Cooperative Navigation for Large Swarms of Munitions in Three-Dimensional Flight

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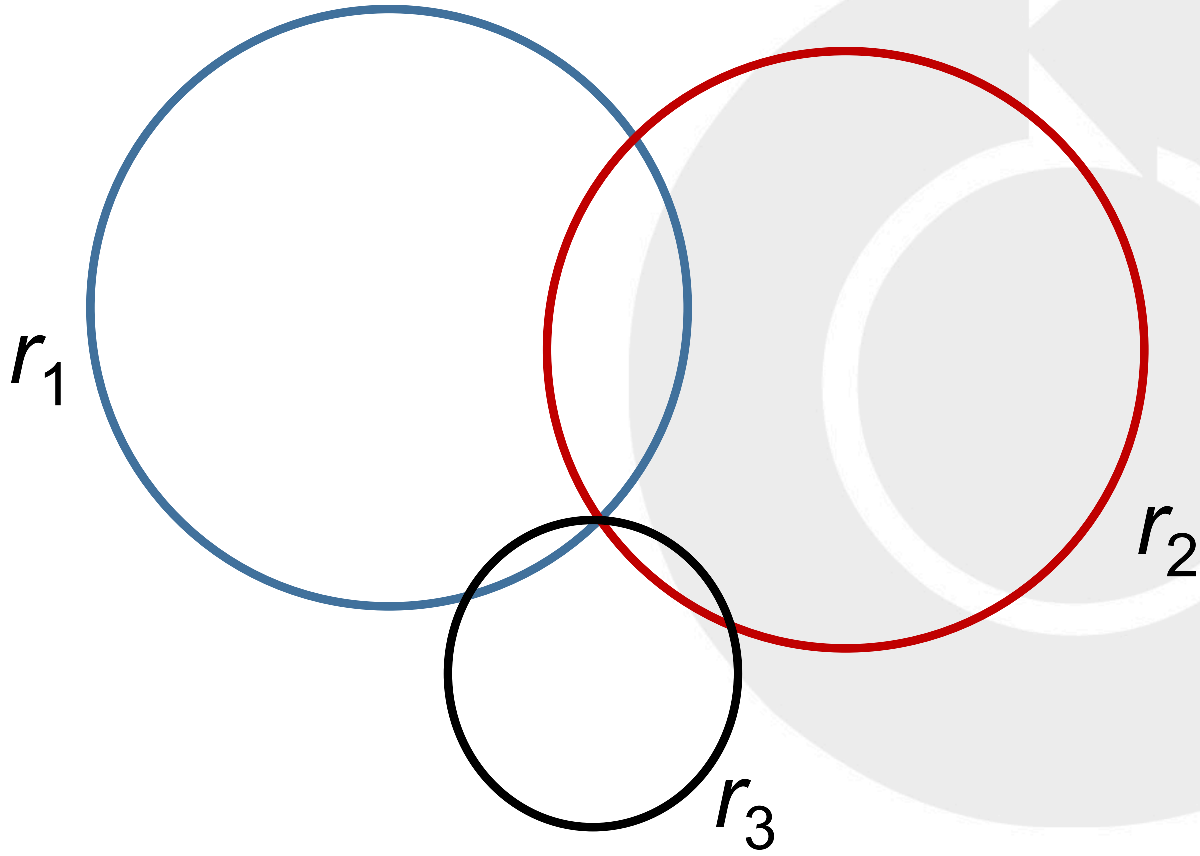
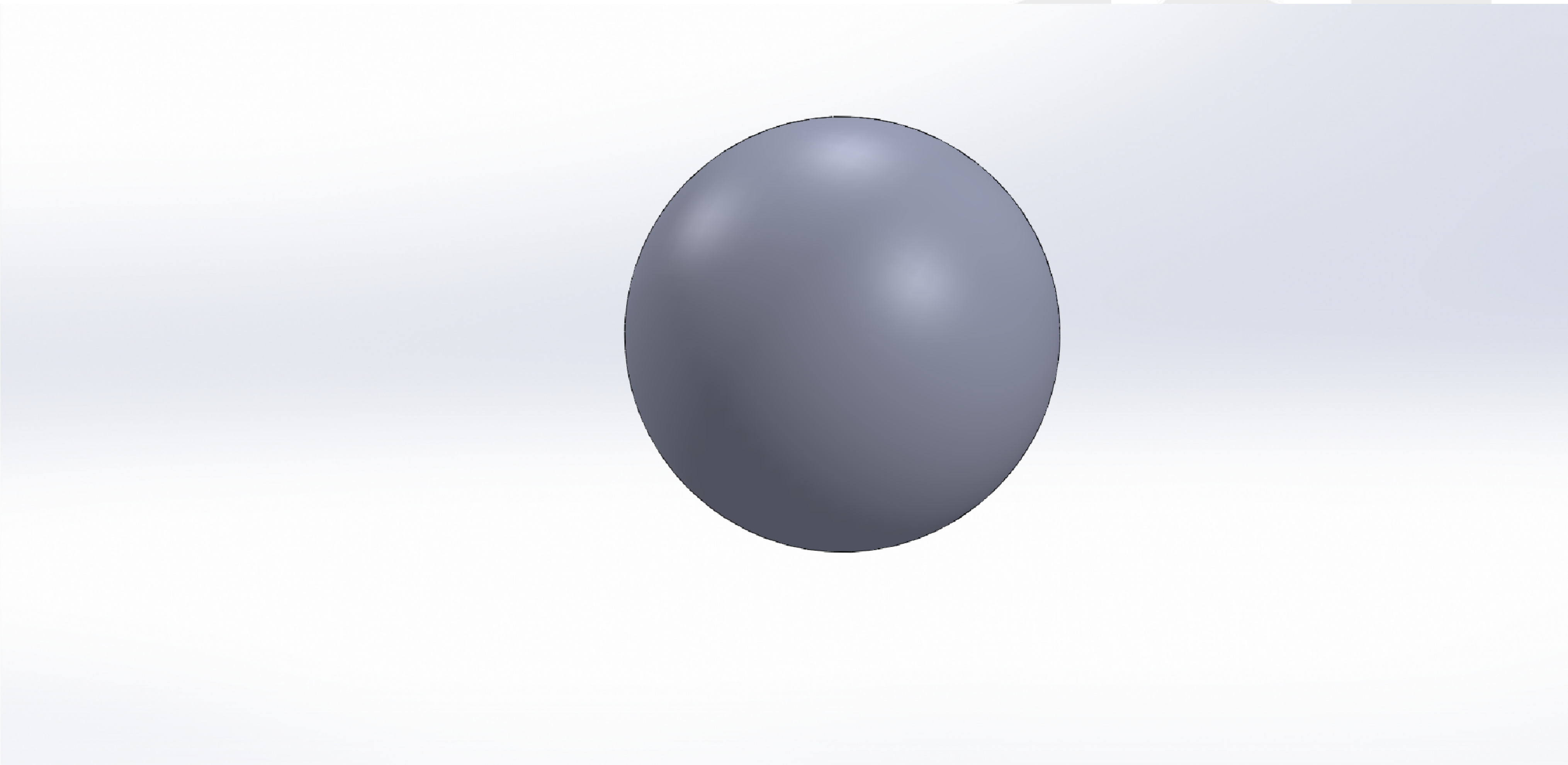
Outline

- Range-only cooperative navigation
- Measurement schemes for formation flight
- MultiBoom High Fidelity Simulation
- Internal Models and Kalman Filters
  - Child Munition Guidance
    - Modified Linear Implementation
  - Centralized Navigation Kalman Filter
- Results
- Conclusions
- Extensions and work in progress

