Units of B field are tesla (T)

$\vec B$ due to a moving charge q

$\vec v$ = velocity of charge q

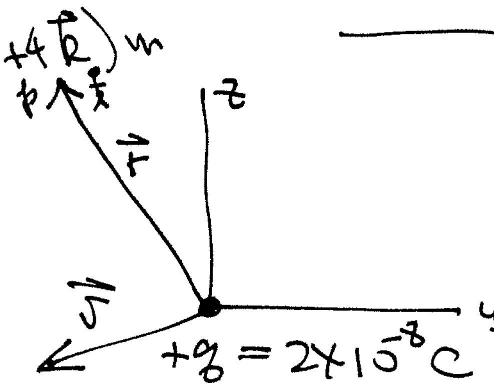
$\vec r$ goes from q to p

$$\boxed{\vec B_p = \frac{\mu_0}{4\pi} q \frac{\vec v \times \vec r}{r^3}}$$

32.4 p. 100

Example:

(MJM)



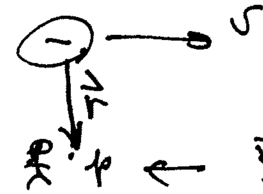
$\vec v = \hat i \; 3\times10^4$ m/s

$$\frac{\vec v \times \vec r}{r^3} = \frac{3\times10^4 \,\text{m/s}\; \hat i \times (3\hat i + 4\hat k)\,\text{m}}{\underbrace{(3^2+4^2)^{3/2}}_{r^3}} = -\hat j \, \frac{12\times10^4 \,\text{m}^2/\text{s}}{125\,\text{m}^3}$$

$$B_p = -\hat j \, \frac{4\pi\times10^{-7}\,\text{N/A}^2}{4\pi} \cdot 2\times10^{-8}\,\text{C} \; \frac{12\times10^4\,\text{m}^2/\text{s}}{125\,\text{m}^3} \cong 0.2\times10^{-11}\,\text{T}\,(-\hat j)$$

Lines of $\vec B$ due to a moving charge q form ~~⊗~~ circular patterns (Fig 32.7, p. 1002)

Example 32.2 p. 1003 electron



← B at p?

$\vec v \times \vec r$ points INTO the page

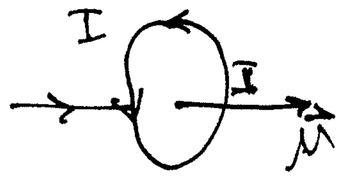
But q is negative for the electron

so B_p due to the electron is

OUT OF THE PAGE

Magnetic dipoles (p. 1008) $\vec{\mu}$ = magnetic dipole moment ②

Dipole moment of a current loop



Wrap fingers of RH around the current I.
Thumb is along the direction of $\vec{\mu}$

$\vec{\mu} = I\vec{A}$ $\vec{A}$ = area vector ($\perp$ to area according to the RH rule)

magnetic moment = current · area

Example (MJM) We have a loop of radius 6mm
It has a magnetic moment
$\mu = 0.56$ A-m^2
What is the current in the loop?

$$\mu = 0.56 \text{ A-m}^2 = I\,\pi\,(.006\text{m})^2$$

$$I = \frac{.56 \text{ A-m}^2}{\pi \times 36 \times 10^{-6} \text{ m}^2} = 4950 \text{ A} \sim 5000 \text{ A}$$

B field on axis far from the loop

$$B_{\text{axis of loop}} = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}} \xrightarrow{z \gg R} \frac{\mu_0 \overbrace{I \pi R^2}^{IA = \mu = \text{magnetic moment}}}{2\pi\ z^3}$$

