ES 202
Fluid and Thermal Systems

Lecture 22:
Isentropic Efficiencies & Simple Power Cycles
(2/3/2003)

Road Map of Lecture 22

- Quiz on Week 7 materials
- Comments on Lab 2
  - overall better write-up than Lab 1
  - some impressive write-ups, very encouraging
- Comments on constant specific heat for argon
  - a deeper look at variable specific heats
- Representation of isentropic and non-isentropic processes on
  - *h*-*s* diagram
  - introduce the limit of best performance
  - notion of isentropic efficiency
  - *T*-*s* diagram for compressor and turbine
- Power cycles

My Lab #2

- **“Water Wall” experiment (5 points)**
  - The “Pendulum” demonstration:
    - the key difference is the change in momentum on a concave versus a flat surface
  - The “Torricelli”-like demonstration:
    - depth, rather than base area, is the controlling factor
  - The “Four-tube” demonstration:
    - recognition of static and stagnation pressure measurements
    - difference between stagnation pressure and static pressure is the dynamic pressure
    - flow velocity increases from large to small tube (difference between stagnation and static measurements increases from large to small tube)
    - as a result, pressure drops from large to small tube (higher static pressure in first tube relative to third tube)
    - stagnation pressure stays almost constant from large to small tube (with a slight drop due to losses; small difference between second and fourth tube)
  - The “Three-coil” demonstration:
    - learn to think in non-dimensional world *(e.g.* ε/D*,* L/D*)*
    - same pressure difference across coil in all cases
    - loss is smaller in shorter and wider coil
    - apart from major loss, there is also minor loss due to continuous change in flow direction

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My Lab #2 (Cont’d)

- **Pipe friction experiment (5 points)**
  - comparison between measured and predicted values on the same plot
  - explanation for the difference
  - question on zero surface roughness assumption
  - try out non-zero values for surface roughness (gives equivalent surface roughness for PVC pipe)
  - discussion of sources of error (manometer reading, timing)
- **Torricelli’s experiment (5 points)**
  - mean and variation in results
  - discharge, velocity and contraction coefficients must be smaller than unity
  - if not, give explanation for the non-physical values
  - discussion of sources of error (unsteadiness, extent of valve opening, measurement of shooting range)

A Deeper Look at Variable Specific Heats

- In classical statistical mechanics, the specific heat at constant specific volume (c_v) can be expressed as:

$$c_v = \frac{n_f}{2} R$$

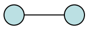
where n_f is the "number of degrees of freedom" of the molecular model

- In ideal gases, the difference between the two specific heats (c_p, c_v) is the specific gas constant:

$$c_p - c_v = R \longrightarrow c_p = \frac{n_f + 2}{2} R$$

- Some examples of molecular models are:
  - smooth sphere model for monatomic gases like helium, argon, *etc.*

  - rigid or flexible dumb-bell model for diatomic gases like oxygen, nitrogen, *etc.*



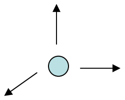
(rigid)

(flexible)

A Deeper Look at Variable Specific Heats (II)

- For the smooth sphere model:

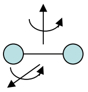
$$n_f = 3$$



which stands for translational motion in 3 spatial coordinates. This model holds true for monatomic gases over a very wide range of temperatures. Hence, a constant specific heat assumption is excellent for monatomic gases.

- For rigid dumb-bell model:

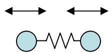
$$n_f = 5$$



which includes translational motion in 3 spatial coordinates and rotational motion along 2 major axes.

- At high temperatures, the rigid dumb-bell model becomes flexible to take into account of possible vibrational motion (2 additional degrees of freedom: KE + PE):

$$n_f = 7$$



A Deeper Look at Variable Specific Heats (III)

- The transition from the rigid dumb-bell model to the flexible dumb-bell model for diatomic gases occurs gradually over a temperature range.

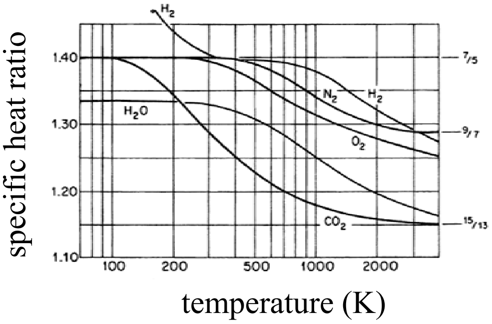


Figure F.1 in "Compressible Fluid Dynamics" by Thompson

For monatomic gases, $k = 5/3$ at all temperatures.

- The molecular model for more complex molecules (tri-atomic or higher) are more sophisticated and their dependency of modal excitation on temperature is much more complicated. But the crude picture has been sketched here.

Adiabatic Devices: Energy Analysis

- For most steady-state devices, (for examples, compressors, turbines, nozzles, diffusers), the flow process between the inlet and outlet state is often modeled as adiabatic because
  - heat transfer is not the primary function of these devices (not true for heat exchangers)
  - flow process at design condition is often fast compared with the heat transfer process.
- For these devices, the energy balance can be reduced to a simple form:

$$\underbrace{\cancel{\frac{dE}{dt}}}_{\text{steady state}} = \underbrace{\cancel{\dot{Q}_{\text{in}}}}_{\text{adiabatic}} + \dot{W}_{\text{in}} + \dot{m}_{\text{in}}\left(h + \frac{v^2}{2} + gz\right)_{\text{in}} - \underbrace{\dot{m}_{\text{out}}}_{= \dot{m}_{\text{in}} \text{ (due to mass conservation)}}\left(h + \frac{v^2}{2} + gz\right)_{\text{out}}$$

$$\Longrightarrow \quad \dot{W}_{\text{in}} = \dot{m}\left[\left(h + \frac{v^2}{2} + gz\right)_{\text{out}} - \left(h + \frac{v^2}{2} + gz\right)_{\text{in}}\right]$$

