Examples for Lecture 21 (1/30/03)

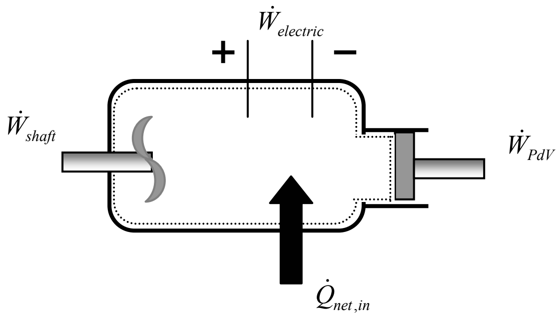
Objectives:

(A) Demonstrate that entropy generation (irreversibilities) within a system always hurts the performance of a system, i.e. reduces the work output or increases the required work input for any process of a closed system.

(B) Demonstrate the relationship between heat transfer and entropy change for an internally reversible process of a closed system and how changes in entropy can be related to changes in other properties (Tds-relations.)

Consider the system shown in the figure and assume that the following assumptions apply:

(1) Closed system,
(2) System contains a simple, compressible substance,
(3) Heat transfer on the boundary of the system occurs at only one temperature, T_b, and
(4) Changes in kinetic and gravitational potential energy are negligible.



A.1 Starting with the general rate-form of the conservation of energy and lumping all of the work transfers of energy together into a single term $\dot{W}_{net,in}$, use the available information about the system to develop the applicable rate-form of the conservation of energy equation for this specific system. Clearly show how you apply the assumptions to simplify the general energy balance.

A.2 Starting with the general rate-form of the entropy accounting equation and recognizing that all the heat transfer occurs at a single temperature T_b, develop the applicable rate-form of the entropy accounting equation for this specific system. Clearly show how you apply the assumptions to simplify the general entropy accounting equation.

A.3 Starting with your entropy balance (A.2), solve the entropy balance for the heat transfer rate to the system.

A.4 Now substitute your expression for the heat transfer rate (A.3) into your energy balance (A.1) to eliminate the heat transfer rate from the energy balance. Now rewrite your equation so that the work (power) term is isolated on the left-hand side of the equation.

A.5 Now consider *two* systems undergoing “identical” processes except for that one process is internally *reversible* and the other is internally *irreversible*:

$$\text{Irreversible Process} \quad \rightarrow \quad \dot{W}_{net,in} = \left[\frac{dU_{sys}}{dt} - T_b \frac{dS_{sys}}{dt} \right] + T_b \dot{S}_{gen}$$

$$\text{Int. Reversible Process} \quad \rightarrow \quad W_{net,in}\Big|_{rev} = \left[\frac{dU_{sys}}{dt} - T_b \frac{dS_{sys}}{dt} \right] + T_b \underbrace{\cancel{\dot{S}_{gen}}}_{=0}$$

Assuming that the terms inside the brackets are identical, i.e, the rate of change of the state of the system is identical, subtract the "reversible" equation from the "irreversible" equation and see how the two different power terms are related.

A.6. What can you conclude about the relationship between the "reversible" work, the "irreversible" work, and the entropy production for a system?

B.1 Now starting with the original energy and entropy balances developed in Parts A.1 and A.2 rewrite these equations on a differential basis, e.g. start out as though you were going to integrate the equations with respect to time but don't.

B.2 Now assume that the process of interest is internally reversible. How does the assumption of internally reversible process influence the following:

The entropy production rate for the system is ____________.

All intensive properties are spatially ________________; thus

$$T_b = T\,; \qquad m_{\text{sys}} = m = \rho V\,; \qquad S_{\text{sys}} = S = ms\,; \qquad U_{\text{sys}} = U = mu\,;$$

The only reversible work mechanism for a simple, compressible system is ________________________ work.

For a finite change in state this work mode can be written as the integral $W_{\text{in}} = -\int_1^2 P\,d\cancel{V}$

What would this look like for a differential change of state?

$\delta W_{\text{in}} =$

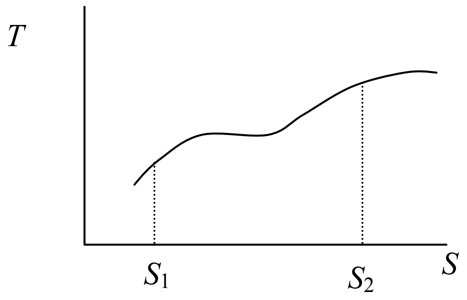
Using this information the differential energy balance and entropy balance become

B.4 **Entropy and Heat Transfer.** Note that the equation above gives a definition for a differential change in entropy. What would it be for a finite change in entropy?

$$dS = \left.\frac{\delta Q}{T}\right|_{rev} \qquad \rightarrow \qquad \Delta S =$$

This can also be turned around to relate the heat transfer in an *internally reversible* process to the area on a *T-S* diagram.

$$\left.\delta Q_{in}\right|_{rev} = TdS \qquad \rightarrow \qquad \left.Q_{in}\right|_{rev} = \int_1^2 TdS$$



How is this related to the graph at right?
What happens when *T* remains constant?
What happens when *S* remains constant?

B.5 Now return to the differential forms of the energy and entropy balance undergoing an internally reversible process: $dU = \delta Q\big|_{rev} - PdV$ and $TdS = \delta Q\big|_{rev}$. Combine these equations by eliminating the heat transfer term.

Can you write it on a per unit mass basis, i.e. in terms of *u*, *s,* and *v*?

What's the significance of the resulting equation?

> Although initially derived for an internally reversible process of a closed system, it now provides a useful relationship between the *pure* intensive properties *T* and *P* and the *specific* intensive properties *s, u,* and *υ.* Because it only involves values of *P* and *T* and differential changes in *s, u,* and *υ*, it can be integrated between any two end states regardless of the process. For example, given sufficient information to evaluate the integrals, changes in *u* or *s* could be calculated as follows:

B.6 Starting with the definition of specific enthalpy, $h = u + P\upsilon$, and the equation , develop the following equation $Tds = dh - \upsilon dP$.

B.7 The two equations that result are known collectively as the *T*ds or Gibbs equations for a simple compressible substance