$$B_{\text{on axis } z \gg R} = \frac{\mu_0}{2\pi} \frac{(\mu)}{z^3}$$

$(\mu) \leftarrow$ magnetic moment

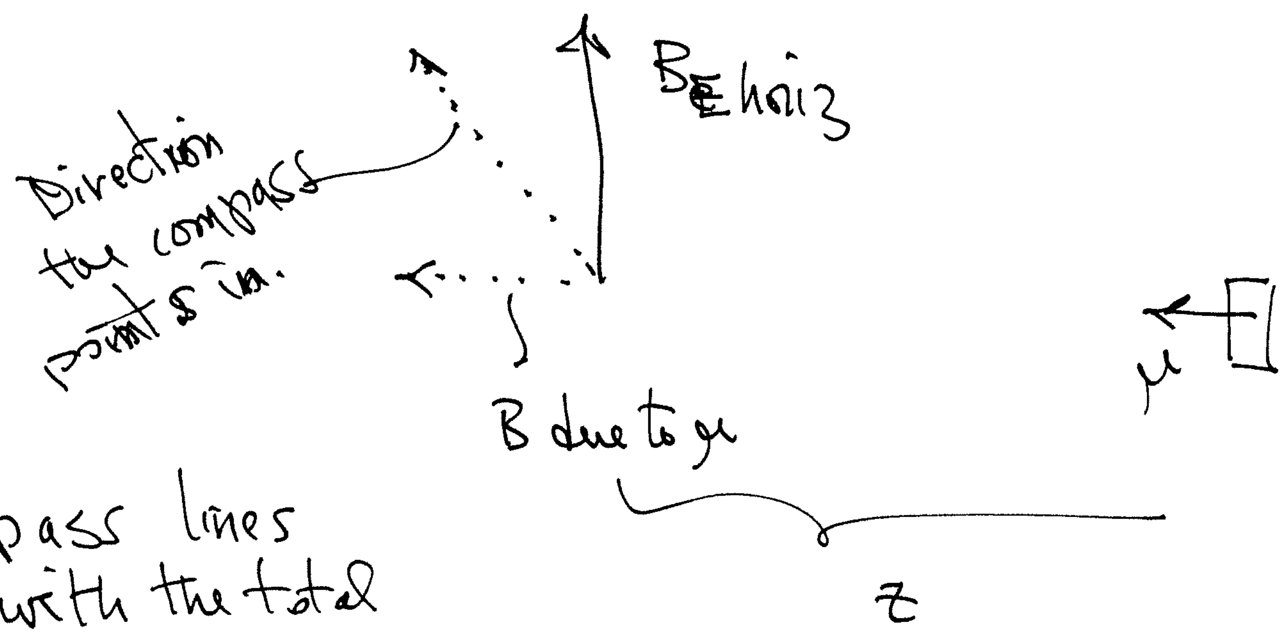
$z^3 \leftarrow$ distance^3

Finding the horizontal component of Earth's magnetic field

(Not in the book)

1) Line up a compass pointing North (it is lined up with $B_{E,horiz}$)

2) Bring up a magnet of known μ along an E-W direction



(Compass lines up with the total B field)

3) When the compass rotates 45°, $B_{E\,horiz}$ is equal to $B_{\text{due to }\mu}$

4) Knowing z and μ, Calculat $B_{E\,horiz}$

Ampere's Law (a 2nd way of calculating B)
(1st ~~way~~ was Biot-Savart Law)

$$\int_{\text{curve } C} \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed by } C}$$

32.14
~~32.13~~
p. 1013

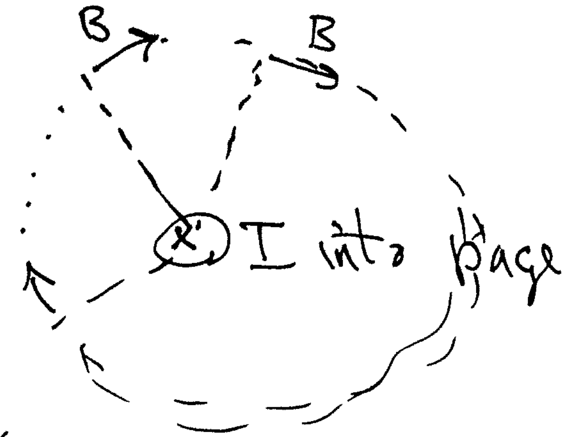
DOT PRODUCT

$d\vec{s}$ follows curve C
(Does not follow current I!)

