Force on a charge q moving w/ speed $\vec{v}$ in a $\vec{B}$ field

$\Rightarrow \vec{F} = q\,\vec{v} \times \vec{B}$

F is in Newtons, q is coulombs
v in m/s, B in tesla

$1\,N = 1\,C\,\frac{m}{s}\,T = 1\left(\frac{C}{s}\right)T\,m$

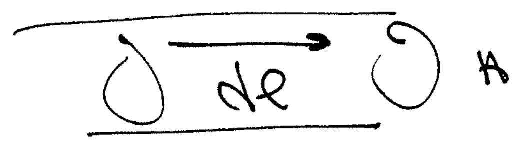
$1\,N = 1\,(\text{Ampere})\,\text{Tesla-meter}$

---

or $d\vec{F} = dq\,\vec{v} \times \vec{B}$

If the charge dq is in a current-carrying wire

$dF = dq\underbrace{\left(\frac{d\vec{l}}{dt}\right)}_{\vec{v}} \times B$



$\vec{v} = \frac{d\vec{l}}{dt}$ in the wire

OR
move the dt under dq:

$$d\vec{F} = \left(\frac{dq}{dt}\right) d\vec{l} \times \vec{B}$$

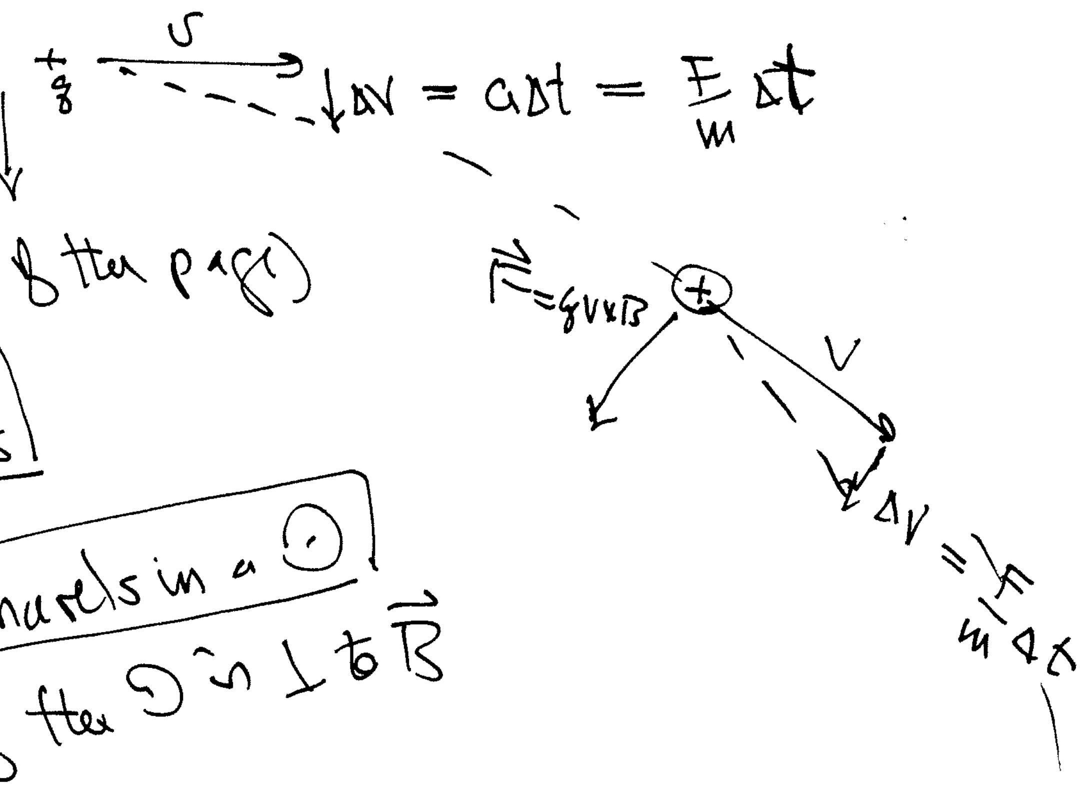
↑ current: $1\,A = 1\,C/s$

$\Rightarrow d\vec{F} = I\,d\vec{l} \times B$ Force on an element $d\vec{l}$ of wire w/ current I

$\Rightarrow \vec{F} = I\,\vec{l} \times \vec{B}$ Force on a straight section of wire $\vec{l}$

Forces on moving charges

A. Circular motion



$\vec F = q\vec v \times \vec B$

(B is out of the page)