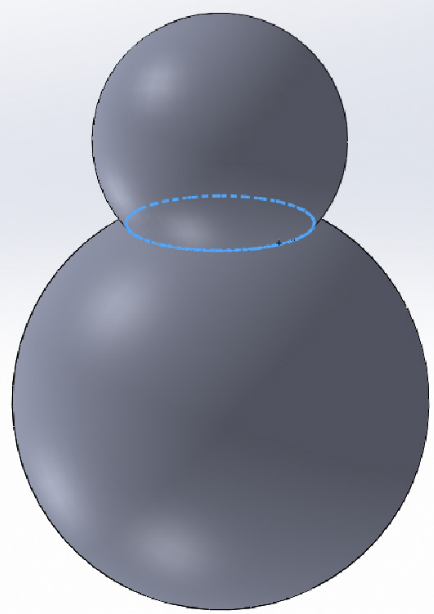
Range-only triangulation, 2D



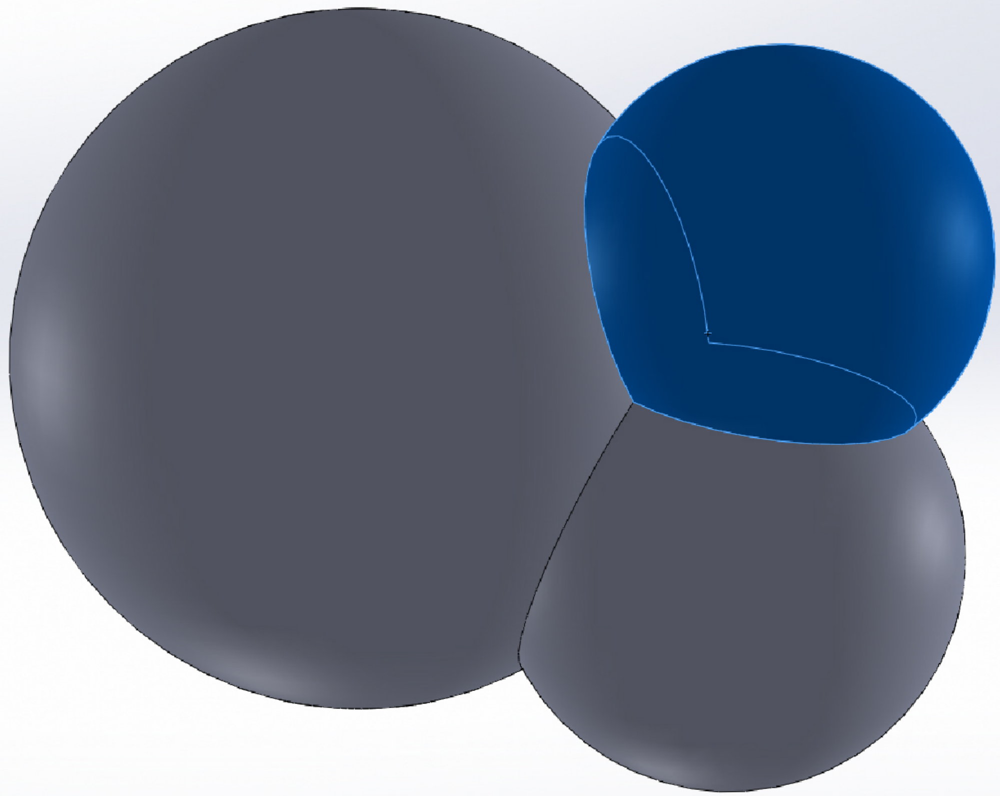
Range-Only triangulation, 3D



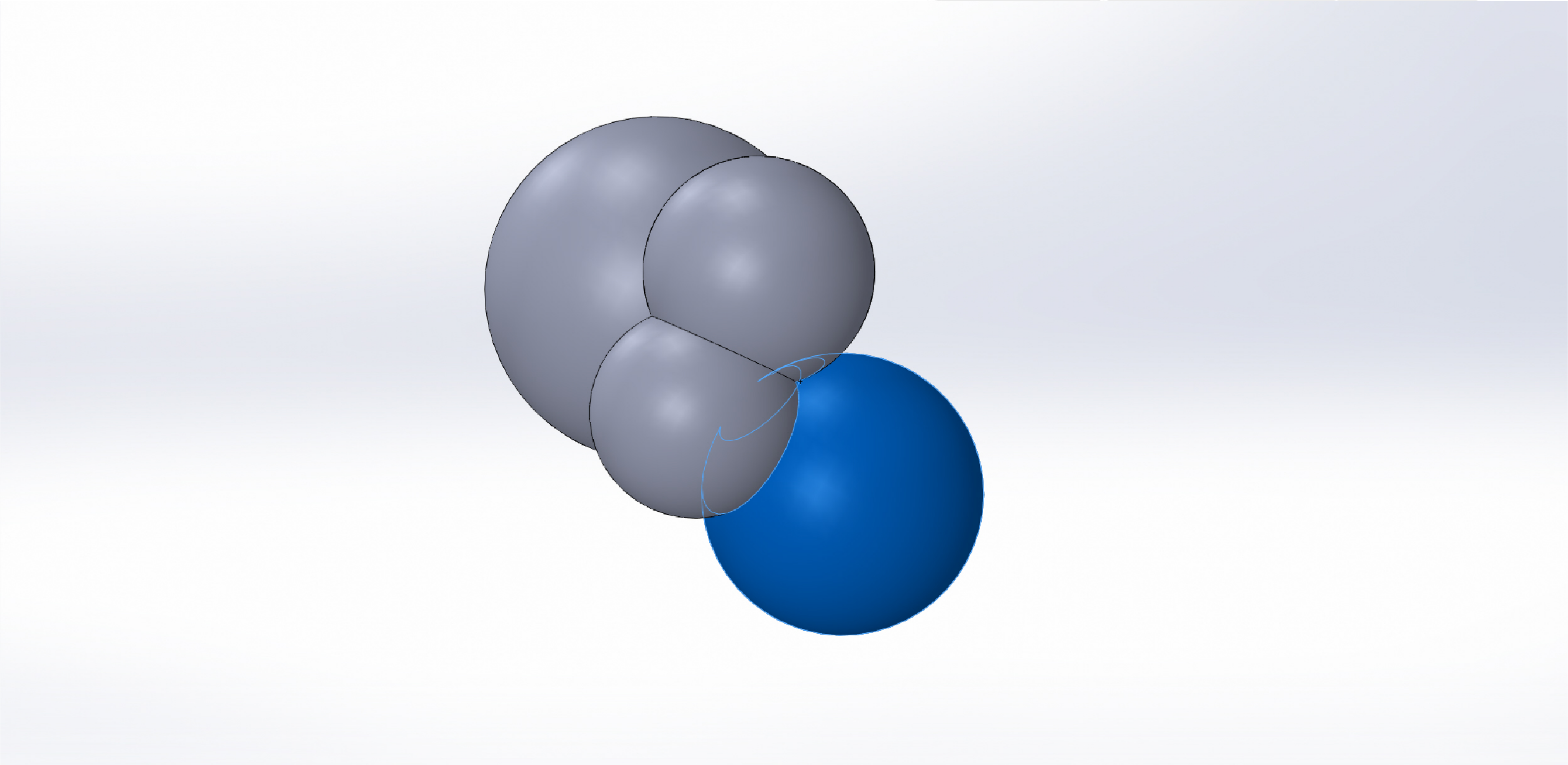
Range-Only triangulation, 3D



Range-Only triangulation, 3D

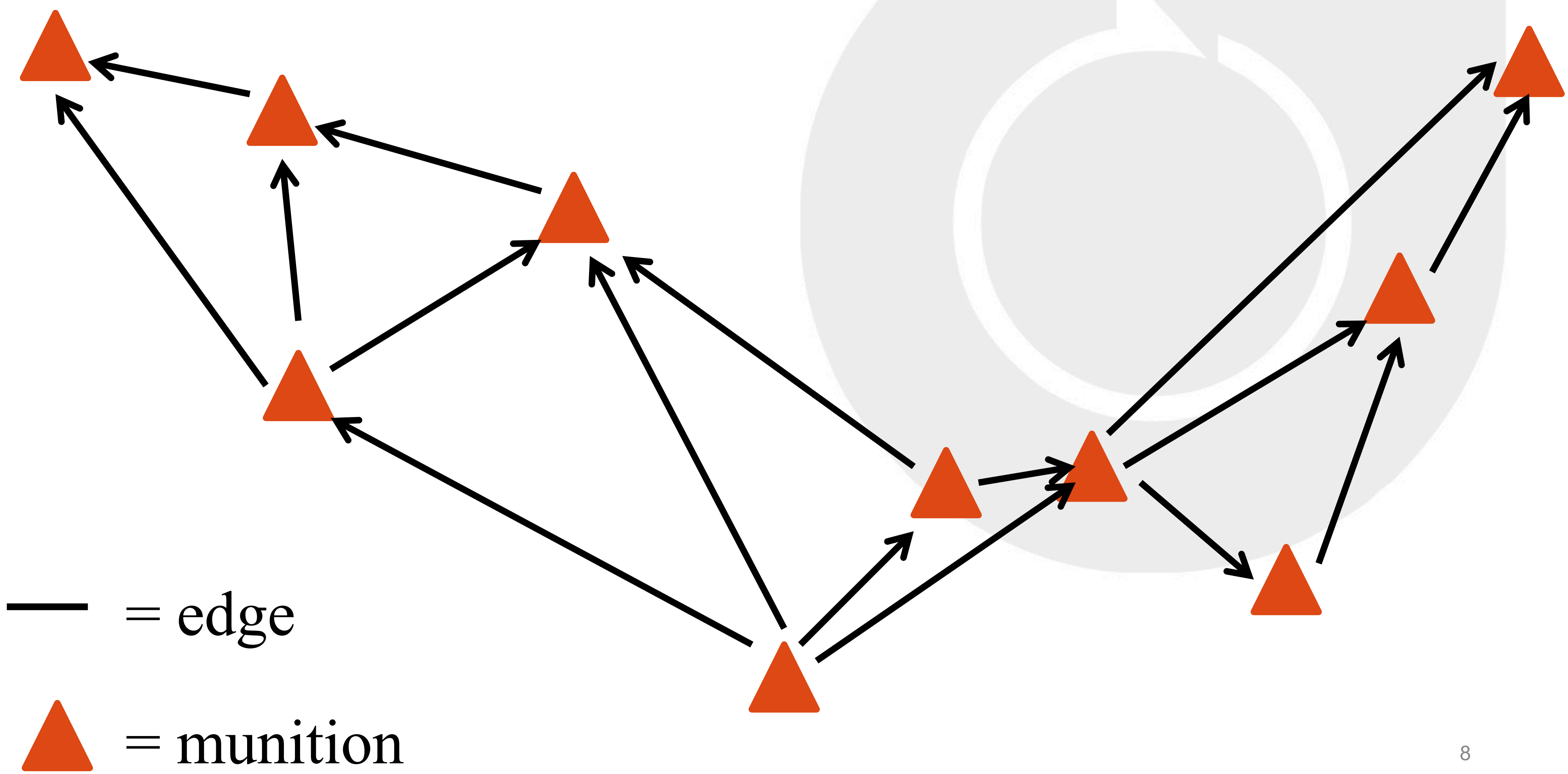


Range-Only triangulation, 3D



Intra-formation Measurement Schemes

- Undirected graph (directed later for sensitivities)



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Measurement Schemes for Formation Flight

- Use a formation of 10 projectiles
- Represent edges of the graph as matrix sparsity pattern

Scheme A

1										
2	X									
3	X	X								
4	X	X	X							
5	X	X	X	X						
6		X	X	X	X					
7			X	X	X	X				
8				X	X	X	X			
9					X	X	X	X		
0						X	X	X	X	
	1	2	3	4	5	6	7	8	9	0

Measurement Schemes for Formation Flight

Scheme B

1										
2	X									
3	X	X								
4	X	X	X							
5		X	X	X						
6			X	X	X					
7				X	X	X				
8	X				X	X	X			
9	X	X				X	X	X		
0	X	X	X				X	X	X	
	1	2	3	4	5	6	7	8	9	0

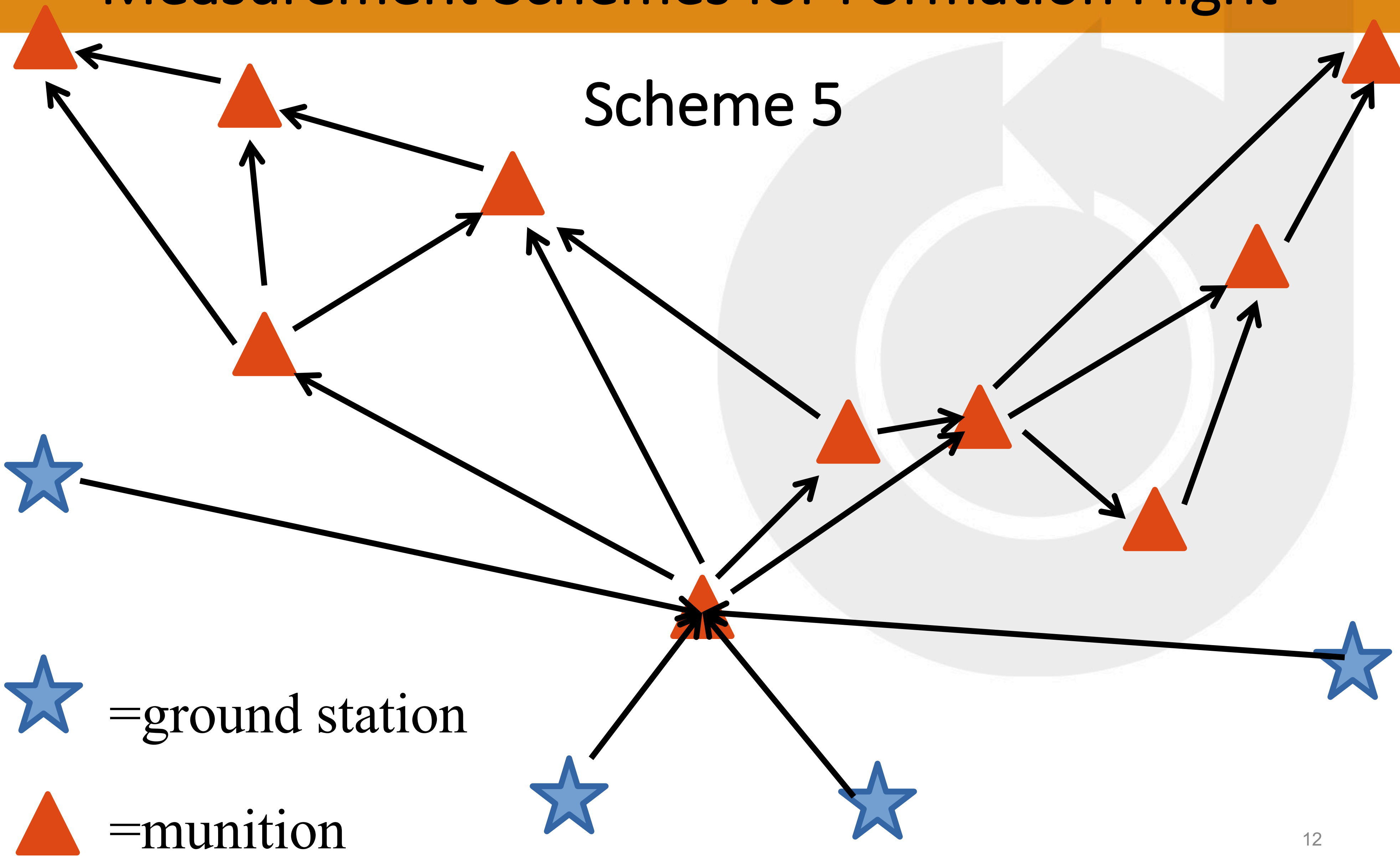
Measurement Schemes for Formation Flight

Scheme C

1										
2										
3	x									
4	x	x								
5		x	x							
6	x		x	x						
7	x	x		x	x					
8	x	x	x		x	x				
9	x	x	x	x		x	x			
0	x	x	x	x	x		x	x		
	1	2	3	4	5	6	7	8	9	0

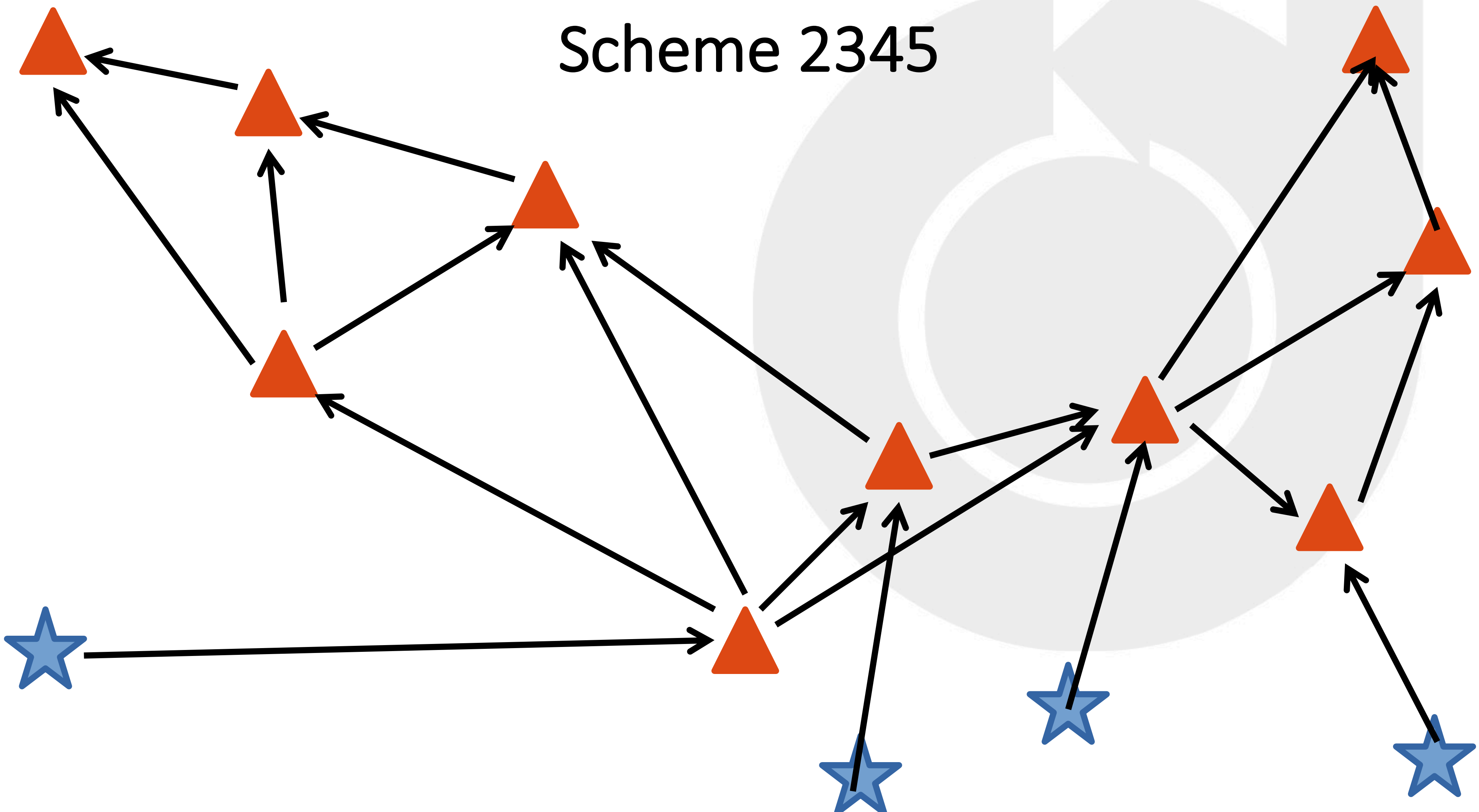
Measurement Schemes for Formation Flight

