotted as a function of time for the Cu3 slab at $\theta = 31^\circ$ and $\theta = 51^\circ$ in figure 6. A linear relationship between the two variables can be seen for all three experiments, and the slope of the straight lines gives the value of the terminal speeds of the sliding magnet, which increases with θ. This figure highlights that the terminal speed for both magnets at $\theta = 31^\circ$ is similar although the mass of M1 magnet is 4.6 times that of the C magnet. In order to calculate the damping coefficient and the kinetic friction coefficient, the ratio $v_T/\cos\theta$ is plotted as a function of $\tan\theta$ in figure 7. This figure shows a good fit of the experimental data ($r > 0.999$), which could confirm that our basic theoretical treatment of the experimental results is correct. The estimated values of b and μ are listed in table 3. Further insights into the interpretation of magnetic braking come from the analysis of these values. The lower damping coefficient for the A magnet (lower diameter) in comparison with the M1 magnet for a slab with the same thickness is noteworthy. The ratio of the b values for both

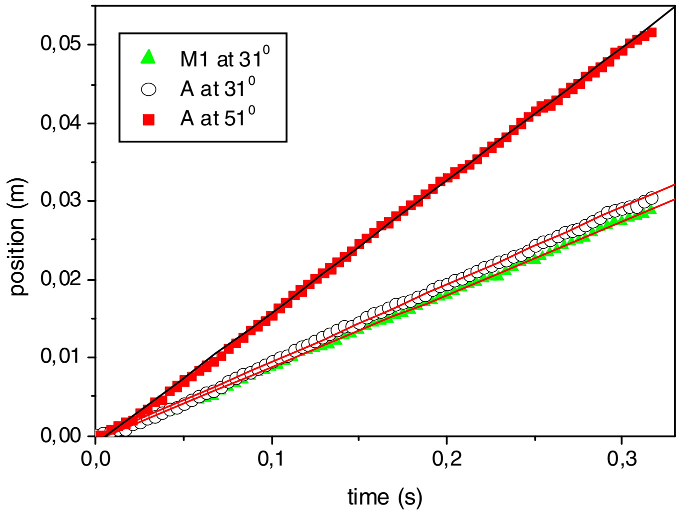

Figure 6. Magnet position versus time for a Cu3 track.

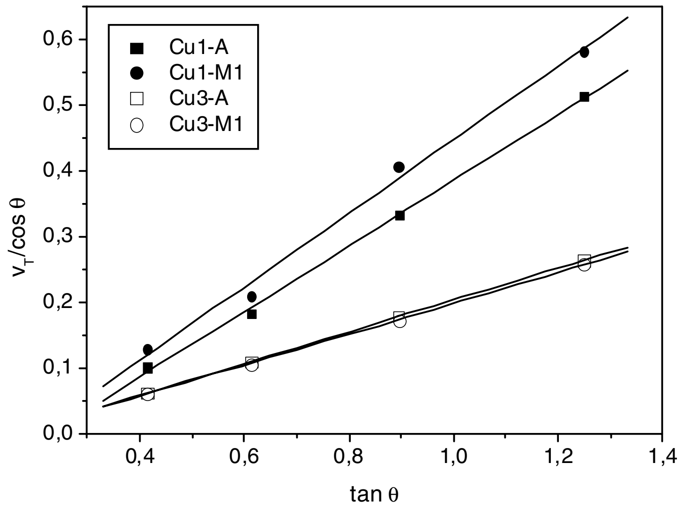

Figure 7. Plot of $v_T/\cos\theta$ as a function of $\tan\theta$ for A and M1 magnets.

Table 3. Damping coefficient (b) and kinetic friction coefficient (μ) values as a function of the magnet used in the experiment.

System (track–magnet)	b (N s m^{-1})	μ
Cu1–A	0.039 ± 0.001	0.22 ± 0.02
Cu3–A	0.081 ± 0.001	0.21 ± 0.01
Cu1–M1	0.194 ± 0.001	0.20 ± 0.04
Cu3–M1	0.378 ± 0.001	0.20 ± 0.01

magnets for a slab with a thickness of 1 mm (0.20) is similar to that for a slab with a thickness of 3 mm (0.21). The mean value obtained for the kinetic friction coefficient is in line with that calculated from the study of the sliding magnet on the plastic slab without a metal base.