$\boxed{q\vec v \times \vec B \text{ pushes } q \text{ sideways}}$

$\boxed{\text{charge travels in a } \odot}$

plane of the ⊙ is ⊥ to $\vec B$

accel in ⊙ is $v^2/r = a$

$\vec v \perp \vec B$ → $F = qvB = ma = \frac{mv^2}{r}$

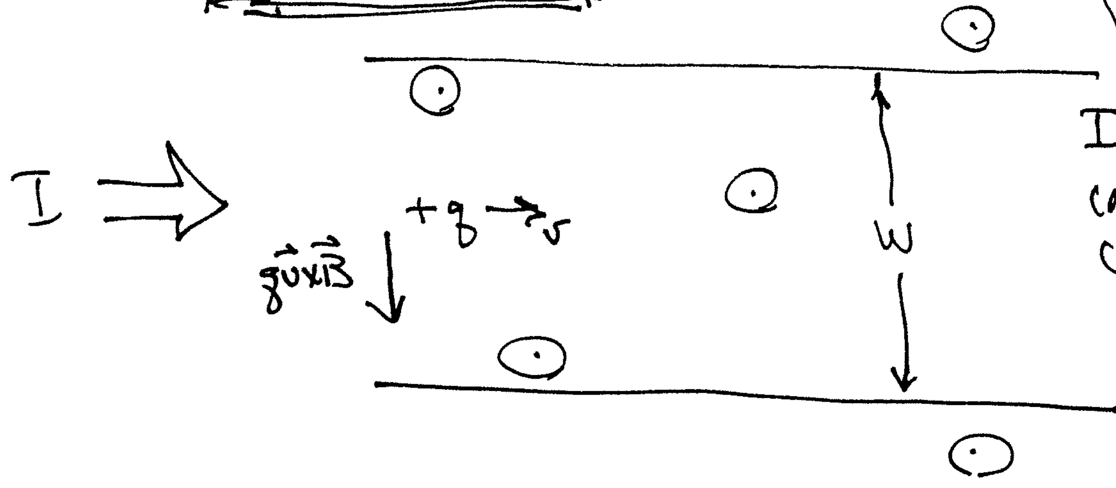
$$\boxed{qvB = \frac{mv^2}{r}}$$

$r = \frac{mv}{qB}$ ← radius of circle

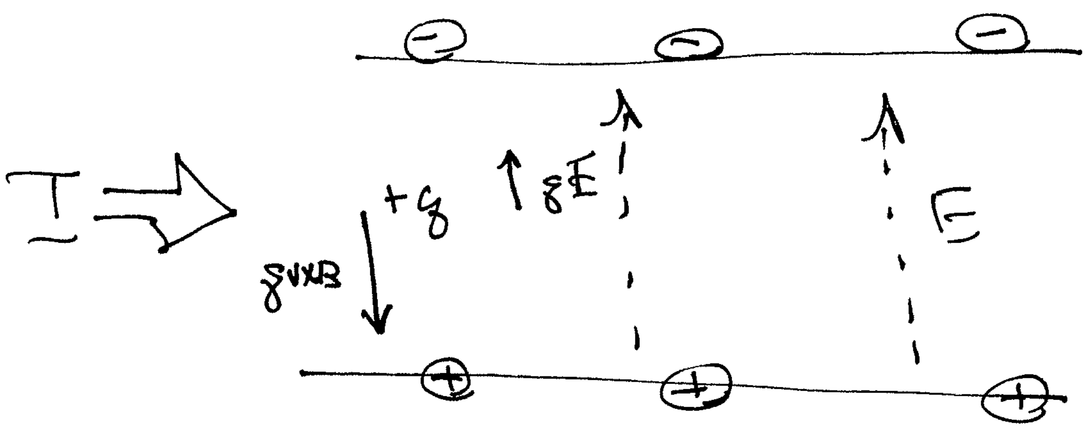
Forces on moving charges

B. $\boxed{\text{Hall effect}}$

B is out of the page.



If ⊕ charges carry the current I they get pushed DOWN by $q\vec{v}\times\vec{B}$



Some $+q$ accumulate on bottom and some $-q$ are left on top

This creates an electric field $\vec{E}$ which points up

If $q\vec{v}\times\vec{B} > q\vec{E}$ $\quad +q$ continue to be pushed down until