Scheme 5



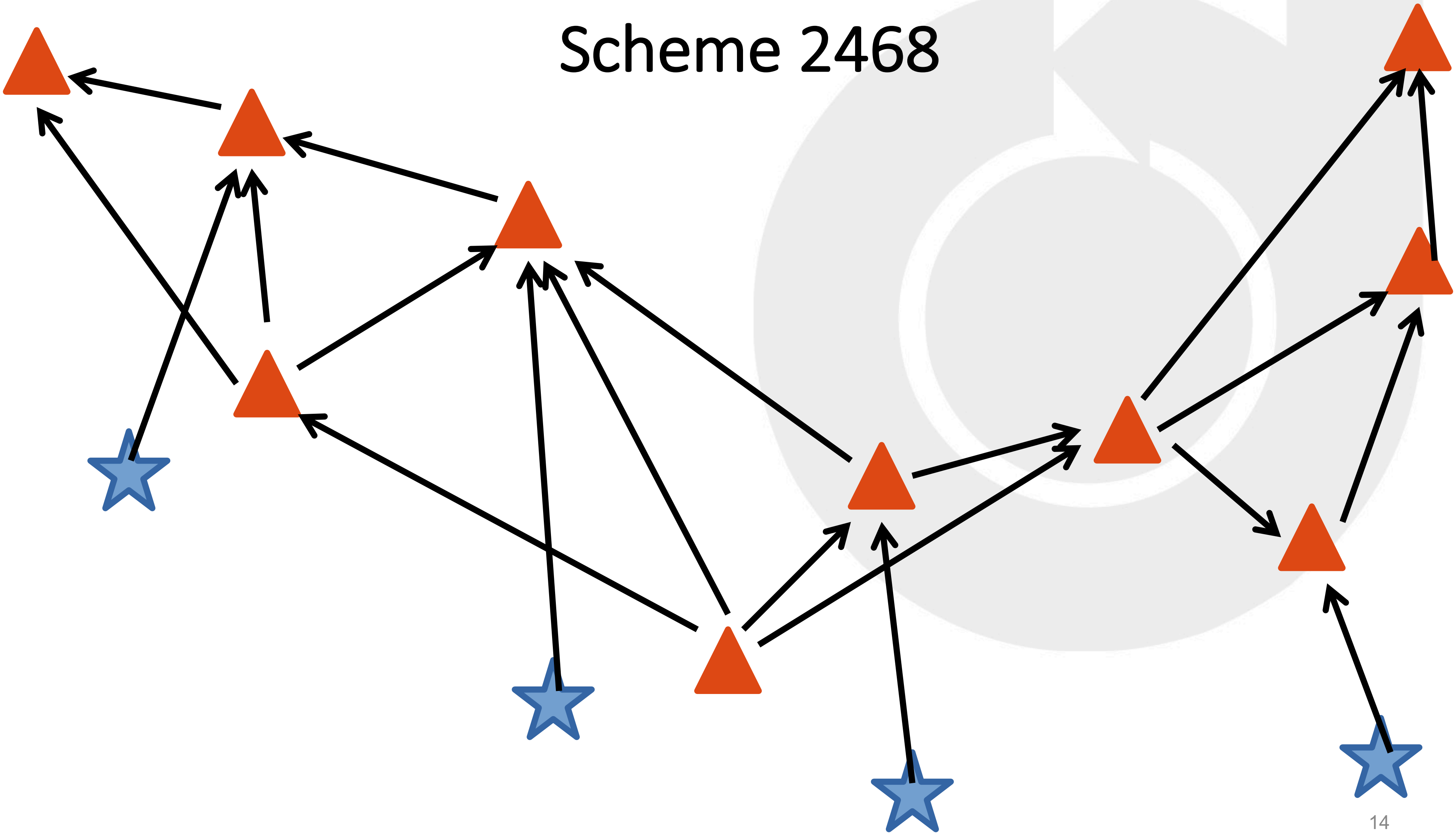
Measurement Schemes for Formation Flight

Scheme 2345



Measurement Schemes for Formation Flight

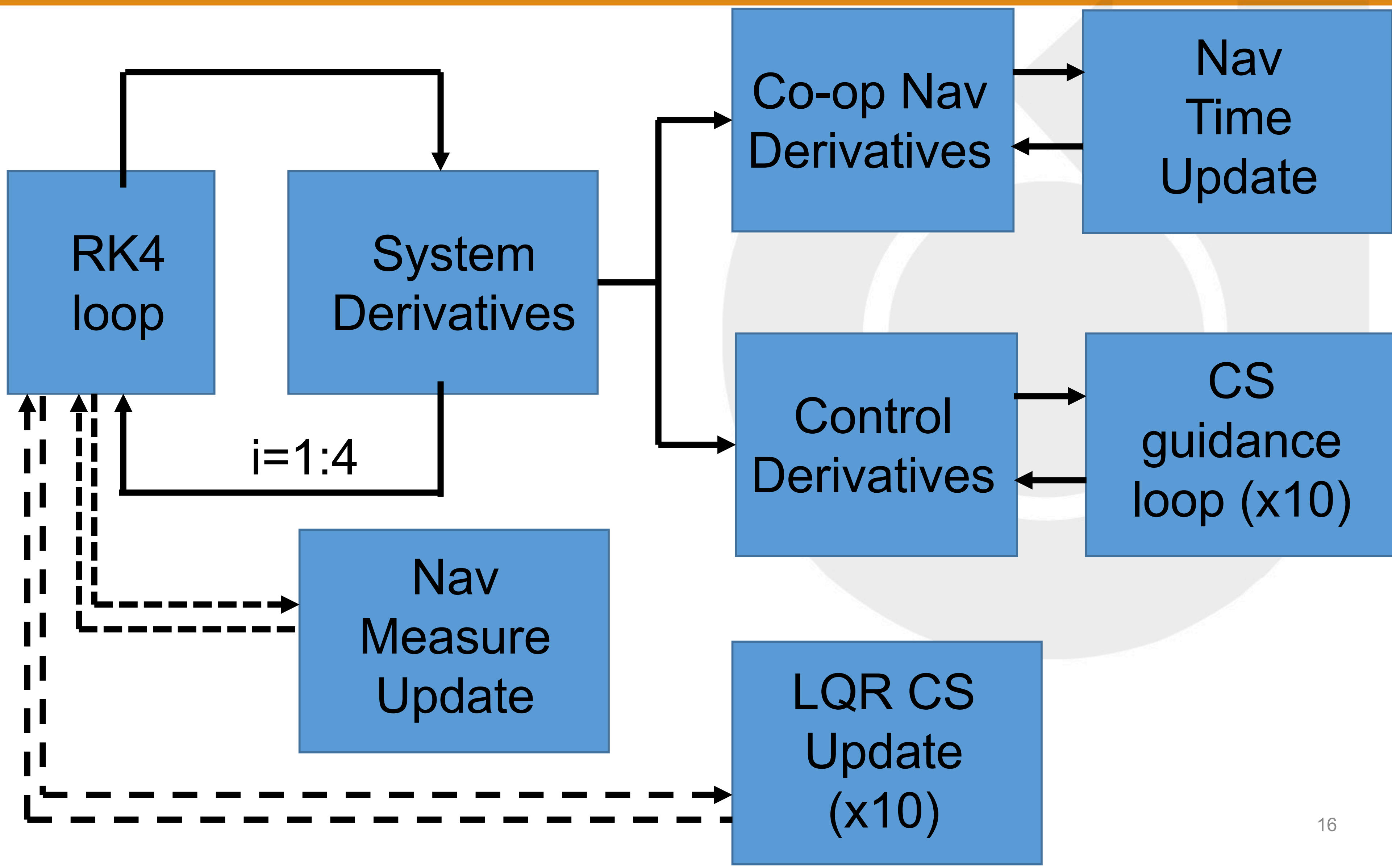
Scheme 2468



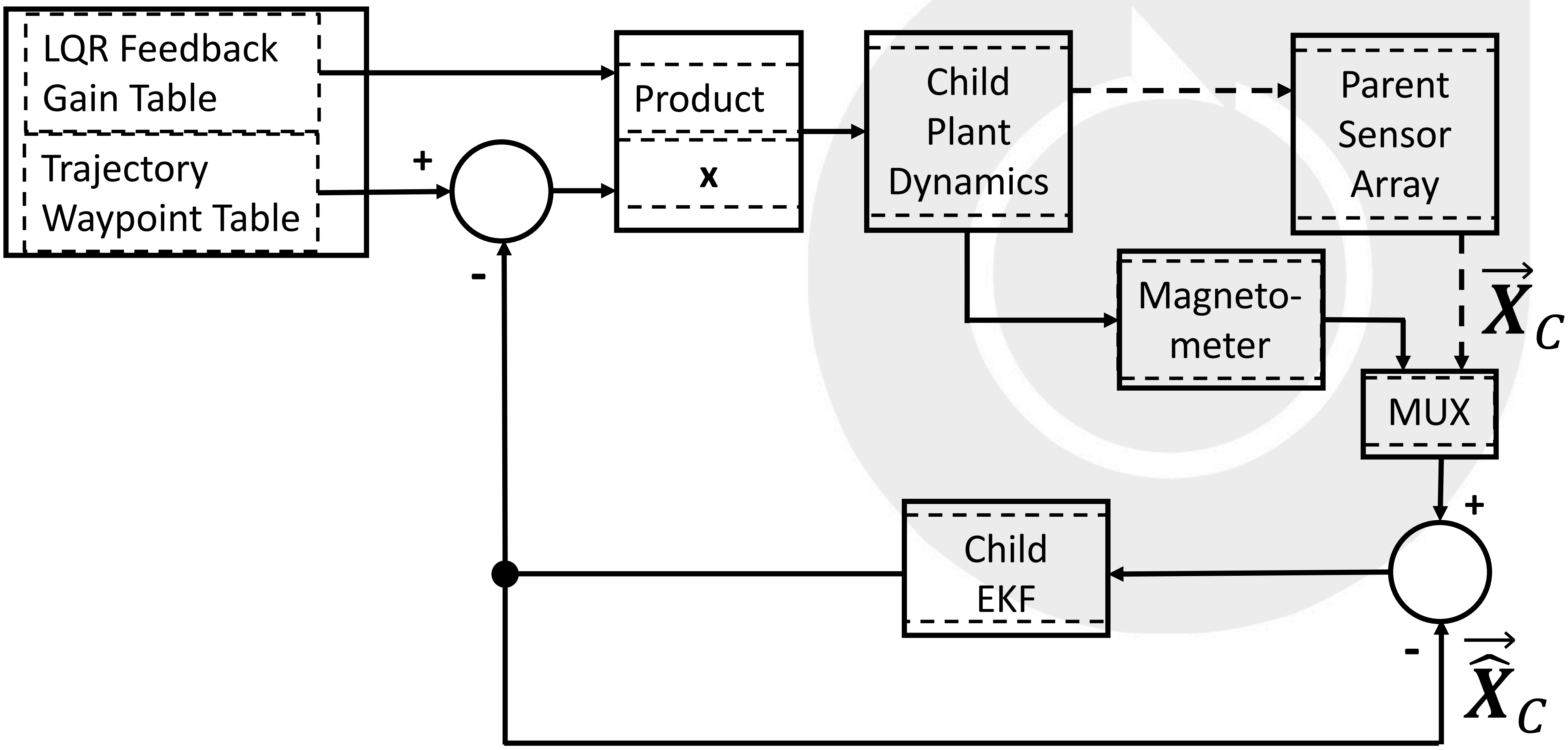
MultiBoom High Fidelity Simulation

- Fortran 95 code using structures
- Bodies modeled in 6DOF with quaternions
- Bodies easily added to the formation
- Can add physical connections and aero surfaces
- Modules added:
  - Cooperative Navigation time update
  - Cooperative Navigation measurement update
  - Child EKF derivatives
  - LQR tracking control

MultiBoom modifications—"daisy chain”



Child munition functional block diagram



Internal Models: Child Guidance system

$$
\begin{aligned}
\dot{x} &= V c_\theta c_\psi \\
\dot{y} &= V \psi c_\theta + v \\
\dot{z} &= -V s_\theta + w c_\theta \\
\dot{\phi} &= p + r t_\theta \\
\dot{\theta} &= q \\
\dot{\psi} &= r \sec(\theta) \\
\dot{V} &= -\Gamma V - g s_\theta \\
\dot{v} &= -A v - V r + b_1 u_2 \\
\dot{w} &= -A w + V q + g c_\theta - b_1 u_1 \\
\dot{p} &= \Xi V + \Omega p \\
\dot{q} &= C w + E q - \frac{I_{xx}}{I_{yy}} p r + b_2 u_1 \\
\dot{r} &= -C v + \frac{I_{xx}}{I_{yy}} p q + E r + b_2 u_2
\end{aligned}
$$