Adiabatic Devices: Energy Analysis (II)

- If the changes in kinetic energy and potential energy can be further neglected (commonly assumed in compressor and turbine analyses; NOT in nozzle and diffuser), the rate of work input will be directly related to
  - the mass flow rate;
  - change in enthalpy

$$\longrightarrow \quad \dot{W}_{in} = \dot{m}\left(h_{out} - h_{in}\right)$$

- Based on the above result, the change in enthalpy between the inlet and outlet state in a compressor or a turbine can be interpreted as directly related to the work input per mass flow:

$$\longrightarrow \quad W_{in} = \frac{\dot{W}_{in}}{\dot{m}} = h_{out} - h_{in}$$

Adiabatic Devices: Entropy Analysis

- For these adiabatic devices, the entropy balance can also be reduced to a simple form:

$$\underset{\text{steady state}}{\cancel{\frac{dS}{dt}}} = \underset{\text{adiabatic}}{\cancel{\frac{\dot{Q}_{in}}{T}}} + \dot{m}_{in}\, s_{in} - \underbrace{\dot{m}_{out}}_{=\dot{m}_{in} \ \text{(due to mass conservation)}} s_{out} + \dot{S}_{gen}$$

$$\longrightarrow \quad \dot{S}_{gen} = \dot{m}\left[s_{out} - s_{in}\right]$$

- Based on the above result, the change in entropy between the inlet and outlet states can be interpreted as the entropy generation per mass flow:

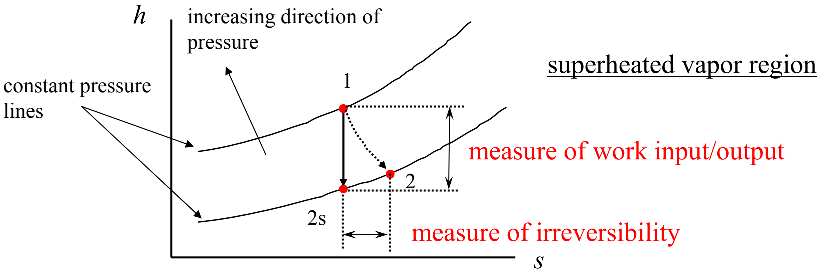
$$\longrightarrow \quad S_{gen} = \frac{\dot{S}_{gen}}{\dot{m}} = s_{out} - s_{in}$$

- For a steady, reversible, adiabatic process, it is also isentropic:

$$\longrightarrow \quad s_{out} = s_{in}$$

The *h-s* Diagram

- Based on the previous energy and entropy analyses, it is learned that:
  - the change in enthalpy between the inlet and outlet state is related to the work input/output per mass flow;
  - the change in entropy between the inlet and outlet state is related to the entropy generation per mass flow (irreversibility).
- It is informative to represent the process path on a *h-s* diagram:



Isentropic Efficiency

- Central theme: the reversible, adiabatic (*i.e.* isentropic) process sets the limit of best performance of any system
- It also sets the reference for the definition of efficiency of compressors and turbines
- For compressors and pumps, work is done on the system:

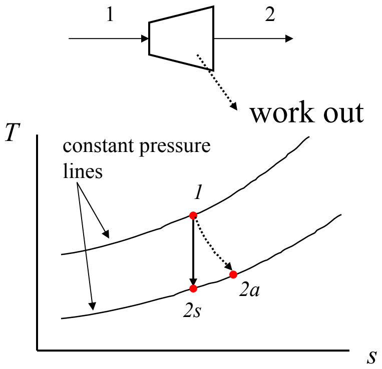
$$\eta_{\text{comp}} = \frac{\dot{W}_{\text{ideal}}}{\dot{W}_{\text{actual}}}$$

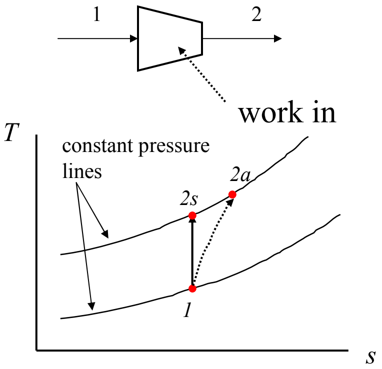
- For turbines, work is done by the system:

$$\eta_{\text{turb}} = \frac{\dot{W}_{\text{actual}}}{\dot{W}_{\text{ideal}}}$$

The *T-s* Diagram

- For ideal gases, the *h-s* and *T-s* diagrams are very similar.
- Turbine
- Compressor



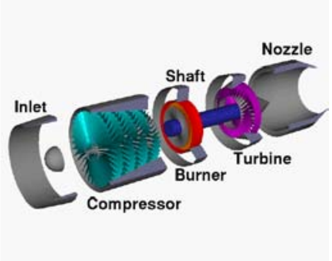


Differences Between Actual and Ideal States

- With reference to the *T-s* diagrams on the previous slide, a few observations are noteworthy:
  - the outlet pressure is the same for both the actual *(2a)* and ideal *(2s)* states (system specification);
  - the outlet temperature is different between the actual *(2a)* and ideal *(2s)* states. Their difference is the penalty one needs to pay for the irreversibilities.
  - the outlet state *(2a)* is always to the right of the inlet state *(1)* because entropy is generated during the process.

Power Cycles

- The integration of turbines, compressors, heat exchangers and combustors can generate power.
- For example, the gas turbine engine:
  - identify the different components



- The process path of any thermodynamic cycles form a closed loop on any phase/property diagram.
- The direction of process path determines what kind of cycle it is (extracting power versus refrigeration, *etc*.)