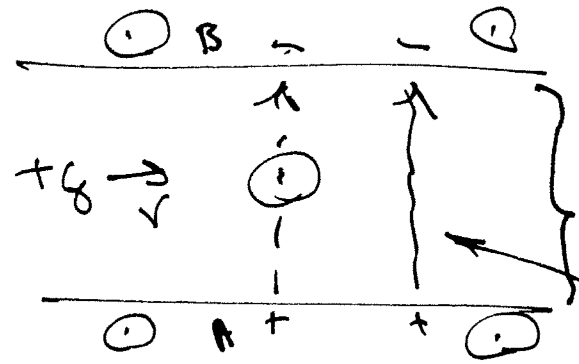
$$-q\vec{v}\times\vec{B} = q\vec{E}$$

At this point no more q get pushed ~~down~~ down

$$\vec{F} = q\vec{v}\times\vec{B} + q\vec{E} = 0$$

Hall effect

+q carries current ⇒



B out of page

$\vec{E} = -\vec{v}\times\vec{B}$

+q pile up at the bottom

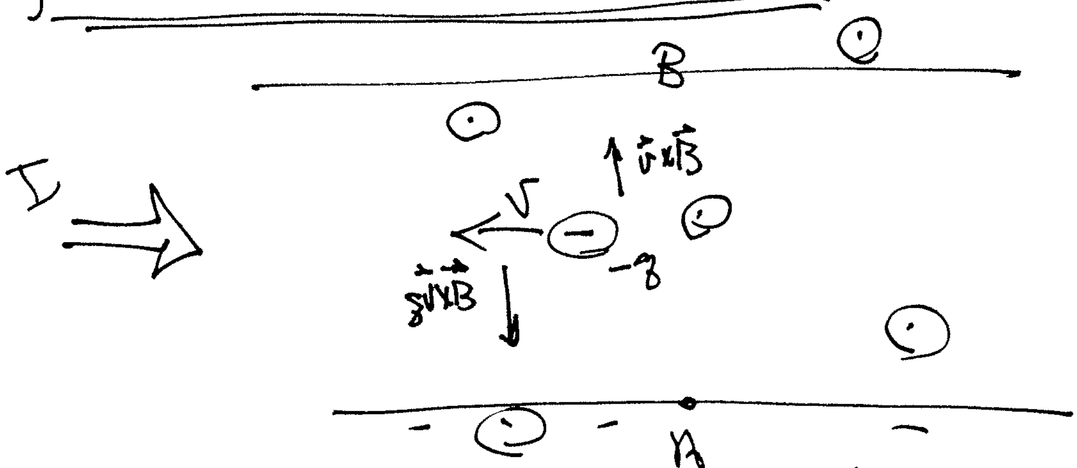
E_{Hall} points upward $\vec{E}_{Hall} = -\vec{v}\times\vec{B}$

$V_A - V_B$ = 'hall voltage' $= w E_{hall} = w v B$

width of sample

drift velocity

If −q carries the current:



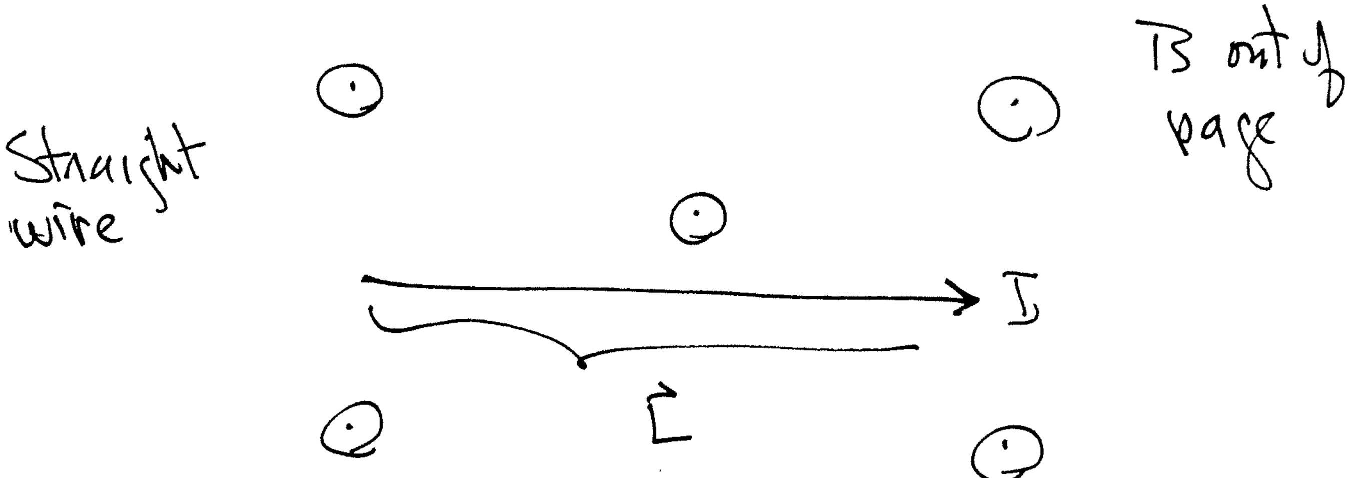
B out of page

$\vec{v}\times\vec{B}$ points up BUT $q\vec{v}\times\vec{B}$ points DOWN! (q is negative)