Internal Models: Child EKF

$$\mathbf{K}_{ekf} = \mathbf{P}_{ekf}\mathbf{H}_{ekf}^{T}\mathbf{R}_{ekf}^{-1}$$

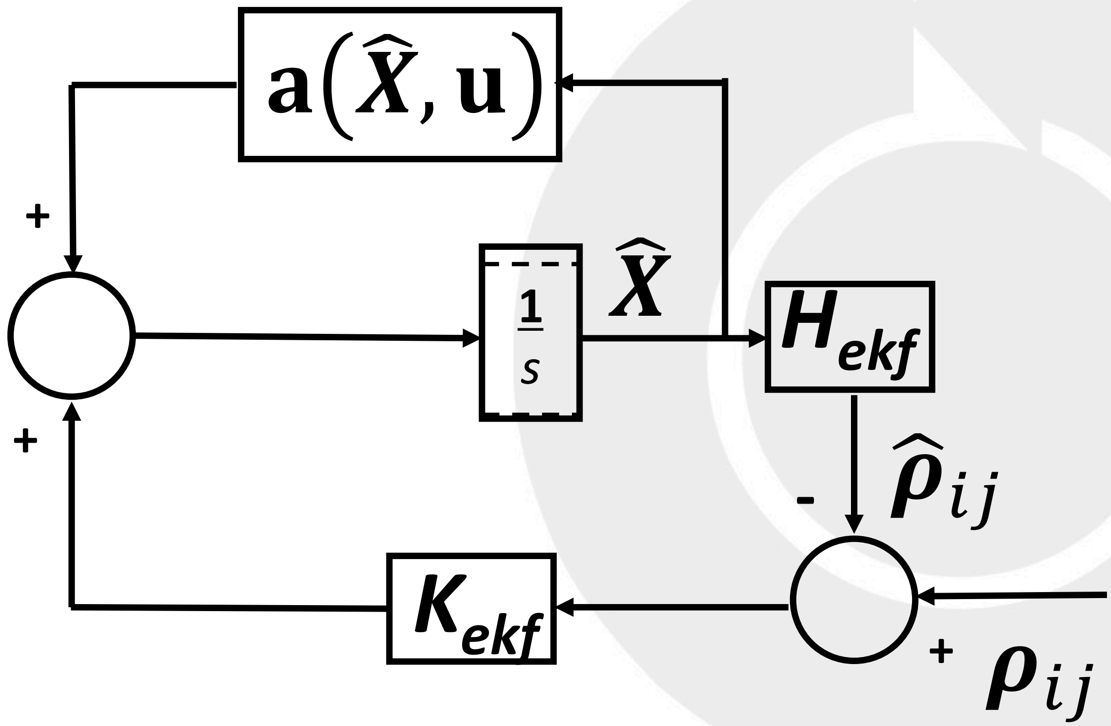
$$\dot{\mathbf{P}}_{ekf} = \mathbf{A}_{ekf}\mathbf{P}_{ekf} + \mathbf{P}_{ekf}\mathbf{A}_{ekf}^{T} + \mathbf{G}_{ekf}\mathbf{Q}_{ekf}\mathbf{G}_{ekf}^{T} - \mathbf{P}_{ekf}\mathbf{H}_{ekf}^{T}\mathbf{R}_{ekf}^{-1}\mathbf{H}_{ekf}\mathbf{P}_{ekf}$$

$$\dot{\hat{\mathbf{x}}}^{+} = \dot{\hat{\mathbf{x}}}^{-} + \mathbf{K}_{ekf}(\mathbf{z} - \hat{\mathbf{z}})$$

$$\mathbf{H}_{ekf} = \begin{bmatrix} \mathbf{I}_{5\times 5} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{A}_{ekf} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$$

Parent Extended Kalman Filter



Internal Models: Parent EKF

$$\dot{\hat{x}}_i^- = V_i c_{\theta i} c_{\psi i}$$

$$\dot{\hat{y}}_i^- = V_i \psi_i c_{\theta i}$$

$$\dot{\hat{z}}_i^- = -V_i s_{\theta i}$$

$$\dot{\mathbf{P}}_{ekf} = \mathbf{G}_{ekf}\mathbf{Q}_{ekf}\mathbf{G}_{ekf}^T$$

$$\begin{aligned}
\mathbf{K}_{ekf} &= \mathbf{P}_{ekf}\mathbf{H}_{ekf}^T\left(\mathbf{H}_{ekf}\mathbf{P}_{ekf}\mathbf{H}_{ekf}^T + \mathbf{R}_{ekf}\right)^{-1} \\
\mathbf{P}_{ekf}^+ &= \left(\mathbf{G}_{ekf} - \mathbf{K}_{ekf}\mathbf{H}_{ekf}\right)\mathbf{P}_{ekf} \\
\hat{\mathbf{x}}^+ &= \hat{\mathbf{x}}^- + \mathbf{K}_{ekf}(\mathbf{z} - \hat{\mathbf{z}})
\end{aligned}$$

Where

$$\hat{\mathbf{z}}_k = h_k = \hat{\rho}_{ij}$$

$$\rho_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}.$$

$$\mathbf{H}_{ekf}(k, 1 + 3(i-1)) = \frac{\partial h_k}{\partial \hat{x}_i} = -(\hat{x}_j - \hat{x}_i)/\hat{\rho}_{ij}$$

$$\mathbf{H}_{ekf}(k, 2 + 3(i-1)) = \frac{\partial h_k}{\partial \hat{y}_i} = -(\hat{y}_j - \hat{y}_i)/\hat{\rho}_{ij}$$

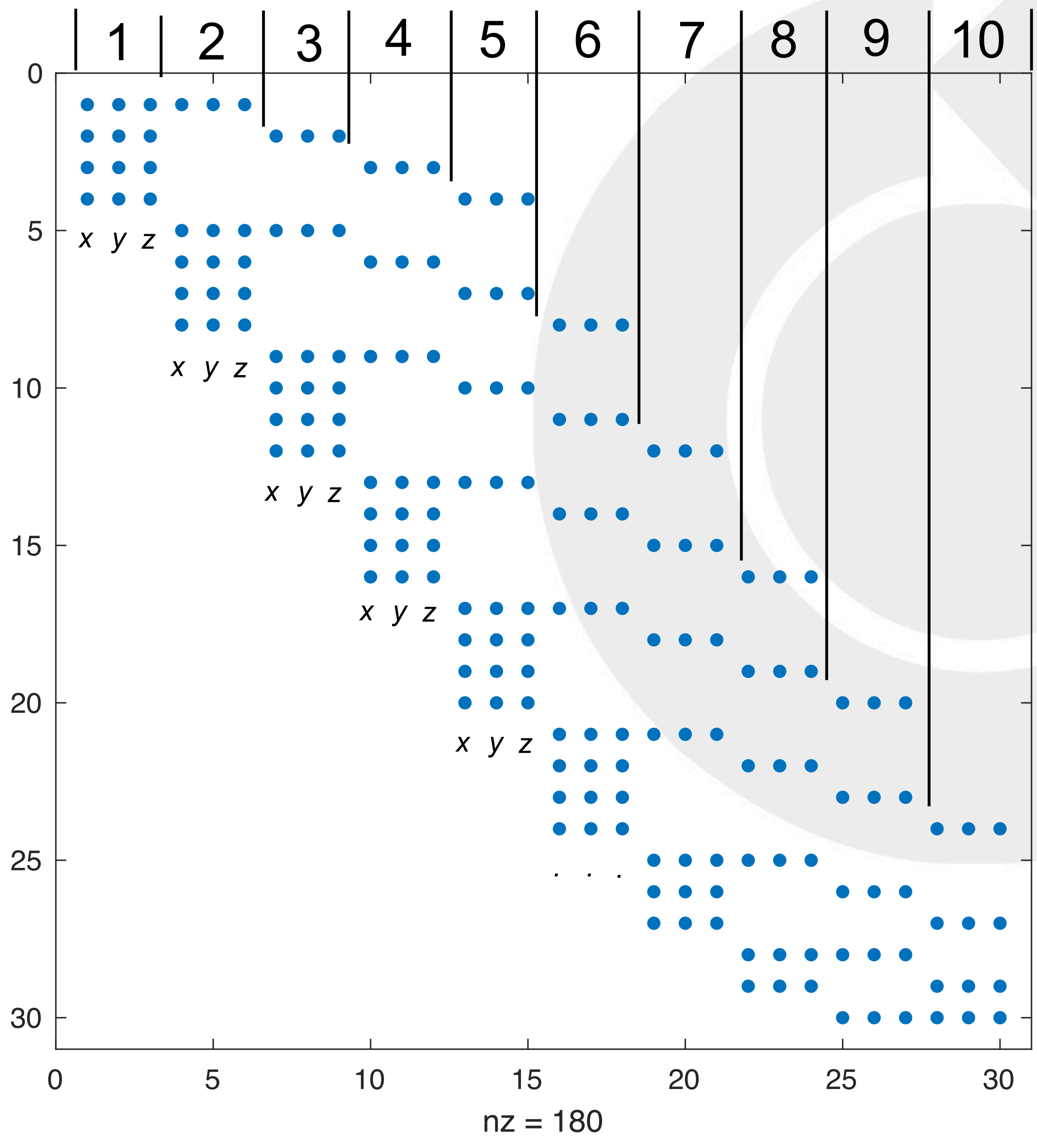
$$\mathbf{H}_{ekf}(k, 3 + 3(i-1)) = \frac{\partial h_k}{\partial \hat{z}_i} = -(\hat{z}_j - \hat{z}_i)/\hat{\rho}_{ij}$$

$$\mathbf{H}_{ekf}(k, 1 + 3(j-1)) = -\mathbf{H}_{ekf}(k, 1 + 3(i-1))$$

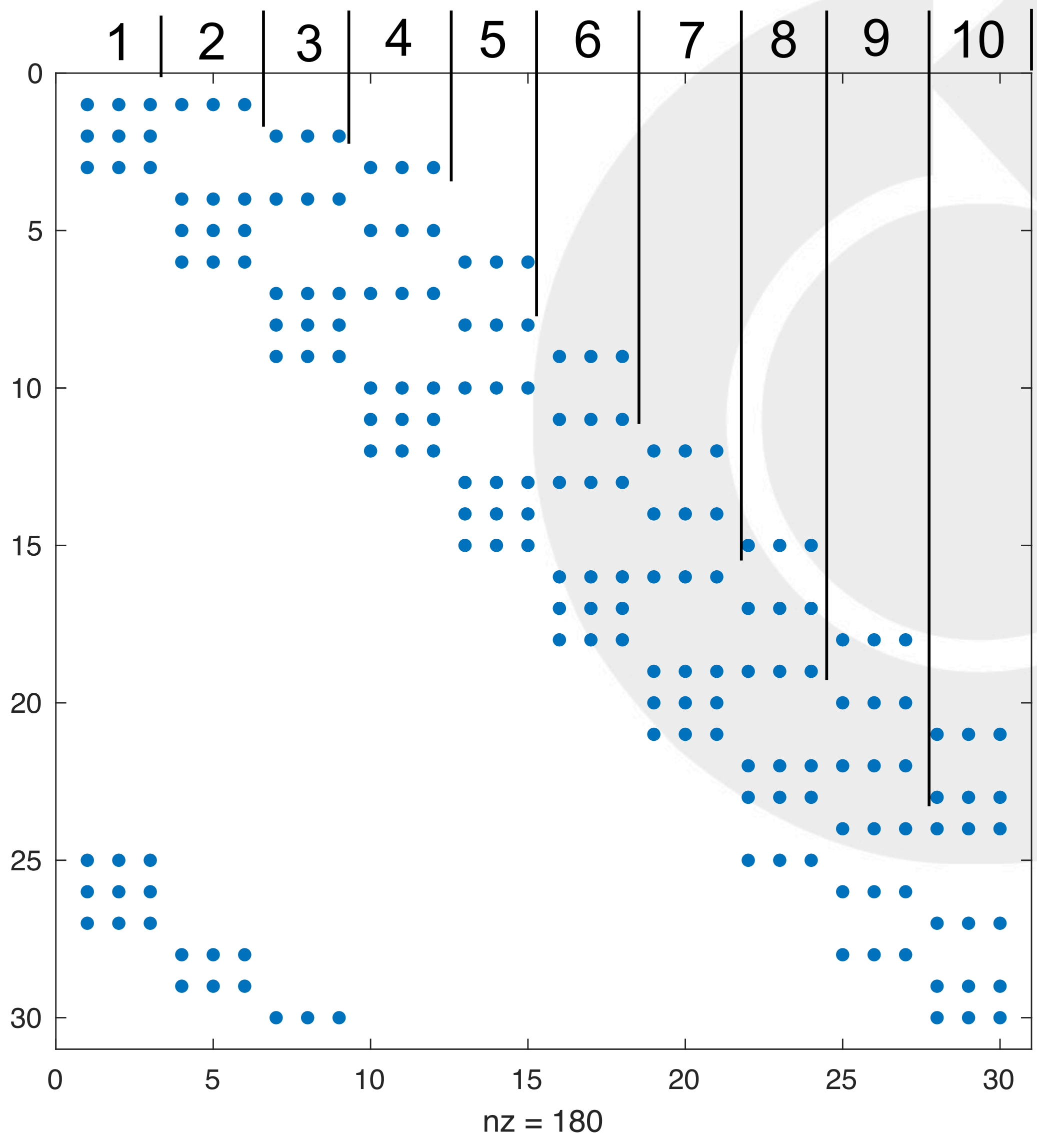
$$\mathbf{H}_{ekf}(k, 2 + 3(j-1)) = -\mathbf{H}_{ekf}(k, 2 + 3(i-1))$$

