ES 202
Fluid and Thermal Systems

Lecture 19:
Models Versus General Substances
(1/27/2003)

Road Map of Lecture 19

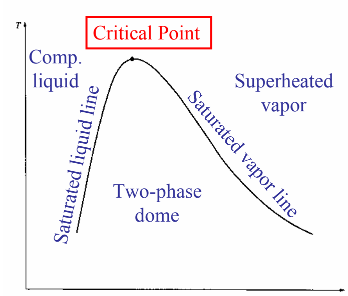
- Quiz on Week 6 materials
- Real gas versus ideal gas
  - notion of reduced coordinate
  - definition of compressibility factor
  - Z-chart
- Ideal gas model
  - change in specific internal energy and specific enthalpy
  - change in specific entropy
  - Gibbs equation and its interpretation
  - variation of specific heats

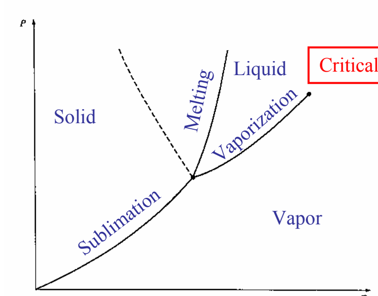
Real Gas Versus Ideal Gas

- Recall ideal gas as a simplified (yet powerful) model for real gas behavior
- Its original derivation assumes negligible mutual interaction between gas molecules. Hence, it is expected to work well for gases under low pressure.
- But, the next logical question will be: “How low is low?” or “Against what standard is low pressure measured with respect to?”
- To answer this question, we need to recall the phase diagrams of a general substance.

Critical State and Reduced Coordinate

- Recall the phase diagrams of a general substance:





- Base on the thermodynamic properties associated with the critical point, a non-dimensional reduced coordinate (a π group) can be defined for each substance: reduced pressure: $P_R = \dfrac{P}{P_{cr}}$, reduced temperature: $T_R = \dfrac{T}{T_{cr}}$

Compressibility Chart

Compressibility Factor:

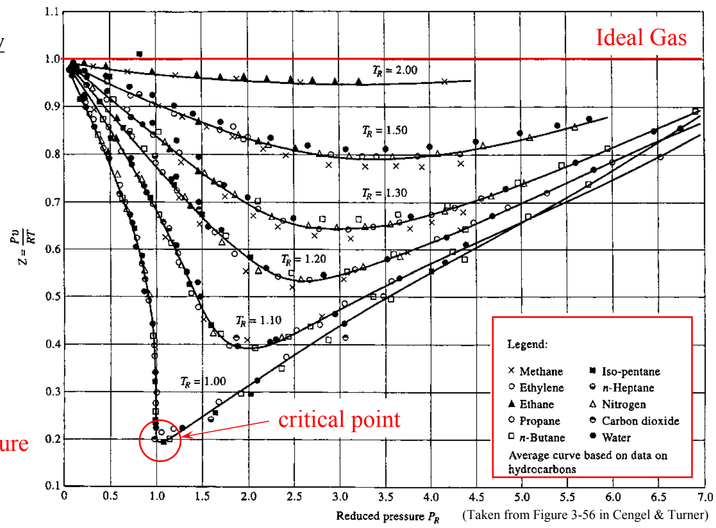
$$Z = \frac{P\,v}{R\,T}$$

Ideal Gas:

$$Z = 1$$

Good for:

- low pressure
- high temperature



(Taken from Figure 3-56 in Cengel & Turner)

Revisit Ideal Gas Specific Heats

- In general,

$$u = u(T, v) \longrightarrow du = \underbrace{\left(\frac{\partial u}{\partial T}\right)_v}_{c_v} dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

Definition of c_v

$$h = h(T, P) \longrightarrow dh = \underbrace{\left(\frac{\partial h}{\partial T}\right)_P}_{c_p} dT + \left(\frac{\partial h}{\partial P}\right)_T dP$$

Definition of c_p

- For an ideal gas, the specific internal energy *(u)* , hence, specific enthalpy *(h)* are functions of temperature only.
- For an ideal gas, the change in specific internal energy and specific enthalpy can be simplified as:

$$du = c_v\,dT \quad , \quad dh = c_p\,dT$$

Entropy Variation in Ideal Gas

- Introduce the Gibbs equation for a general substance:

$$Tds = du + Pdv$$

or $$Tds = dh - vdP \qquad (h = u + Pv)$$

- Interpretation:

$$ds = \left(\frac{\delta q}{T}\right)_{\text{int,rev}} \longrightarrow Tds = (\delta q)_{\text{int,rev}}$$

  – for a simple compressible system, $Pdv = (\delta w)_{\text{int,rev}}$

- For an ideal gas, the Gibbs equation reduces to a simpler form.

Variation in Specific Heats

- In general, the specific heats (c_v, c_p) are NOT true constants. They vary (increase) slightly with temperature even for ideal gases.
- Afterall, it is the change in properties that matters (their absolute values depend on the chosen reference state.)
- For an ideal gas with finite temperature change:

$$\Delta u = \int du = \int c_v dT \quad , \qquad \Delta h = \int dh = \int c_p \, dT$$

$$\Delta s = \int ds = \int c_v \frac{dT}{T} + \int R \frac{dv}{v} \qquad \text{or} \qquad \Delta s = \int c_p \frac{dT}{T} - \int R \frac{dP}{P}$$

- Different ways to approximate the integrals:
  – direct integration (c_v and c_p as functions of T)
  – divide and conquer
  – "average" specific heats

  } geometrical interpretation