−q get forced to the bottom instead of +q

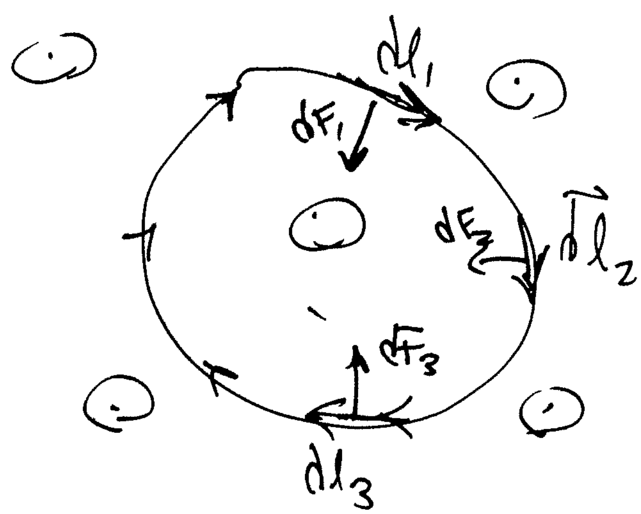
Since − charges are on the bottom $V_A - V_B < 0$: the sign of the hall voltage tells the sign of the charge carriers

Forces on wires (see p. 1)



$$\boxed{F_{\text{on length } L} = I\,\vec{L}\times\vec{B}} \text{ (down)}$$

Loop of ~~wire~~ wire in uniform $\vec{B}$ field



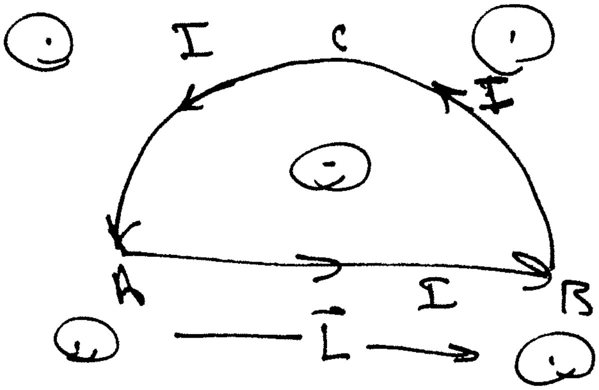
$$\begin{aligned}
\text{Force on loop} &= d\vec{F}_1 + d\vec{F}_2 + d\vec{F}_3 + \ldots \\
&= I\,(d\vec{l}_1\times B + d\vec{l}_2\times B + d\vec{l}_3\times B \ldots) \\
&= I\,(d\vec{l}_1 + d\vec{l}_2 + d\vec{l}_3 + \ldots)\times\vec{B} \\
&= 0!
\end{aligned}$$

$\boxed{\text{The vector sum of all } d\vec{l} \text{ is zero.}}$

Force on loop is zero in uniform $\vec{B}$ field

Force on wire:

Exercise: show that force on curved part BCA

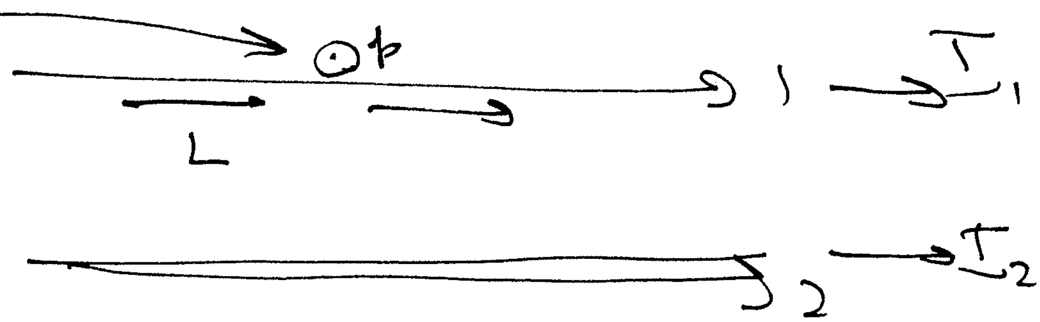


D-shaped loop in a uniform B field

$$\vec{F}_{BCA} = -I\,\vec{L}\times\vec{B}$$

Force between parallel wires

B at p due to wire 2 is out of paper



Force on wire 1 from B at p due to 2 is

$$F_{\text{on 1 fm 2}} = I_1(\underbrace{\vec{L}\times\vec{B}_{\text{from 2}}})$$

down: wires attract.