$$\mathbf{H}_{ekf}(k, 3 + 3(j-1)) = -\mathbf{H}_{ekf}(k, 3 + 3(i-1))$$

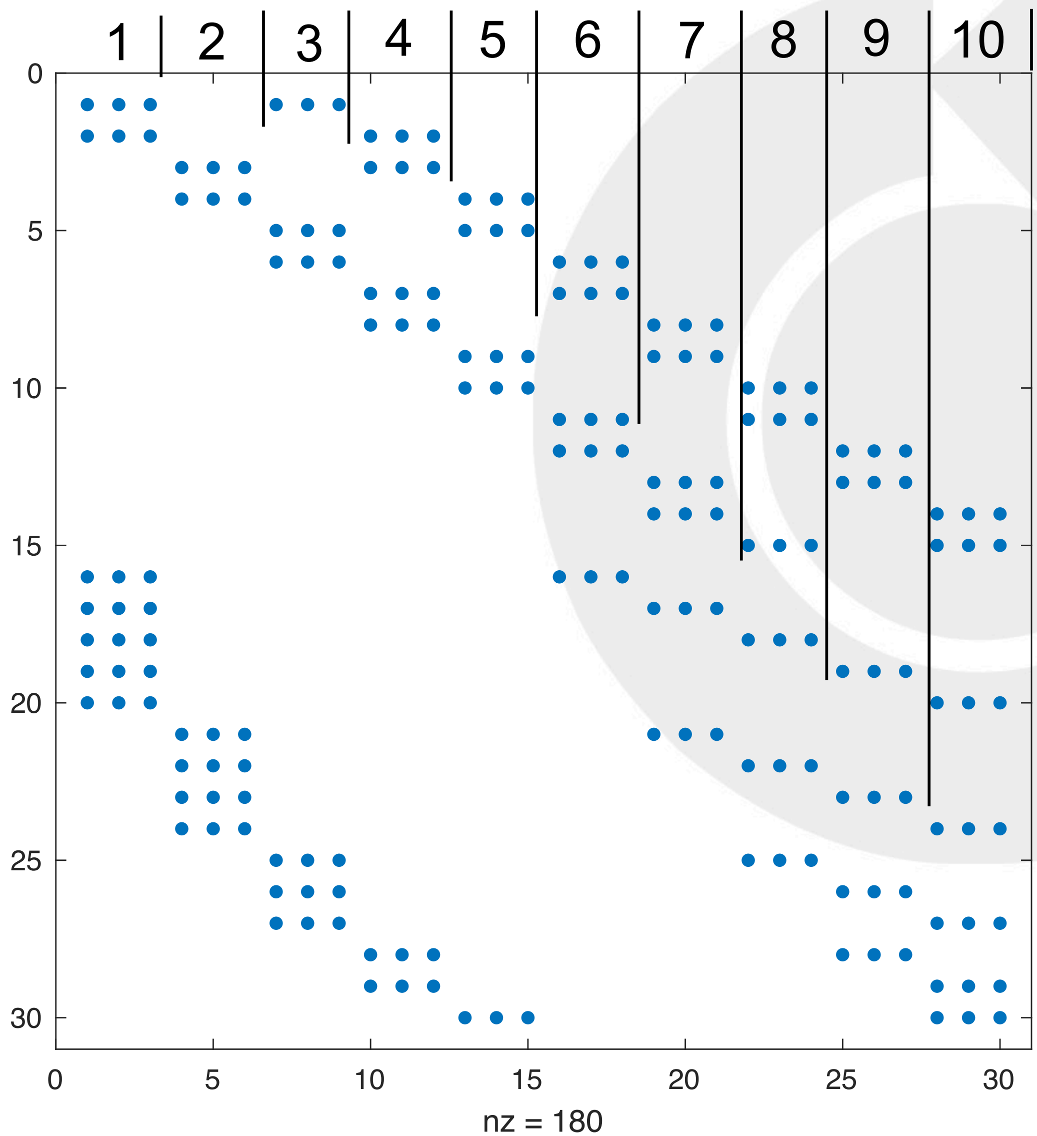
H_{ekf} sparsity, scheme A



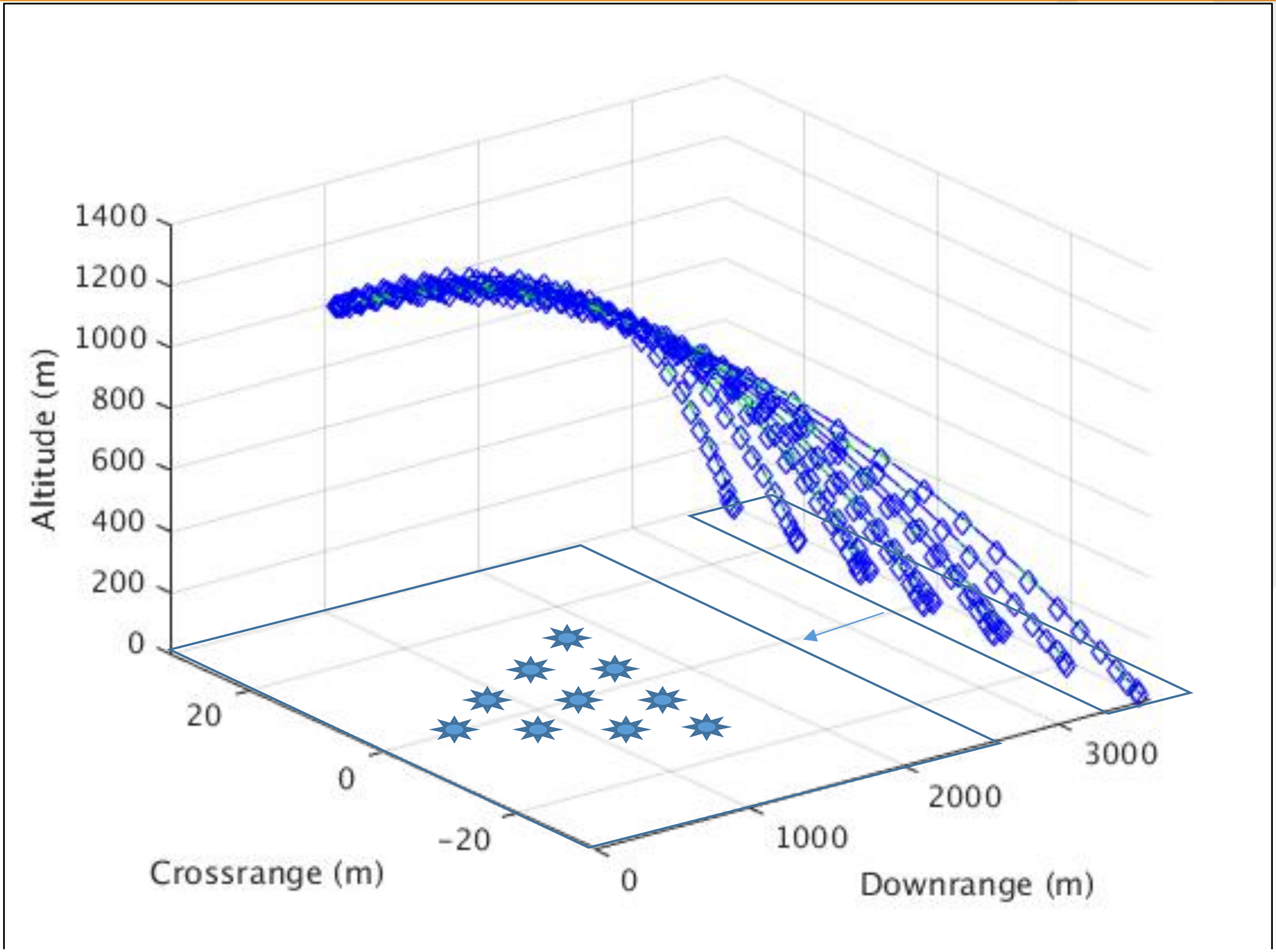
H_{ekf} sparsity, scheme B



H_{ekf} sparsity, scheme C

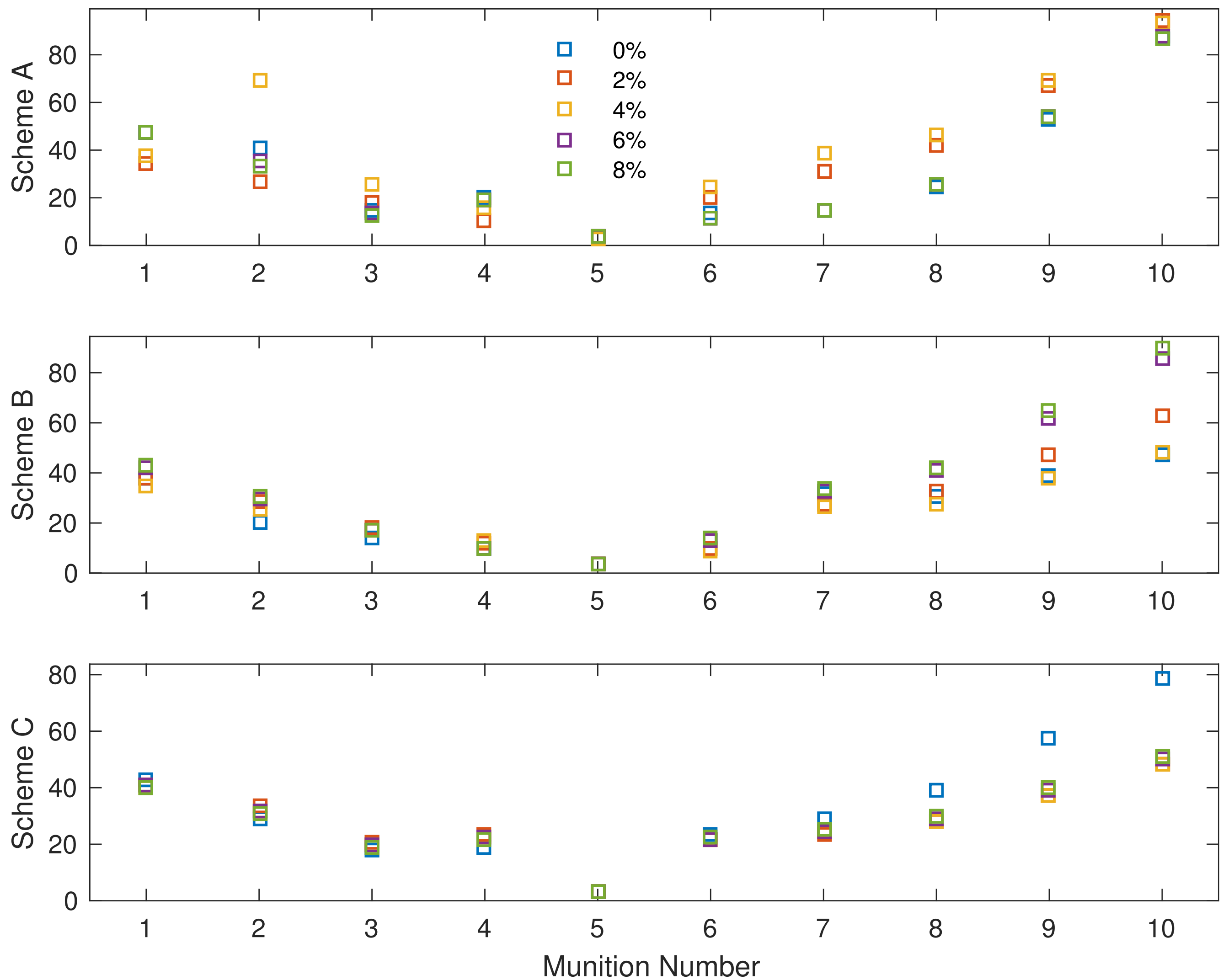


Scenario

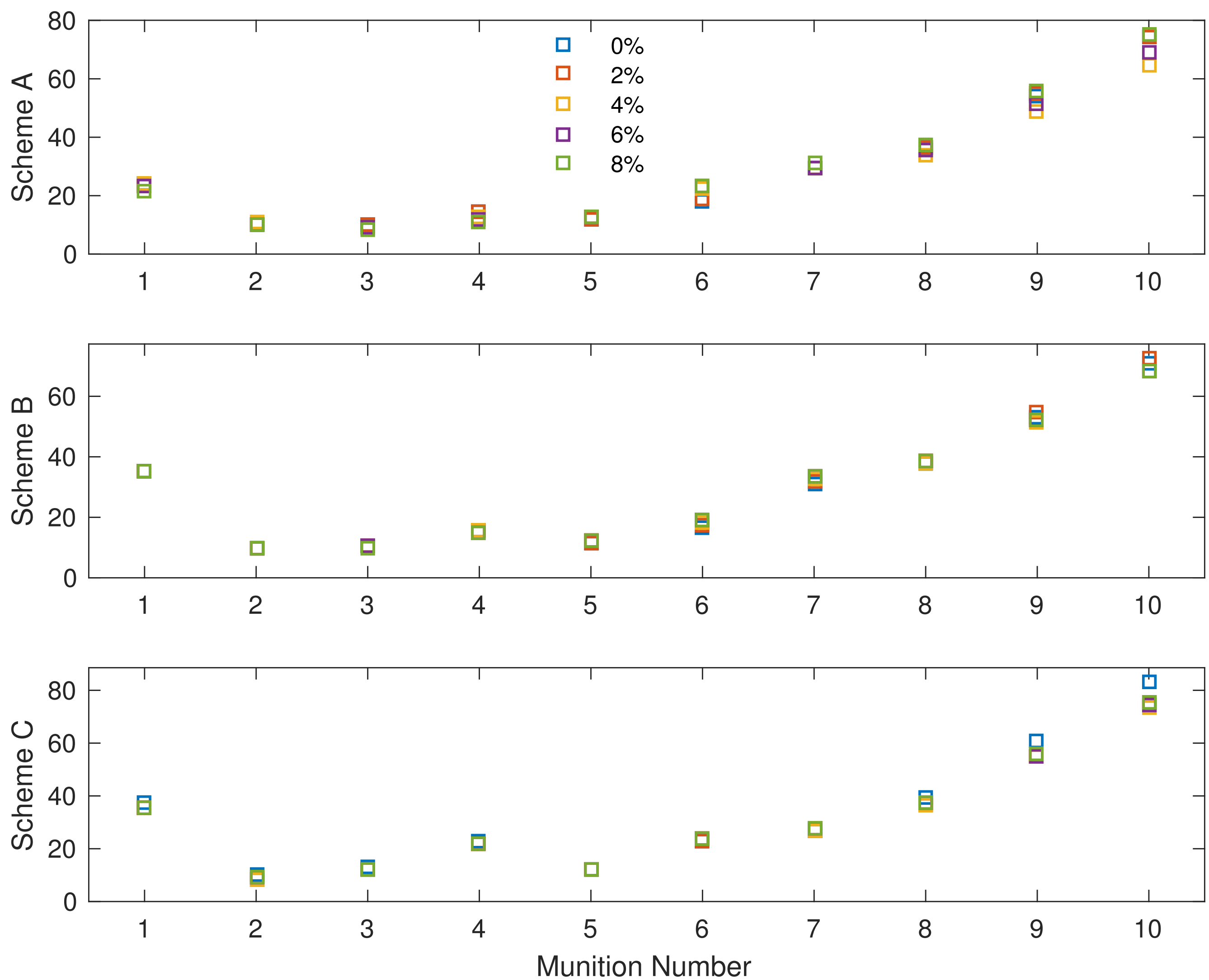


1400
1200
1000
800
600
400
200
0
Altitude (m)
20
0
-20
Crossrange (m)
0
1000
2000
3000
Downrange (m)