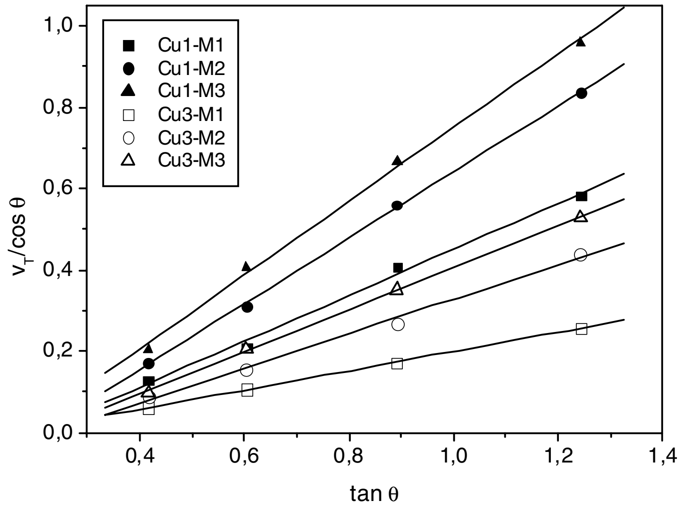

Figure 8. Plot of $v_T/\cos\theta$ as a function of tan θ for M1, M2 and M3 magnets.

Table 4. Damping coefficient (b) and kinetic friction coefficient (μ) values as a function of the magnet used in the experiment.

System (track–magnet)	b (N s m^{-1})	μ
Cu1–M1	0.194 ± 0.001	0.20 ± 0.04
Cu3–M1	0.378 ± 0.001	0.20 ± 0.01
Cu1–M2	0.195 ± 0.003	0.20 ± 0.02
Cu3–M2	0.370 ± 0.002	0.23 ± 0.05
Cu1–M3	0.204 ± 0.007	0.18 ± 0.03
Cu3–M3	0.367 ± 0.006	0.22 ± 0.02

It is again observed that as the thickness of the slab increases, the damping coefficient also increases due to the increasing eddy currents.

4.3. Influence of the mass of the magnet

We present here the results of several experiments in which the mass of the magnet was changed maintaining constant the rest of the properties of the magnet. For this purpose, we loaded the magnet with different amounts of Pb pellets. Figure 8 shows the ratio $v_T/\cos\theta$ as a function of tan θ. Again, almost straight lines can be fitted to the experimental points ($r > 0.999$). This figure reveals that, for the same slab, the terminal speed increases as the mass of the magnet increases. The calculated parameters from figure 8 are tabulated in table 4. Note that, as expected, if we compare magnets that have the same magnetic field strength and geometry, then the damping coefficient will be the same.

5. Summary

In this paper, we have shown the results of a thorough study on the motion of a magnet sliding on an inclined conducting plane. The experiment can be performed with easily obtainable materials and uses widely available high-tech devices (a high-speed digital camera and a personal computer) for data acquisition. This experiment, which is also characterized by its use of freely available video analysis software (Tracker), is particularly suitable for laboratory activities aimed at university undergraduates. Tracker is an open-source video analysis tool that has allowed us to introduce new exercises into the introductory physics laboratory. Indeed, as students can easily download Tracker to their own computer, they can also use it for extended homework assignments. The proposed activity provides students with the opportunity to perform quantitative experiments that help them, amongst other aspects, to develop fundamental skills such as data analysis and competence in instrumental calibration. This experiment has been specifically designed to help students better understand kinematic graphs. Moreover, it allows a more in-depth interpretation of eddy current brake, an important phenomenon based on electromagnetic induction. The time required for the measurements is relatively short and the cost of the experiment is low. We expect students to use the video experiment as complementary material. In this sense, students should be able to use it at home, at their own pace. Likewise, the observed influence of experimental conditions on the terminal speed of the sliding magnet is consistent with the general inference from the electromagnetic theory.

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