Parallel wires attract

Antiparallel wires repel

$$\text{Force/length} = I_1 B_{\text{from 2}} = I_1\left(\frac{\mu_0 I_2}{2\pi d}\right)$$

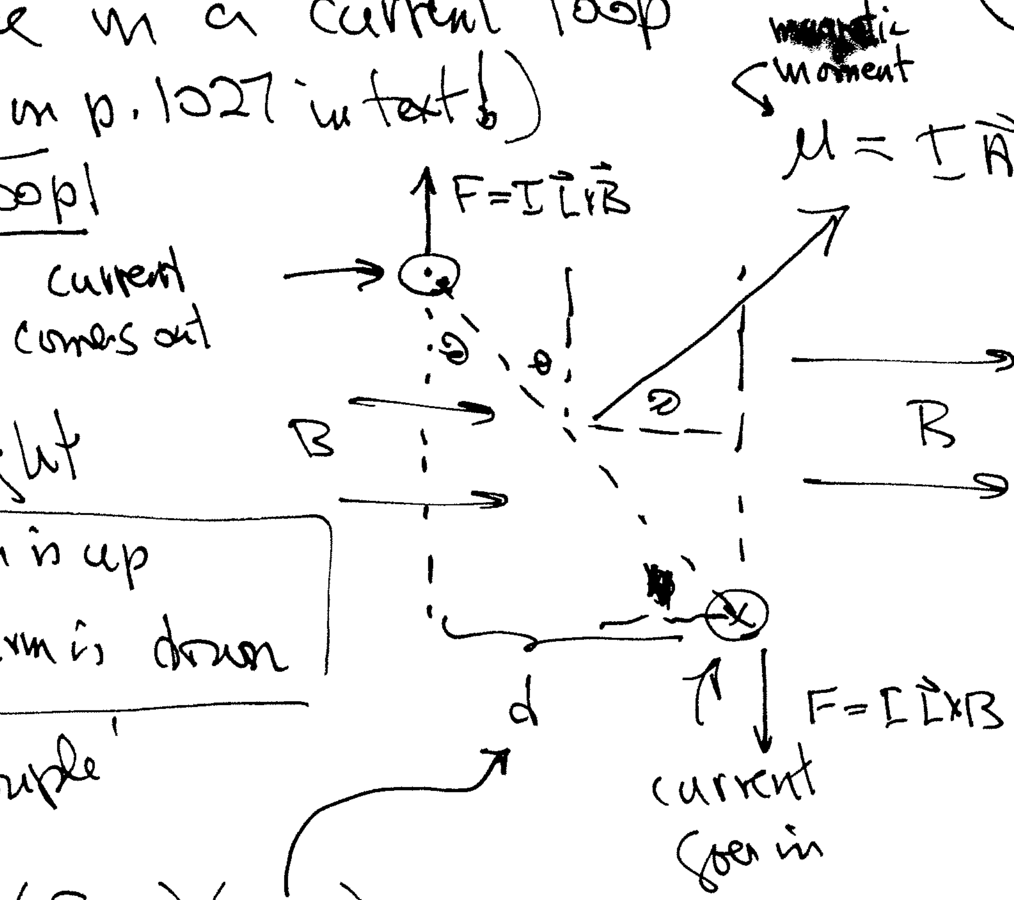
$$\boxed{F/l = \frac{\mu_0 I_1 I_2}{2\pi d}}$$

(32.26 p. 1025

Torque on a current loop

(Nice diagram on p. 1027 in text!)

Square loop of side L



$\vec{B}$ goes to right

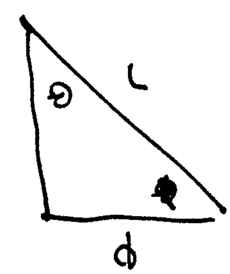
$\vec{F}$ on top arm is up
F on bottom arm is down

these form a 'couple'

$$\text{torque} = (\text{force})(\text{arm})$$

$$= (ILB)(\underbrace{d}_{L\sin\theta})$$

$d = L\sin\theta$



$$\tau = IBL^2\sin\theta = \underbrace{(IL^2)}_{\mu = IA} B\sin\theta$$

causes CW rotation

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$ causes CW rotation

$$\vec{\tau} = \vec{p}\times\vec{E}$$ for electric dipoles

DIPOLES ALWAYS ~~TRY~~ TO ALIGN WITH THE FIELD