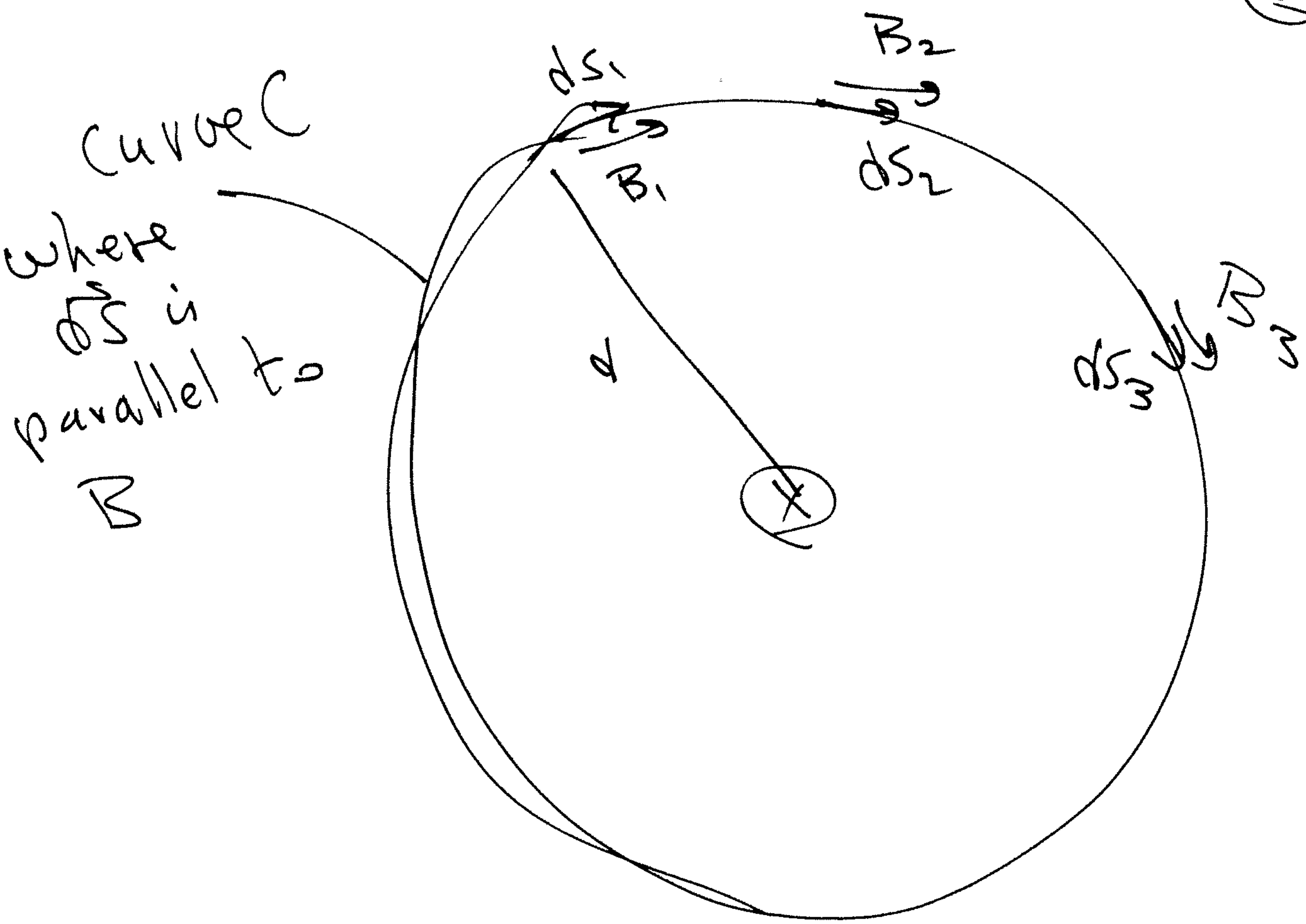
Simplest Ampere's Law calculation pp. 1012–1013

Long straight wire pointing into the page

$\vec{B}$ surrounding a long straight wire goes in concentric ⊙s around the wire
Fig 32.14. p. 1006
Fig 32.5 p. 1000
Fig 32.2 p. 999



For Ampere's law we select a curve C so that $d\vec{s}$ is everywhere parallel to B.
This means C is a ⊙ of radius d Fig 32.23 p. 1013



$\cancel{\int d\vec{s}_1}$ $\int \vec{B}\cdot d\vec{s} = \vec{B}_1\cdot d\vec{s}_1 + B_2\cdot ds_2 + \ldots$

