

ES 202

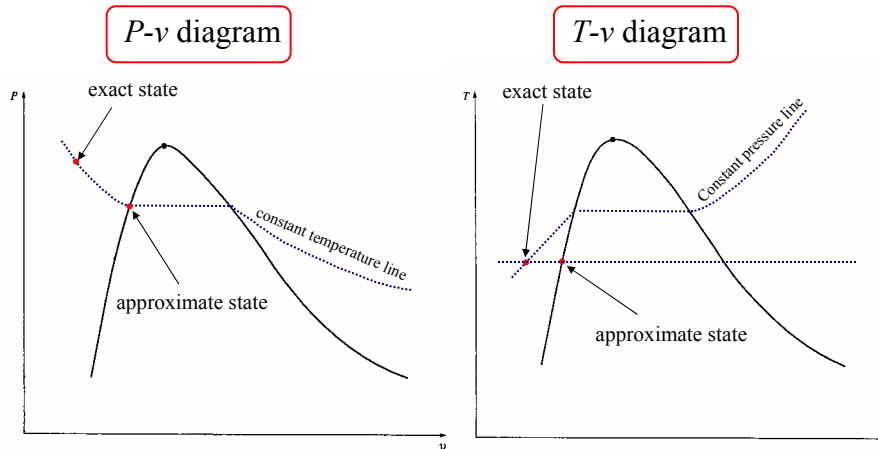
Fluid and Thermal Systems

Lecture 20: Isentropic Processes (1/28/2003)

Road Map of Lecture 20

- Comments on Quiz 3
 - saturated liquid ($x = 0$) and saturated vapor ($x = 1$)
 - good job on interpolation problem
 - weak on Compressed Liquid Approximation (**quality is undefined** in compressed liquid region)
 - constant pressure and temperature curves on phase diagrams (shape and direction)
- Supplement to Lecture 19
- Property variation in an ideal gas: variable specific heats
 - Gibbs equation
 - graphical interpretation
 - newly defined variables
- Isentropic processes
 - When is “constant entropy” a good assumption?
 - entropy change for an **ideal gas** with **constant** specific heats (**special** case)
 - entropy change for an **ideal gas** with **variable** specific heats (**general** case)
- Examples

Interpretation of **CLA** on Phase Diagrams



CLA stands for **C**ompressed **L**iquid **A**pproximation

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Supplement to Lecture 19

- Critical-point properties:

<u>Substance</u>	<u>Temperature</u>	<u>Pressure</u>
Air	132.5 K	3.77 MPa
Water	647.3 K	22.09 MPa

(extracted from Table A-1 in Cengel & Turner)

- Relationship between c_v and c_p
 - Which one has a higher value?
 - What is the reason for the difference?
 - Apply your reasoning to the Ideal Gas Model and Incompressible Substance Model

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Property Variation in an Ideal Gas

- Recall the **Gibbs equation** (relationship between changes in properties)

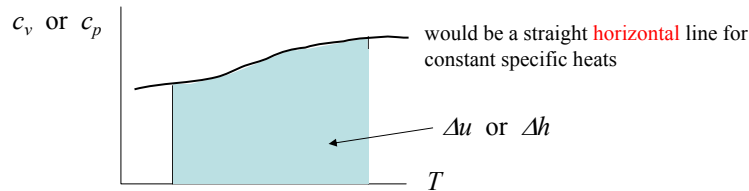
$$Tds = du + Pdv \quad \text{or} \quad Tds = dh - vdP$$

- For an **ideal gas** with finite temperature change:

$$\Delta u = \int du = \int c_v(T) dT \quad , \quad \Delta h = \int dh = \int c_p(T) dT$$

$$\Delta s = \int ds = \int c_v(T) \frac{dT}{T} + \int R \frac{dv}{v} \quad \text{or} \quad \Delta s = \int c_p(T) \frac{dT}{T} - \int R \frac{dP}{P}$$

- How to evaluate the integrals? (graphical interpretation)



An Easy Way Out

- For **variable specific heats**, due to the frequent usage of the integrals, its value (from a common reference point) is tabulated (*e.g.* Table A-17 in Cengel & Turner)

- A new variable is defined:

$$s^0 = \int_0^T c_p(T) \frac{dT}{T}$$

which is the **temperature** dependent part in entropy change

- By making use of this newly defined variable, the entropy difference between any two states can be easily expressed as:

$$s_2 - s_1 = s_2^0 - s_1^0 - R \log \left(\frac{P_2}{P_1} \right)$$

contribution due to pressure change

Constant Property Processes

- constant **pressure** (isobaric; piston-cylinder model)
- constant **volume** (rigid system boundaries)
- constant **temperature** (sufficient time for heat transfer with environment)
- constant **entropy** (reversible, adiabatic)
- They serve as good **models** for complex problems.
- When is “constant entropy” a **good assumption**?
 - process time scale **short** compared with heat transfer time scale (*i.e.* heat transfer rate **much slower** than other processes in the problem)
 - for example, rapid compression/expansion process

Isentropic Processes

- Recall the **Gibbs equation** for an **ideal gas**:

$$ds = c_v(T) \frac{dT}{T} + R \frac{dv}{v} \quad \text{or} \quad ds = c_p(T) \frac{dT}{T} - R \frac{dP}{P}$$

- For an **isentropic process** ($ds = 0$),
 - you only need to know **one more** thermodynamic property to fix the state
 - the Gibbs equation can be reduced to:

$$c_v(T) \frac{dT}{T} = -R \frac{dv}{v} \quad \text{or} \quad c_p(T) \frac{dT}{T} = R \frac{dP}{P}$$

- The relationship between **temperature**, **pressure** and **specific volume** can be obtained by direct integration of the above equations.

Isentropic Processes (II)

- For the **special** case of **constant specific heats**, direct integration yields:

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{k-1} \longrightarrow T v^{(k-1)} = \text{constant}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \longrightarrow T P^{(1-k)/k} = \text{constant}$$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{v_1}{v_2}\right)^k \longrightarrow P v^k = \text{constant}$$

where $k = c_p / c_v > 1$

- The results confirm the previous claim that if you know **one more** thermodynamic property (temperature, pressure or specific volume), you know everything else!

Isentropic Processes (III)

- For the **general** case of **variable specific heats**, we can recall the newly defined variable s^0 :

$$s^0 = \int_0^T c_p(T) \frac{dT}{T}$$

which further defines two new variables (useful for isentropic analysis):

Relative pressure

$$P_r = \exp\left(\frac{s^0}{R}\right) \longrightarrow \frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}}$$

Relative specific volume:

$$v_r = \frac{T}{P_r} \longrightarrow \frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}}$$