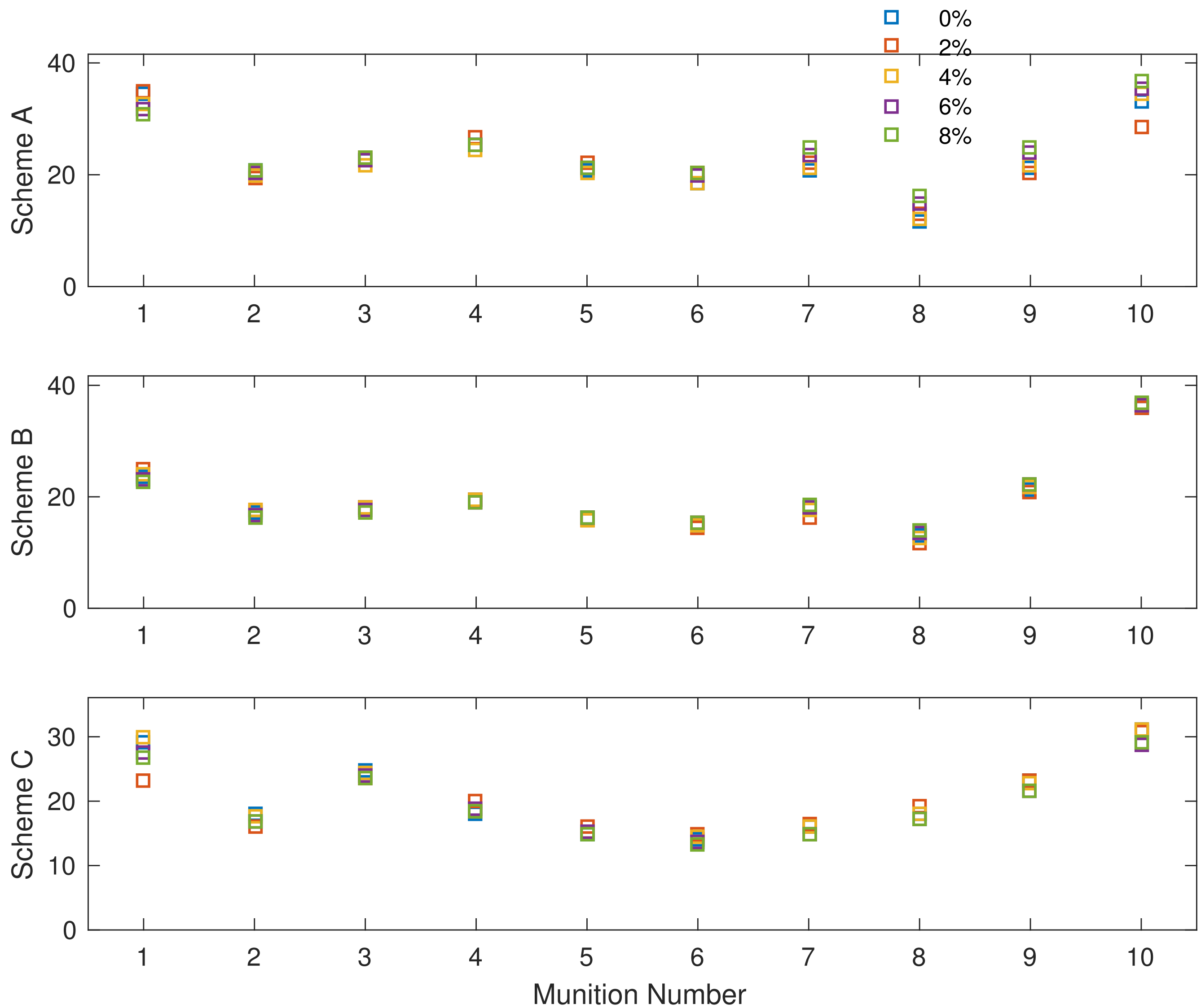
Results RMS estimation error SL – Scheme 5



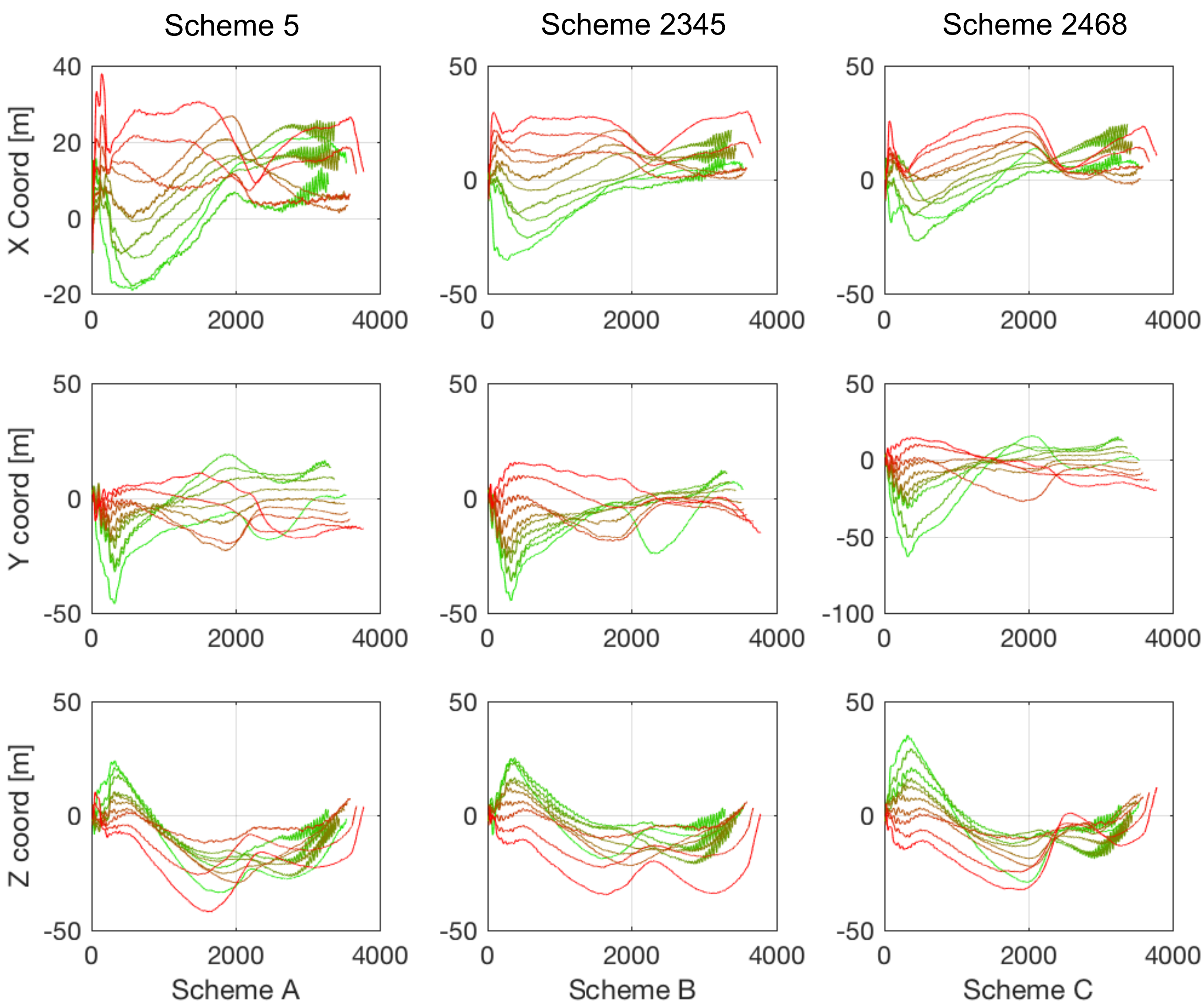
Results – Scheme 2345



Results – Scheme 2468



Results—String Stability



Terminal Estimation Error Scheme 2468C

Noise Intensity	e_x [m]		e_y [m]		e_z [m]	
	min	max	min	max	min	max
0%	0.3	23.3	0.02	18.34	0.13	12.1
2%	0.6	26.1	0.86	21.9	0.47	14.8
4%	0.4	23.6	0.11	18.57	0.08	12.0
6%	0.5	24.4	0.21	19.4	0.06	12.3
8%	0.6	24.3	0.26	19.3	0.03	12.2

Conclusions

- Nine measurement topologies tested in a simulation
- Centralized EKF used with range-only measurements
- Wide distribution of measurements advantageous
  - Global vs. Local information
  - Similar to ‘accordion’ effect in long vehicle convoys

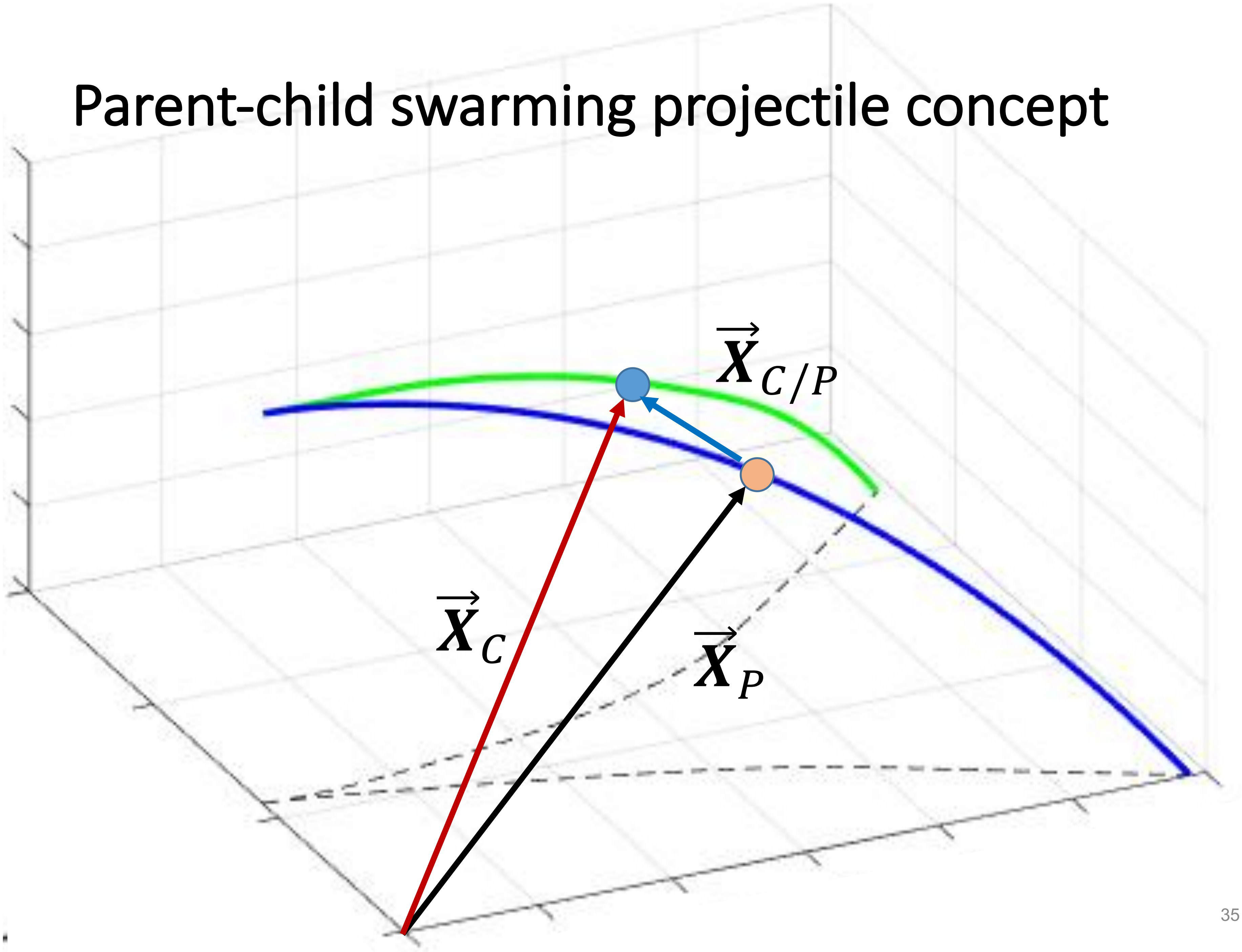
Future Work

- Improved sensor covariance model
- Investigate distributed filtering strategy
- Use slant range measurements directly in Child EKF
  - No intermediate estimation of 3D position coordinates
- A priori design of the measurement topology
  - Use Gauss PS guidance waypoints to choose
  - Find a metric to optimize topology
  - Optimize using GA
  - Using 30 of 45 possible intra-formation measurements