$= B_1 ds_1 \cos\theta_1 + B_2 ds_2 \cos\theta_2 + \ldots$

$\theta_1 = \theta_2 = 0$

since ds is always parallel to B

$\int B\,ds\cos\theta = \int B\,ds$

B is constant

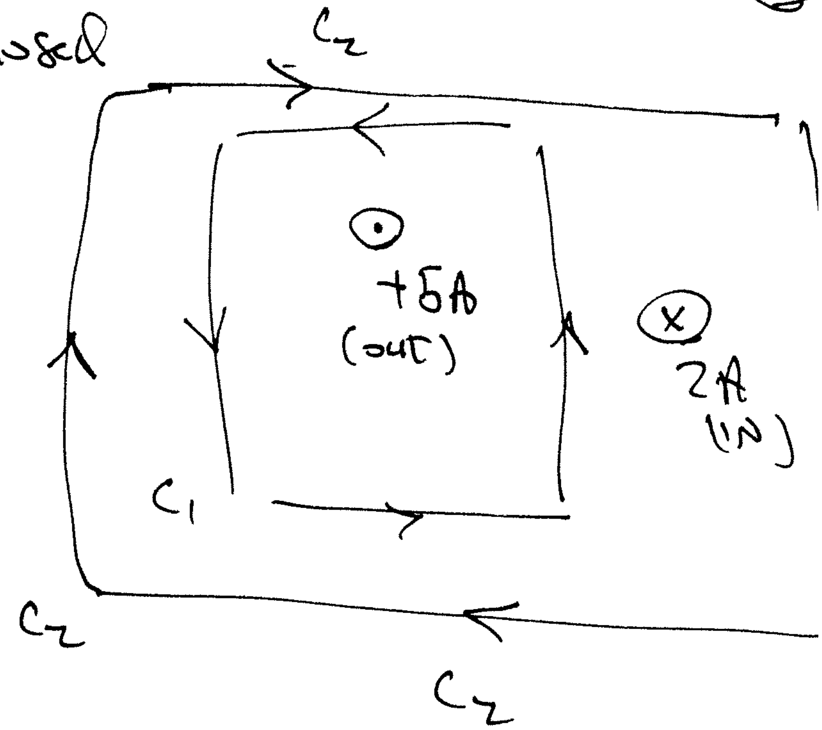
$$\boxed{\int \vec{B}\cdot d\vec{s} = \int B\,ds = B\int ds = B\cdot 2\pi d}$$

Ampere: $\int_C \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enclosed by } c} = \mu_0 I = 2\pi d B$

$$\boxed{B_{\text{long straight wire}} = \frac{\mu_0 I}{2\pi d}}$$

$$\int_C \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$$

follows curve C



$$\int_{C_1} \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enclosed by } C_1}$$

wrap fingers of RH around C_1. Thumb comes out of page.

∴ current through curve C_1 is ⊕ if it comes out of page

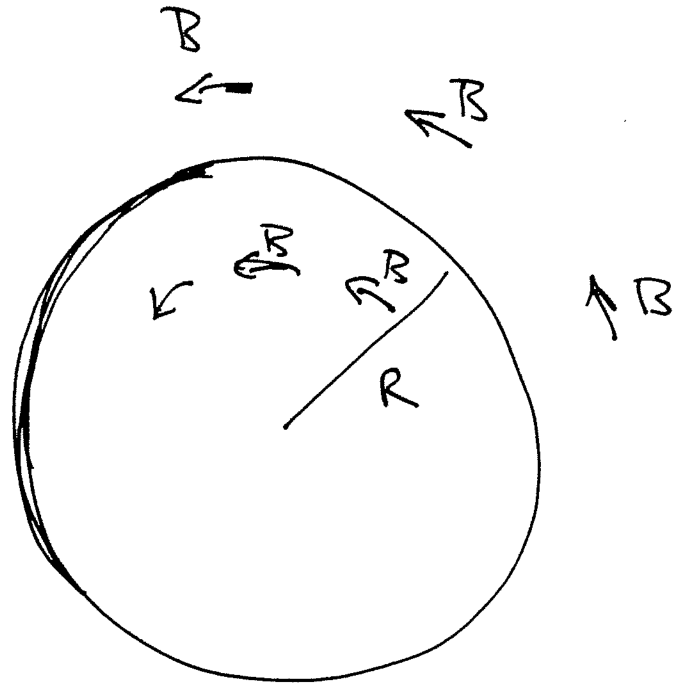
$I_{\text{enclosed by } C_1} = +5A$

C_2 goes CCW so current INTO page counts positive

$I_{\text{enclosed by } C_2} = +2A - 5A = -3A$

FAT WIRE radius R p. 1014

current I in wire comes out of page



$\vec{B}$ outside the wire goes in concentric ○s, and B inside the fat wire also goes in concentric ○s. So we pick a curve C inside which is a circle of radius $r < R$. The integral $\int_C \vec{B} \cdot d\vec{s} = 2\pi r B$ just as it does outside. But now the enclosed current is less than I. The curve C now has area πr^2, and the total area of the wire is πR^2.

So $I_{enclosed} = \frac{\pi r^2}{\pi R^2} I$

Amperes $\int_C \vec{B} \cdot d\vec{s} = 2\pi r B = \mu_0 I \frac{\pi r^2}{\pi R^2}$

$\boxed{B_{INSIDE} = \frac{\mu_0 I r}{2\pi R^2}}$

$r \to 0$ $B_{in} \to 0$

$r \to R$ $B \to \frac{\mu_0 I}{\pi R}$

Fat wire recap

$$\oint_C B\cdot ds = 2\pi r B = \mu_0 I_{encl} = \mu_0 I \frac{\pi r^2}{\pi R^2}$$

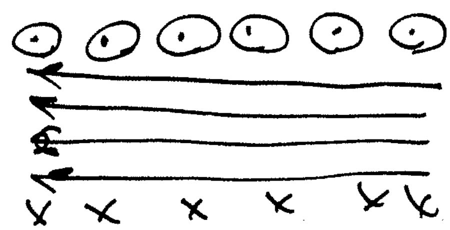
$$B_{IN} = \frac{\mu_0 I r}{2\pi R^2} \to 0 \text{ as } r \to 0$$

$$B_{IN} = \frac{\mu_0 I r}{2\pi R^2} \to \frac{\mu_0 I}{2\pi R} \text{ as } r \to R$$

B_{IN}: inside

Solenoid pp. 1015, 1016

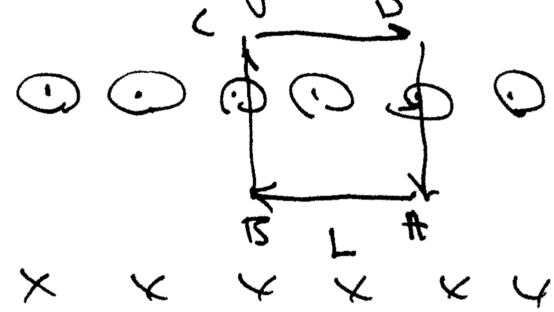
$B \approx 0$



$B \approx 0$

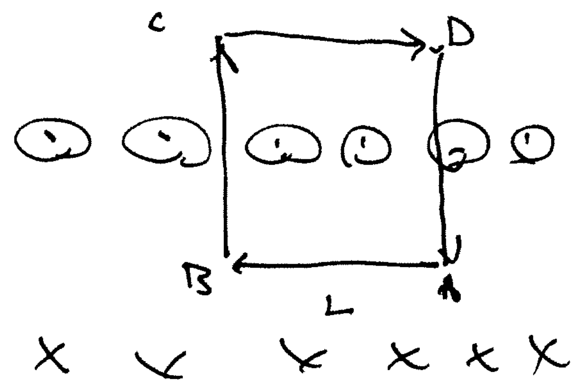
Inside solenoid $\vec{B}$ = constant
Outside solenoid $B \approx 0$

Pick a rectangular curve C



$$\oint B\cdot ds = \underbrace{\int_{AB} B\cdot ds}_{\substack{B \parallel ds \\ BL}} + \underbrace{\int_{BC} B\cdot ds}_{\substack{B \perp ds \\ 0}} + \underbrace{\int_C^D B\, ds}_{B=0} + \underbrace{\int_D^A B\cdot ds}_{\substack{B \perp ds \\ 0}} = B\cdot L$$

Solenoid (finishing up)



- $\int \vec{B} \cdot d\vec{s} = BL$

current enclosed?

current enclosed $= \left(\dfrac{\text{current}}{\text{length}}\right)(\text{length})$

$= \underbrace{\left(\dfrac{\text{current}}{\text{turn}}\right)}\left(\dfrac{\text{turns}}{\text{length}}\right)(\text{length})$

- $I_{enc} = I \quad n \cdot L$

Ampere:

$$\int_C \vec{B} \cdot d\vec{s} = BL = \mu_0 (I n L)$$

$$\boxed{B_{solenoid} = \mu_0 n I}$$

n: $\dfrac{\text{turns}}{\text{length}}$

- If two solenoids have same length & same current but one has twice the turns of the other that one will have twice the B field