$$\binom{45}{30} = 344{,}867{,}425{,}584$$

Work in Progress

Parent-child swarming projectile concept

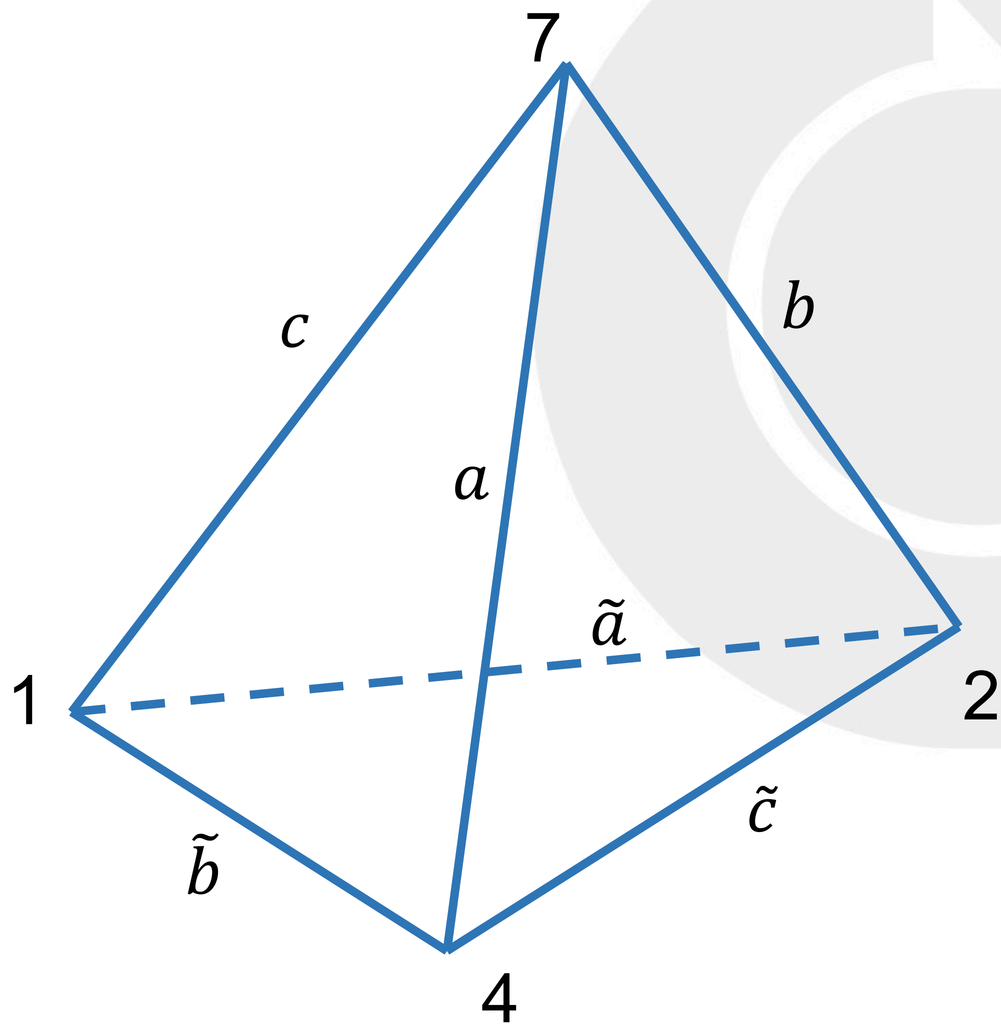


Trajectory Design for Parent / Child Munitions

- Solve a ballistic trajectory using a linearized subset of the MPLT eqns. where V and p are held constant, and c_θ=1, $s_\theta=\theta$.
- Using the linear solution as an initial guess, solve a ballistic trajectory using the MPLT model with t_f held constant and $u_1=u_2=0$.
- Using the non-linear ballistic solution and λ = rand, u_1=rand, u_2= rand, solve for the optimal controlled trajectory using all MPLT eqns.
- Repeat the last step for each child munition, using the parent munition solution as the initial guess. Note that only the target range and target crossrange need to be modified from previous

Toward a Measurement Topology Metric

- Test for non-defective Tetrahedron basis for each projectile



Test for non-defective tetrahedron

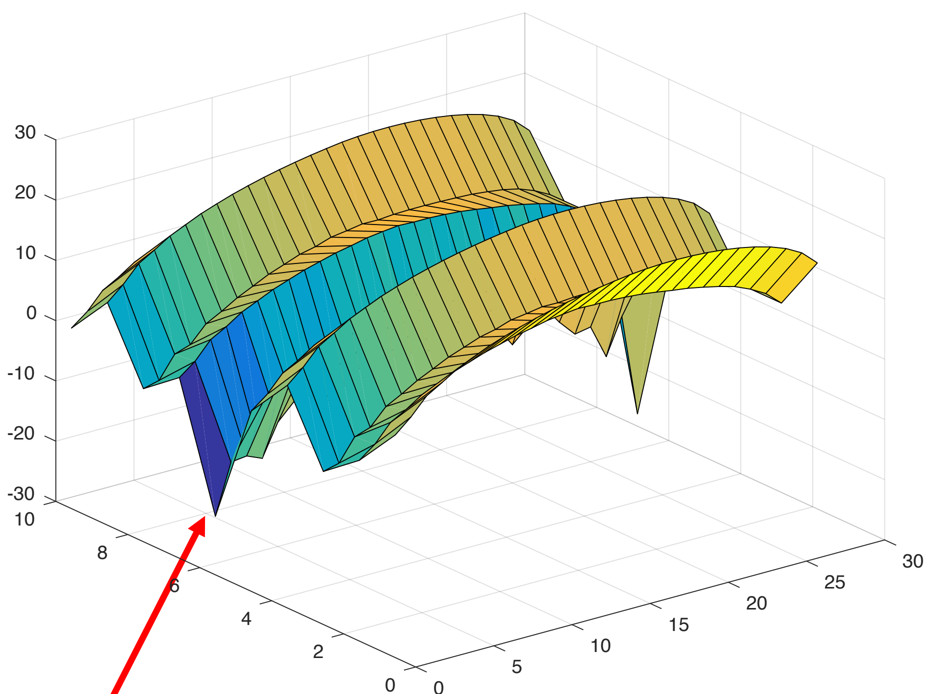
- The sextuple $(a, b, c, \tilde{a}, \tilde{b}, \tilde{c})$ is *Facial*

$$\min(a + b + c, a + \tilde{b} + \tilde{c}, \tilde{a} + b + \tilde{c}, \tilde{a} + \tilde{b} + c) > max(a + \tilde{a}, , \tilde{b} + b, \tilde{c} + c)$$

- The Calyer-Menger determinant is positive

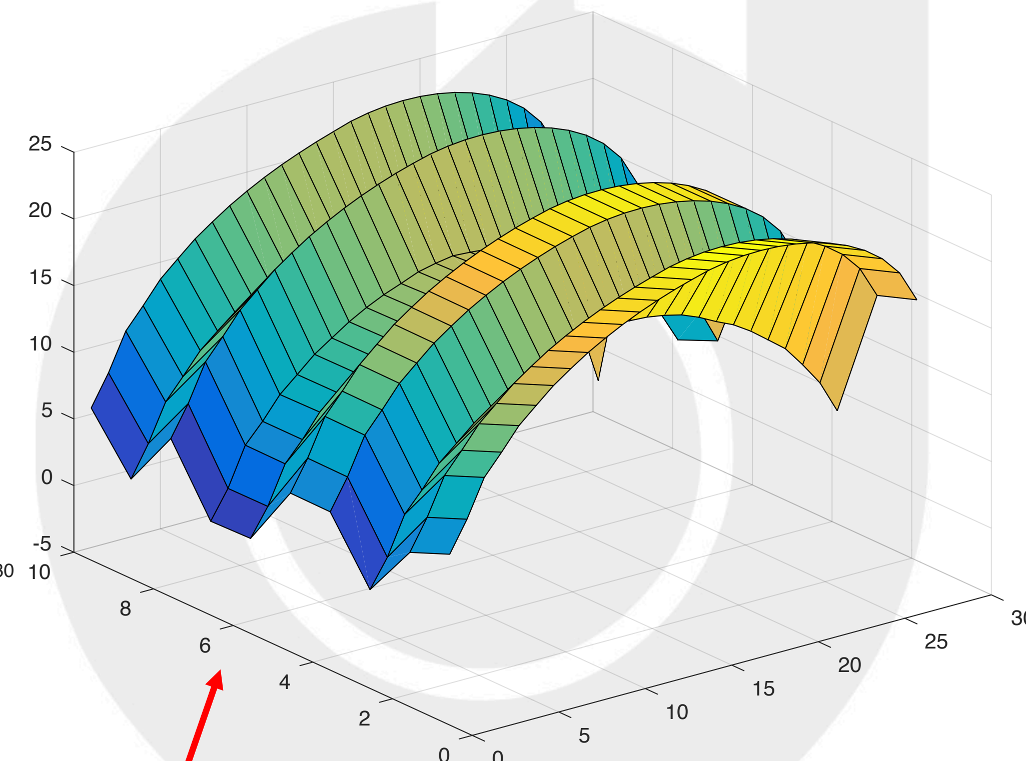
$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & a^2 & b^2 & c^2 \\ 1 & a^2 & 0 & \tilde{c}^2 & \tilde{b}^2 \\ 1 & b^2 & \tilde{c}^2 & 0 & \tilde{a}^2 \\ 1 & c^2 & \tilde{b}^2 & \tilde{a}^2 & 0 \end{vmatrix} > 0$$

log(.) Cayley-Menger determinant



Scheme A

Body 6 tetrahedron approaches defectiveness

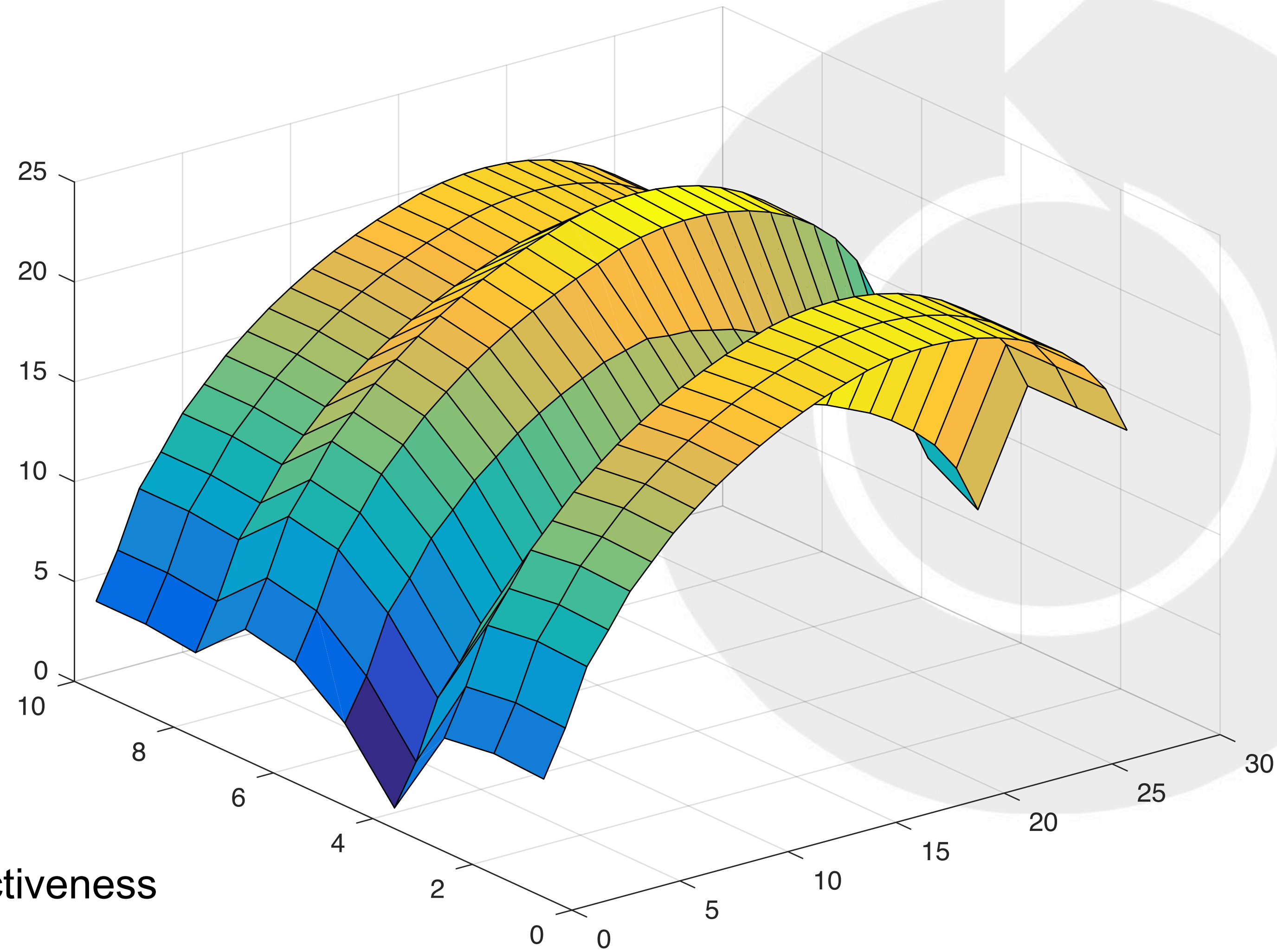


Scheme B

No defectiveness

$$\min_{i,j}(\log(\cdot)) = -0.5873$$

log(.) Cayley-Menger determinant



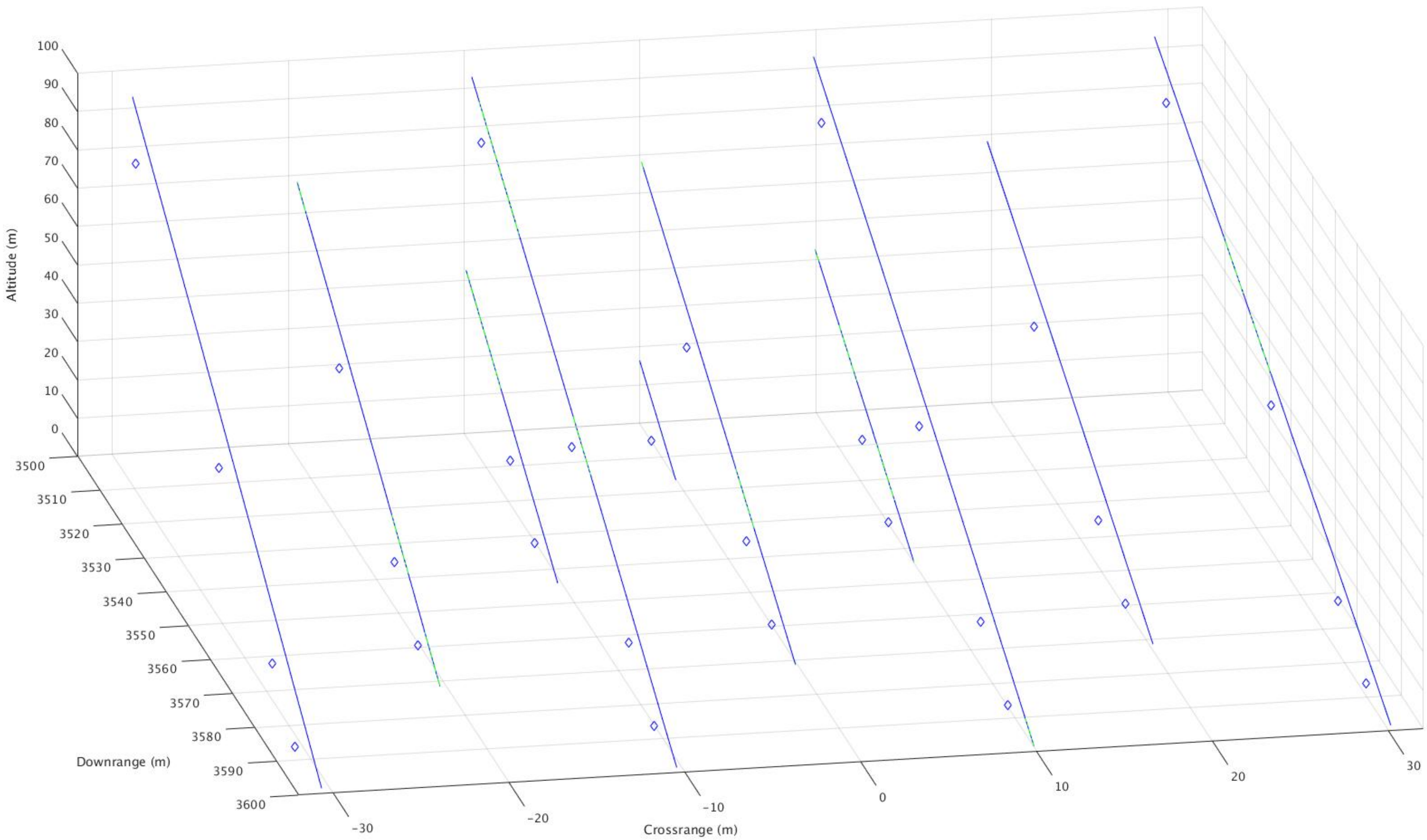
No such defectiveness

$\log(\cdot) > 0 \; \forall \; i, j$

Scheme C

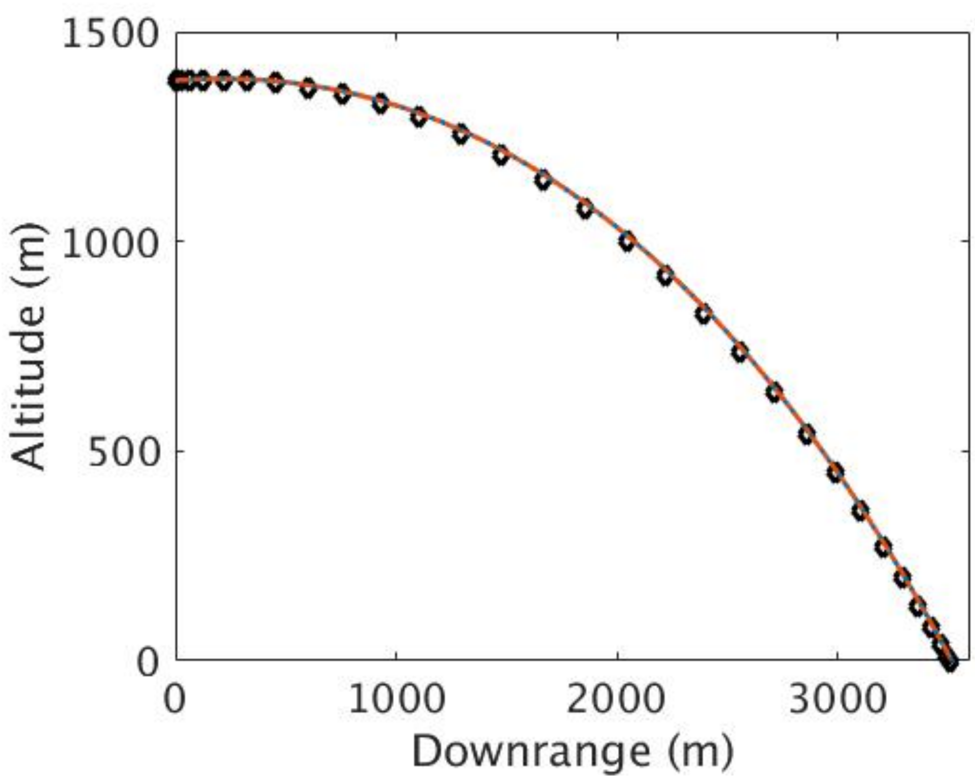
Back up Slides

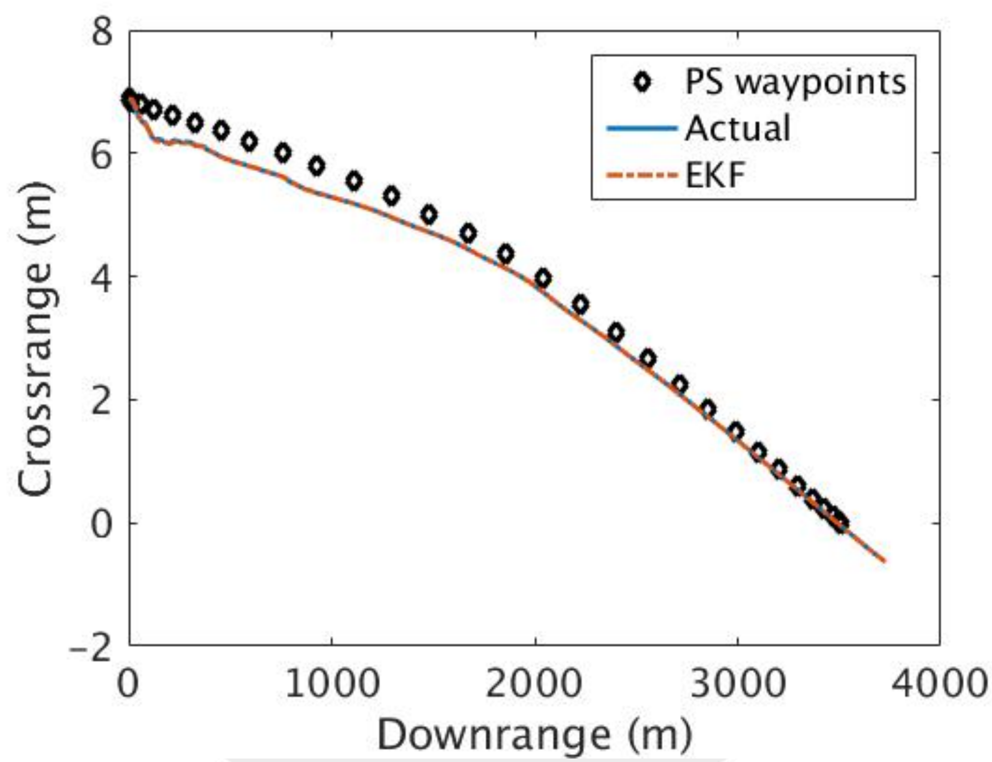
Results



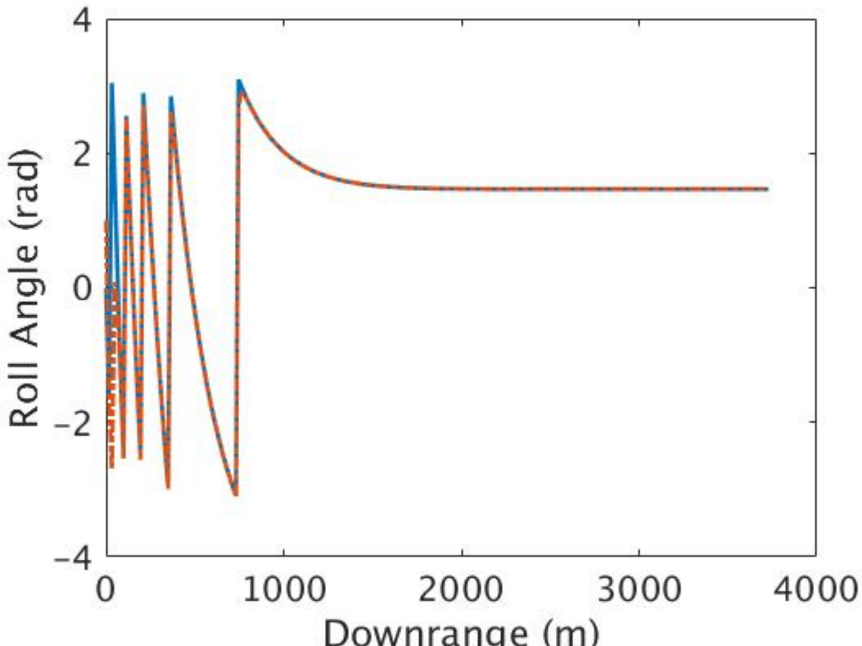
Altitude (m)
100
90
80
70
60
50
40
30
20
10
0
3500
3510
3520
3530
3540
3550
3560
3570
3580
3590
3600
Downrange (m)
-30
-20
-10
0
10
20
30
Crossrange (m)

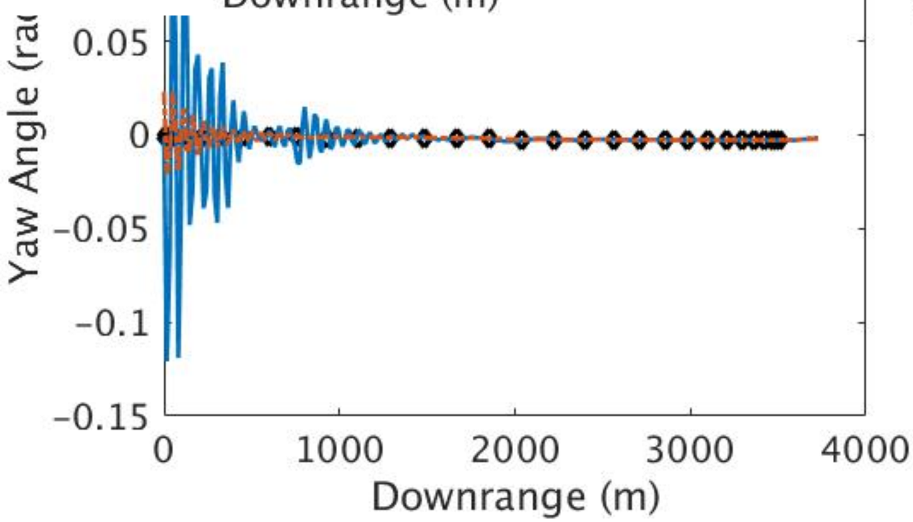
Position with EKF estimates

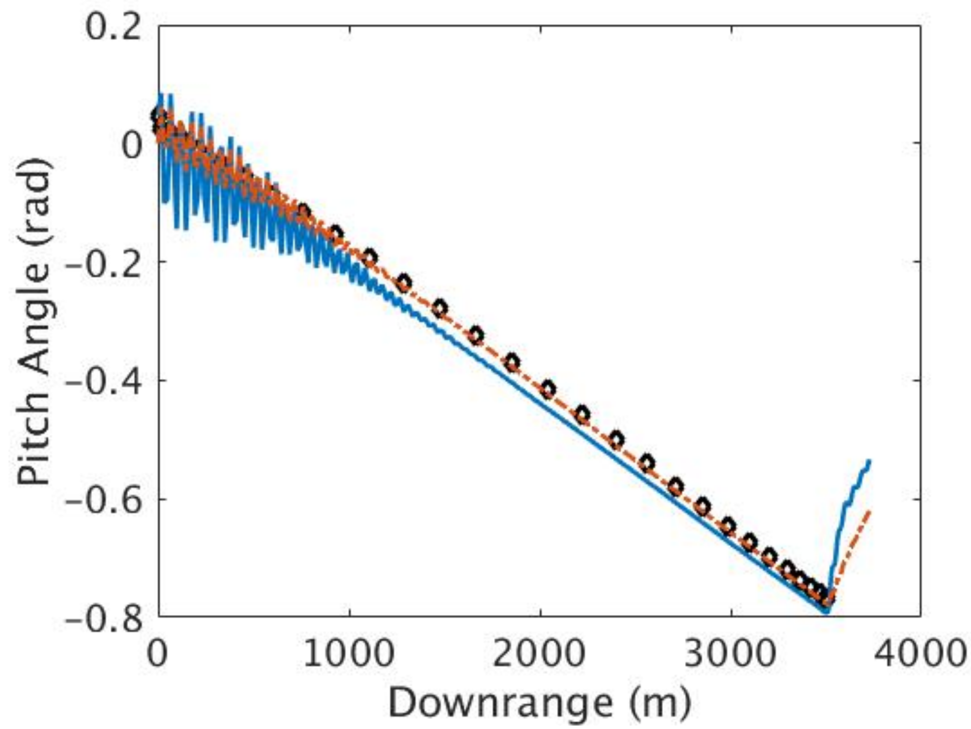




Euler angles with EKF estimates







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